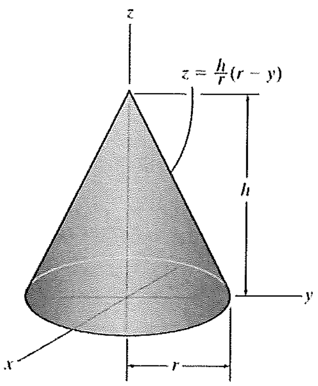
ME 230 Kinematics and Dynamics

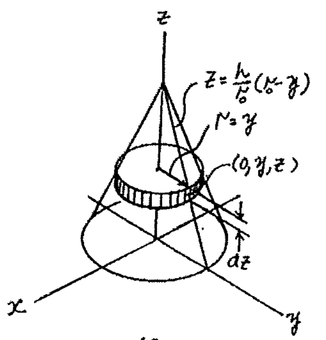
HW # 7 Solution

17-6 The mass of the disk element shown in Fig. is $dm = \rho dV = \rho \pi r^2 dz$. Here $r = y = r_0 - \frac{r_0}{h} z$. Thus, $dm = \rho \pi \left(r_0 - \frac{r_0}{h} z\right)^2 dz$.



The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2} dm\, r^2 = \frac{1}{2}(\rho \pi r^2 dz) r^2$$
$$= \frac{1}{2}\rho \pi r^4 dz = \frac{1}{2}\rho\pi\left(r_0 - \frac{r_0}{h} z\right)^4 dz$$



The mass of the cone:

$$m = \int dm = \int_0^h \rho\pi\left(r_0 - \frac{r_0}{h} z\right)^2 dz$$
$$= \rho\pi\left[\frac{1}{3}\left(r_0 - \frac{r_0}{h} z\right)^3\left(-\frac{h}{r_0}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_0^2 h$$

Integrating dI_z, $I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_0 - \frac{r_0}{h} z\right)^4 dz$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_0 - \frac{r_0}{h} z\right)^5\left(-\frac{h}{r_0}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_0^4 h$$

$$m = \frac{1}{3}\rho\pi r_0^2 h, \quad I_z = \frac{1}{10}(\rho\pi r_0^2 h) r_0^2 = \underline{\frac{3}{10} m r_0^2}$$

17-23

$$m_c = 7.85(10^3)(0.05\pi(0.01)^2) = 0.1233\ kg$$

$$m_p = 7.85(10^3)(0.03\times 0.180)\times 0.02 = 0.8478\ kg$$

$$I_x = \left[\frac{1}{2}(0.1233\times 0.01^2)\right] + \left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.12)^2\right] + \left[\frac{1}{12}(0.8478(0.03)^2 + (0.180)^2) + (0.8478)(0.06^2)\right]$$

$$= 0.00719\ kg\cdot m^2 = 7.19\ g\cdot m^2$$ #

17-27

$$\Sigma F_x = m(a_G)_x;\quad 20 = \left(\frac{600}{32.2}\right)a_G$$

$$a_G = 1.0733\ ft/s^2 = 1.07\ ft/s^2$$ #

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad 20(4) - 600(0.5) + 2N_B(1.5) = \frac{600}{32.2}(1.0733)\times 2$$

$$N_B = 86.7\ lb$$ #

$$+\uparrow \Sigma F_y = m(a_G)_y;\quad 2N_A + 2(86.7) - 600 = 0$$

$$N_A = 213\ lb$$ #

17-33

$+\Sigma M_A = (M_k)_A;$

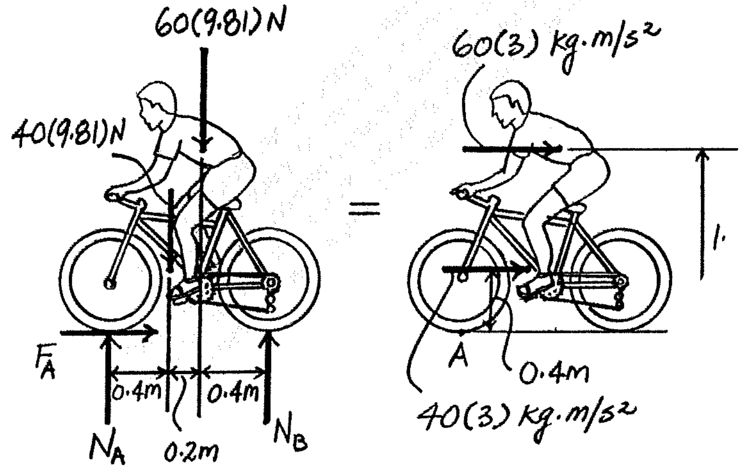
$N_B(1) - 40(9.81)(0.4) - 60(9.81)(0.6)$

$= -60(3)(1.25) - 40(3)(0.4)$

$\underline{N_B = 237.12\,N}$ #

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + 237.12 - 40(9.81)$

$- 60(9.81) = 0$



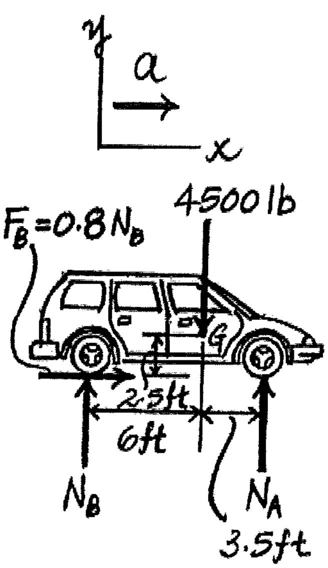
$\underline{N_A = 743.88\,N}$ #

17-38

① $\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 0.8 N_B = \left(\frac{4500}{32.2}\right) a_{max}$

② $+\uparrow \Sigma F_y = m(a_G)_y:\ N_A + N_B - 4500 = 0$

③ $+\Sigma M_G = 0;\ N_A(3.5) + 0.8 N_B(2.5)$
$- N_B(6) = 0$



(a)

Solving eq. ①, ②, ③,

Ans: $N_A = 2.4$ Kip, $N_B = 2.10$ Kip,

$a_{max} = 12.02\ \text{ft/s}^2 = 12.0\ \text{ft/s}^2$

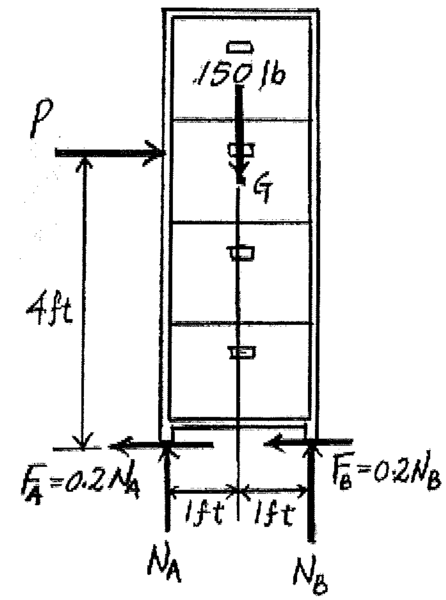
17-43

P: unknown minimum force needed.

Assume the cabinet slides before it tips.

$F_A = \mu_s N_A = 0.2 N_A$

$F_B = \mu_s N_B = 0.2 N_B$



(a)

$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.2N_A - 0.2N_B = 0 \quad (1)$

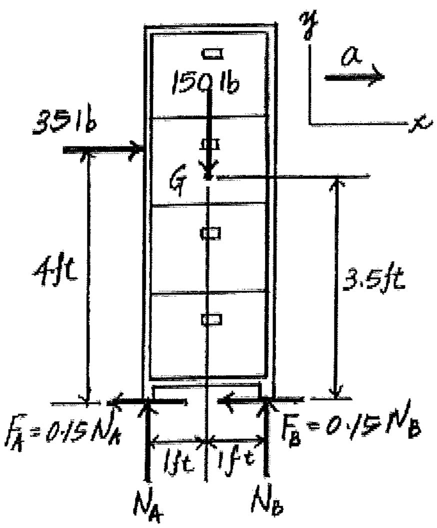
$+\uparrow \Sigma F_y = 0, \quad N_A + N_B - 150 = 0 \quad (2)$

$+\Sigma M_A = 0; \quad N_B(2) - 150(1) - P(4) = 0 \quad (3)$

Solving Eq. (1), (2), (3),

$P = 30 \text{ lb}, \quad N_A = 15 \text{ lb}, \quad N_B = 135 \text{ lb}$

Since $P < 35$ lb, N_A is positive $\Rightarrow$ the cabinet will slide.



$F_A = \mu_k N_A = 0.15 N_A, \quad F_B = \mu_k N_B = 0.15 N_B$

Referring to the FBD,

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 35 - 0.15N_A - 0.15N_B = \left(\frac{150}{32.2}\right)a$

$\xrightarrow{+} \Sigma F_y = m(a_G)_y; \quad N_A + N_B = 150$

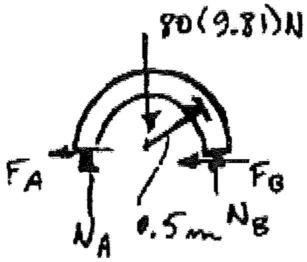
$+\Sigma M_G = 0, \quad N_B(1) - 0.15N_B(3.5) - 0.15N_A(3.5) - N_A \times 1$

$-35(0.5) = 0,$

$a = 2.68 \text{ ft/s}^2, \quad N_A = 26.9 \text{ lb}, \quad N_B = 123 \text{ lb}$ #

17-53

$+\uparrow \Sigma F_y = m(a_G)_y;$

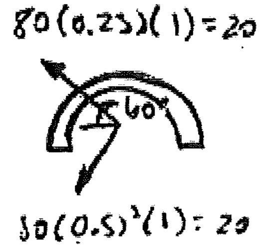


$N_A + N_B - 80(9.81) = 20\sin 60^\circ - 20\cos 60^\circ$

$N_A + N_B = 792.12$

$\circlearrowright + \Sigma M_A = \Sigma (M_k)_A;$

$N_B(1) - 80(9.81)(0.5) = 20\cos 60^\circ(0.2)$
$-20\cos 60^\circ(0.5) + 20\sin 60^\circ(0.2)$



$N_B = 402\,N, \quad N_A = 391\,N$ #

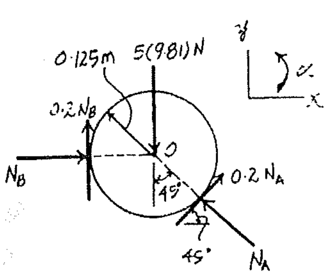
17-74 (see 17-59 next page)

mass moment of inertia $I_o = \frac{1}{2}mr^2 = \frac{1}{2} \times 5 \times 0.125^2$
$= 0.0390625\ kg \cdot m^2$

(1) $\xrightarrow{+} \Sigma F_x = m(a_G)_x;\ N_B + 0.2N_A\cos 45^\circ - N_A\sin 45^\circ = 0$

(2) $+\uparrow \Sigma F_y = m(a_G)_y;\ 0.2N_B + 0.2N_A\sin 45^\circ + N_A\cos 45^\circ - 5(9.81) = 0$

(3) $\circlearrowleft + \Sigma M_o = I_o\alpha;\ 0.2N_A \times 0.125 - 0.2N_B(0.125) = 0.0390625\alpha$



Solving Eq (1), (2), (3),

$N_A = 51.01\,N, \quad N_B = 28.85\,N,$

$\alpha = 14.2\ rad/s^2$ #

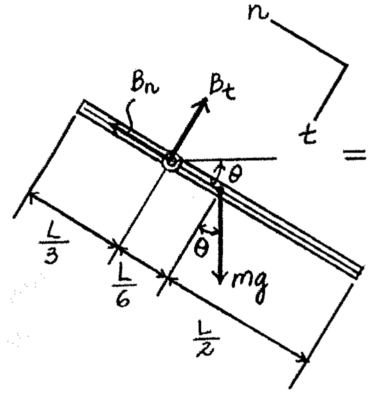
17-59 The rod rotate about a fixed axis

$(a_G)_t = \alpha r_G = \alpha\left(\frac{L}{6}\right),\ (a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{6}\right)$

The mass moment of inertia of the rod about G

$$I_G = \frac{1}{12} mL^2$$

$$+\Sigma M_B = \Sigma (M_k)_B;\ -mg\cos\theta\left(\frac{L}{6}\right) = -m\left[\alpha\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right)\alpha,\quad \alpha = \frac{3g}{2L}\cos\theta$$



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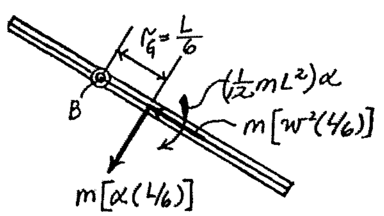


(or $\Sigma M_B = I_B\alpha$, where $I_B = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$

$$+\Sigma M_B = I_B\alpha;\ -mg\cos\theta\left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta\ )$$

The force equation of motion along n and t axes:

$$\Sigma F_t = m(a_G)_t;\ mg\cos\theta - B_t = m\left[\left(\frac{3g}{2L}\cos\theta\right)\left(\frac{L}{6}\right)\right]$$

$$B_t = \frac{3}{4}mg\cos\theta$$

$$\Sigma F_n = m(a_G)_n;\ B_n - mg\sin\theta = m\left[\omega^2\left(\frac{L}{6}\right)\right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg\sin\theta$$

Kinematics: $\int \omega\, d\omega = \int \alpha\, d\theta,\ \int_0^\omega \omega\, d\omega = \int_0^\theta \frac{3g}{2L}\cos\theta\, d\theta,\ \omega = \sqrt{\frac{3g}{L}\sin\theta}$

when $\theta = 90°$; $\omega = \sqrt{\frac{3g}{L}}$; $B_t = \frac{3}{4}mg\cos 90° = 0$

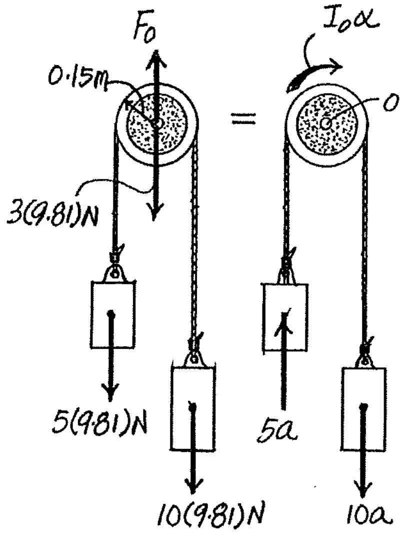
$$B_n = \frac{1}{6}m\left(\frac{3g}{L}\right)L + mg\sin 90° = \frac{3}{2}mg$$

$$F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg$$ #

17-79

Angular acceleration of the pully:

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.667a$$



Mass moment of inertia of the pully:

$$I_O = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15)^2 = 0.03375\ \text{kg}\cdot\text{m}^2$$

$$\circlearrowleft + \Sigma M_O = \Sigma (M_k)_O;\quad 5(9.81)(0.15) - 10(9.81)(0.15)$$
$$= -0.03375(6.667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973\ \text{m/s}^2 = 2.97\ \text{m/s}^2$$

\#

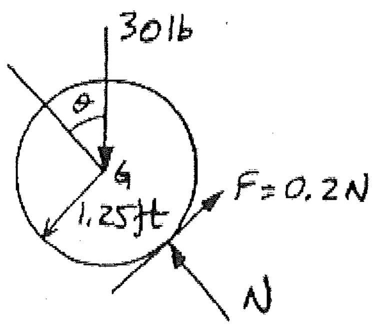
17.95

① $+\swarrow \Sigma F_x = m(a_G)_x;\ 30\sin\theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25\alpha)$

② $+\nwarrow \Sigma F_y = m(a_G)_y;\ N - 30\cos\theta = 0$

③ $\circlearrowleft + \Sigma M_G = I_G\alpha;\ 0.2N(1.25) = \left[\frac{30}{32.2}(0.6)^2\right]\alpha$

solving ①, ②, ③.



$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

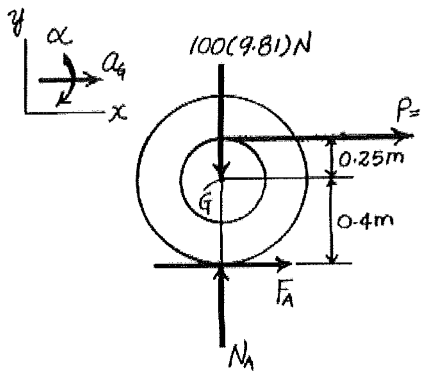
$$30\sin\theta = 32.042\cos\theta$$

$$\tan\theta = 1.068$$

$$\theta = 46.9^\circ$$

\#

17-98



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 600 + F_A = 100 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G \alpha; \quad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assuming no slipping: $a_G = 0.4\alpha$

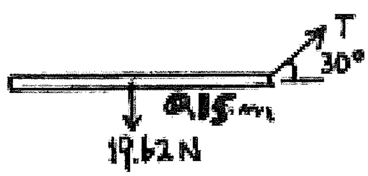
$\underline{\alpha = 15.6 \text{ rad/s}^2} \#$

$a_G = 6.24 \text{ m/s}^2, \; N_A = 981\text{N}, \; F_A = 24\text{N}$

Since $(F_A)_{max} = 0.2(981) = 196.2\text{N} > 24\text{N}$

OK

17-102



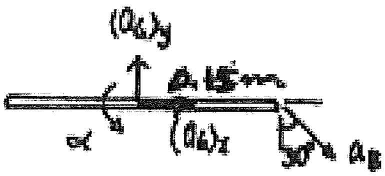
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad T\cos 30° = 2(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad T\sin 30° - 19.62 = 2(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \quad T\sin 30°(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$

$\bar{a}_B = \bar{a}_G + \bar{a}_{B/G}$

$a_B \sin 30° \bar{i} - a_B \cos 30° \bar{j} = (a_G)_x \bar{i} + (a_G)_y \bar{j} + \alpha(0.15)\bar{j}$



$(\xrightarrow{+}) \; (a_B)\sin 30° = (a_G)_x$

$(+\uparrow) \; (a_B)\cos 30° = -(a_G)_y - \alpha(0.15)$

$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$

$\underline{T = 5.61\text{N}} \#$

$(a_G)_x = 2.43 \text{ m/s}^2, \; (a_G)_y = -8.41 \text{ m/s}^2$

$\underline{\alpha = 28.0 \text{ rad/s}^2} \#$

17-109 mass moment of inertia:

$$I_G = mr^2 = 500(0.5^2) = 125\ kg\cdot m^2$$

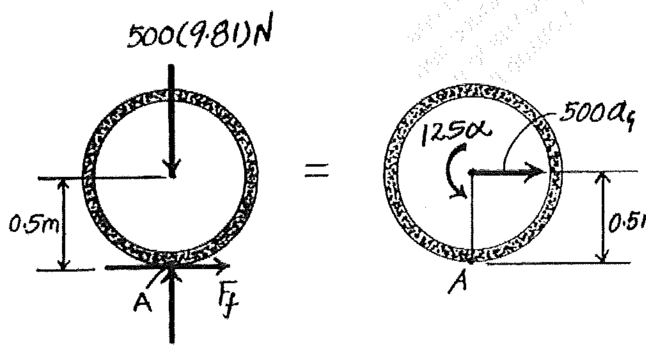
$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\quad 0 = 125\alpha - 500\, a_G(0.5)$$

$$\bar a_G = \bar a_A + \alpha \times \bar r_{G/A} - \omega^2 r_{G/A}$$

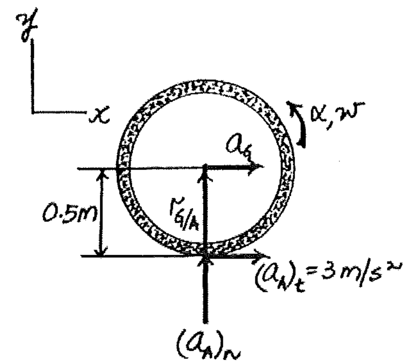
$$\overline{a_G}\,\bar i - 3\bar i + (a_A)_n \bar j + (\alpha \bar k \times 0.5\bar j) - \omega^2(0.5\bar j)$$

$$a_G\,\bar i = (3-0.5\alpha)\bar i + [(a_A)_n - 0.5\omega^2]\bar j$$

$$\begin{cases} a_G = 3 - 0.5\alpha \\ 0 = 125\alpha - 250\, a_G \end{cases} \Rightarrow \begin{cases} a_G = 1.5\ m/s^2 \rightarrow \\ \alpha = 3\ rad/s^2 \end{cases}$$



(a)



(b)