ME 230 Kinematics and Dynamics

Wei-Chih Wang
Department of Mechanical Engineering
University of Washington

- **Instructor:**

  Wei-Chih Wang

  E-mail: abong@u.washington.edu

  Office: MEB 113 or 260

  Phone: 206-543-2479

  Office hours: M,W, F 1:30-2:30PM

- **Teaching Assistant:**

  David Schipf schipf@uw.edu

  Jinyuan Zhang jinyuan@uw.edu

  Paul Murphy pgmurphy@uw.edu

  Kebin Gu kebin@u.washington.edu (grader)

- **Textbook:** R. C. Hibbeler, *Engineering Mechanics: Dynamics*, 13th Ed.
- **Course Website:** **http://courses.washington.edu/engr100/me230**

General Policy

- Homework: Homework will be assigned in class on Wed. Homework for each week is due the following Wednesday (During Class). The homework has usually 10-12 problems per week. Late homework will not be accepted (partial credit will not be given). Homework solution will be available every Wednesday on the web. Please write down your section number on your homework.
- Grading of Homework: Only one or two questions (chosen by the instructor) from the homework (assigned for each week) will be graded – the resulting grade will constitute the grade for that week's homework. Therefore, answer all the questions correctly to get full credit for the homework.
- **Exams:** Exams will be open book and open notes. There will be no alternate exams if you miss any. Exams will include materials covered in the text, class, and homework.

Notes:

- *Homework* be assigned on a *weekly* basis
- Homework should be *hand-written*
- TA will go over the problems with you during Lab Section and answer any questions you have on your homework.
- Solutions to all problems solved in class will be posted on Thursday each week: http://courses.washington.edu/engr100/me230

Grading

Homework	20%
1st Midterm	25%
2nd Midterm	25%
Final Project	30%

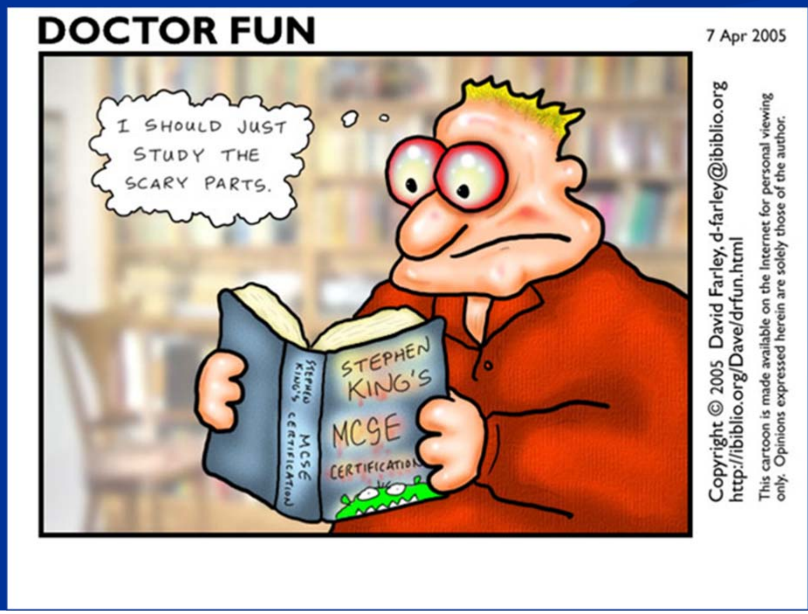
GPA Formula: GPA = (Score-50)/40*(4.0-2.0)+2.0 (94=4.0 and 50=2.0.)

Please make sure…

- You review some maths (i.e. trigonometric identities, derivatives and integrals, vector algebra,)
- … and some STATICS…

UNITS, Vector addition, free body diagram (FDB)

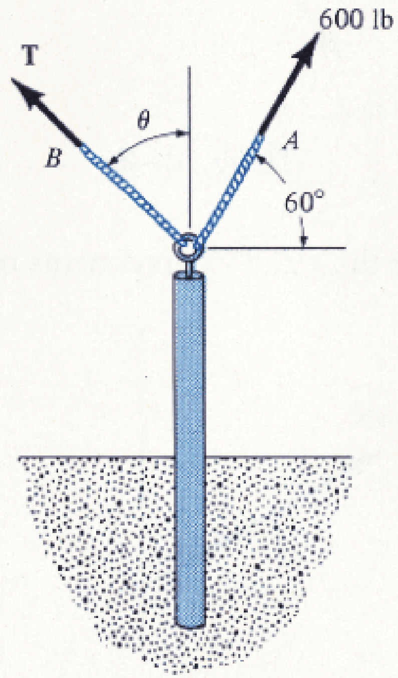
(Hibbeler Statics: Ch. 1,2 and 5)



Examples (1)

1–1. What is the weight in newtons of an object that has a mass of (a) 8 kg, (b) 0.04 g, and (c) 760 Mg?

Examples (2)

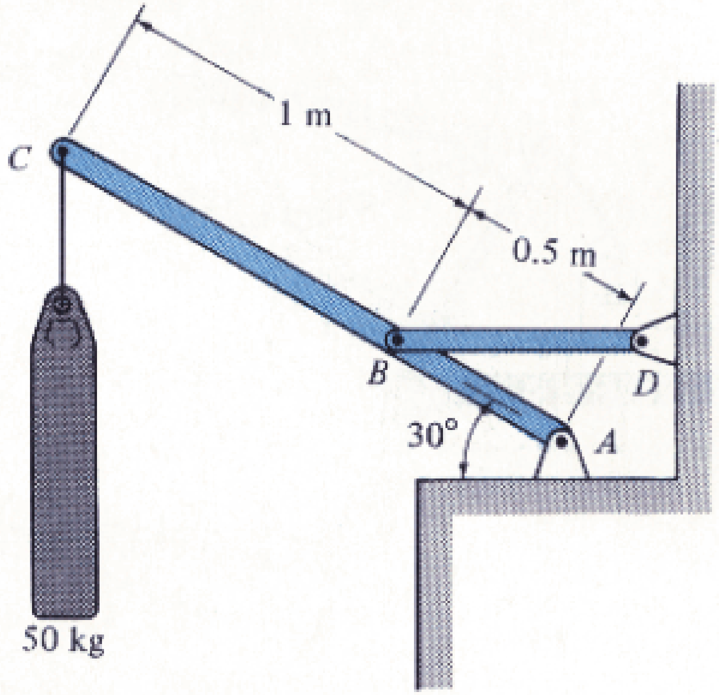


Prob 2–13

2–13. The post is to be pulled out of the ground using two ropes A and B. Rope A is subjected to a force of 600 lb and is directed at 60° from the horizontal. If the resultant force acting on the post is to be 1200 lb, vertically upward, determine the force T in rope B and the corresponding angle θ.

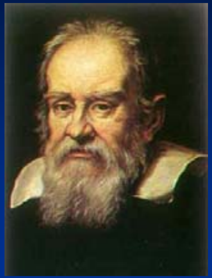
Examples (3)

*5–8. The uniform rod *ABC* supported by a pin at *A* and a short link *BD*.



Prob. 5–8

What is Dynamics?



Important contributors:

Galileo Galilei, Newton, Euler, Lagrange

Mechanics

(The study of how bodies react to forces acting on them)

Study of equilibrium of a body that is at rest/moves with constant velocity ← Statics

Dynamics → Study of accelerated motion of a body

- **Kinematics**: geometric aspects of the motion
- **Kinetics**: Analysis of forces which cause the motion

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An Overview of Mechanics

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium or in constant speed.

Dynamics: The study of force and torque and their effect on a accelerated moving body

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

Mechanics

- Statics – effects of forces on bodies at rest
- Dynamics
  - Theoretically, kinematics and kinetics constitute dynamics.
  - Kinematics – study of motion of bodies without reference to forces which cause the motion
  - Kinetics – relates action of forces on bodies to their resulting motion
  - Kinematics and kinetics almost occur together all the time in practice.

However…

- **From Wikipedia, the free encyclopedia:**

Dynamics is a branch of physics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to *kinematics*, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion. Also why this class is called kinematics and dynamics.

Why is dynamics important?

- Understanding dynamics is key to predicting performance, designing systems, etc.
- The ability to control a system (say, a car) depends upon understanding the dynamics
- It is fundamental to advanced topics, such as fluid mechanics, structural dynamics, or vibration.

Applications of Dynamics

- Modern machines and structures operated with high speed (*acceleration*)
- *Analysis* & *design* of
  - Moving structure
  - Fixed structure subject to shock load
  - Robotic devices
  - Automatic control system
  - Rocket, missiles, spacecraft
  - Ground & air transportation vehicles
  - Machinery
  - Human movement (Biomechanics)

Example: The Coriolis Force

Kinematics: coordinate reference frames matter, as in this merry-go-round

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Example: Car Crush Test

Kinetics: Impact , impulse and moment. Crash Test of a New Mercedes SLS AMG 2010

Example: Three Phase Diamagnetic Levitation Motor

Studying of rotational motion of a motor, kinetics: magnetic forces, Kinematics: rotation speed and angles

Example: Self-Assemble Robots

Block communicate through Wireless Communication

Studying of kinematics and kinetics of a moving robot

Kinetics: forces on latches, kinematics: position tracking

Topics to be covered

Chapter 12:Introduction & Kinematics of a particle

Chapter 13: Kinetics of a particle: Force and Acceleration

Chapter 14: Kinetics of a particle: Work and Energy

Chapter 15: Kinetics of a particle: Impulse and Momentum

Chapter 16: Planar kinematics of a Rigid Body

Chapter 17: Planar kinetics of a Rigid Body: Force and Acceleration

cont'd

Chapter 18: Planar kinetics of a Rigid Body: Work and Energy

Chapter 19: Planar kinetics of a Rigid Body: Impulse and Momentum

Chapter 20 and 21: Three-Dimensional Kinematics of a Rigid Body & Overview of 3D Kinetics of a Rigid Body

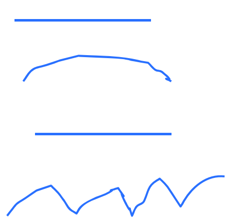
Chapter 22: Vibrations: under-damped free vibration, energy method, undamped forced vibration, viscous damped vibrations

Chapter 12: Kinematics of a Particle

- Chapter 12 introduces the kinematics of a particle
    - kinematics: the study of the geometry of motion (regardless of the forces which cause that motion)
    - particle: a body which can be modeled as having no physical dimensions
- Chapter 12 unfolds by gradually increasing the complexity of our view of this topic, considering different kinds of motion in different coordinate systems

translational motion

- ☑ rectilinear motion: motion in a straight line (12.2, 12.3)
- ☑ curvilinear motion: motion along a curve (12.4-12.8)
- ☑ continuous motion: motion in one direction (12.2, 12.3)
- ☑ erratic motion: motion in different directions (12.3)
- ☑ rectangular components: using Cartesian coordinates (12.2, 12.3, 12.5, 12.6)
- ☑ normal and tangential components: using a "path" coordinate systems (12.7)
- ☑ cylindrical components: using cylindrical/polar coordinates (12.8)
- ☑ absolute motion: using a global/inertial coordinate system (12.9)
- ☑ relative motion: using a local/relative coordinate system (12.10)



Two classes of motion

Coordinates

Particle Kinematic

- **Kinematics of a particle** (Chapter 12)
  - 12.1-12.2

Objectives

Students should be able to:

1. Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path (Continuous motion) (12.2)

Next lecture; Determine position, velocity, and acceleration of a particle using graphs (Erratic motion) (12.3)

Question

1. In dynamics, a particle is assumed to have __________.

   A) both translation and rotational motions

   B) only a mass

   C) a mass but the size and shape cannot be neglected

   D) no mass or size or shape, it is just a point

1. Particles:

Definition: A particle is a body of negligible dimensions.

When the dimensions of a body are irrelevant to the description of its motion, the body can be treated as a particle.

Examples:

(a) An airplane: Yes when analyzing the fleight path from LA to NYC. No when the plane rotates.

(b) A space shuttle: Yes when analyzing the orbit of the shuttle. No when the shuttle turns.

(c) Scott Hamilton: Yes when he skates along the rink. No when he does a double toe-loop.

2. Rigid Bodies:

Definition: A rigid body is a body that does not deform and dimensions of the body are not negligible.

When the deformation is much less than the dimensions of the body to be analyzed and the dimensions of a body are relevant to the description of its motion, the body can be treated as a rigid body.

Examples:

(a) An airplane: Yes when analyzing the rotational motion of the airplane. No when analyzing the vibration of the airplane wings.

(b) The Hubble Telescope: Yes when analyzing the unfolding motion of its solar panels. No when analyzing the vibration of the thermal gitters.

(c) Scott Hamilton: Yes when he does a double toe-loop. No when analyzing the contraction of his muscle.

3. Differences Between Particles and Rigid Bodies:

Particles $\mapsto$ No Rotation $\mapsto$ No Moment Equations

Rigid Bodies $\mapsto$ Rotation Exists $\mapsto$ Moment Equations Are Important.

Therefore, we study the motion of particles first and then rigid bodies.

4. Kinematics:

Definition: The branch of dynamics which describes the motion of bodies without reference to the forces that either cause the motion or are generated as a result of the motion.

Shape of the motion

Content: Acceleration ⇆ Velocity ⇆ Displacement

← Easy (Differentiation), rarely occurs.

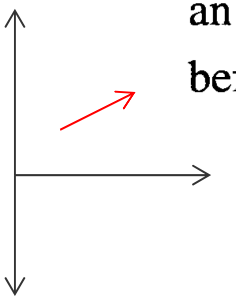
→ Difficult (Integration), Most often.

~~F = ma~~

Examples:

(a) A space shuttle takes off with a constant acceleration *a*. What is the velocity after 10 seconds? What is the height after 20 seconds? What is the velocity when the space shuttle is 1 km above the ground?

(b) A quarter back passes a football with an initial velocity of 3 m/s and at an angle of 40 degrees. How far and how long will the football travel before it lands? How high will the football reach?



<u>We often study kinematics first, because it is easier.</u>

5. Kinetics:

Definition: The study of the relations between unbalanced forces and the change in motion that they produce.

$F = ma$ ✓

Content: Forces ⇦⇨ Newton's Second Law

Work & Energy ➡ Integration of Newton's Law wrt Displ.

Impulse & Momentum ➡ Integration of Newton's Law wrt Time.

Examples:

(a) A space shuttle takes off with a thrust of T (in MN). What is the acceleration of the space shuttle?

(b) A quarter back applies a 10-lb force to a football at an angle of 40 degrees in one second. What is the final velocity when the football is released?

Kinematics of a Particle

Type:

- Constrained Motion: Pendulum, roller coaster, swing.
- Unconstraint motion: Football trajectory, balloon in air

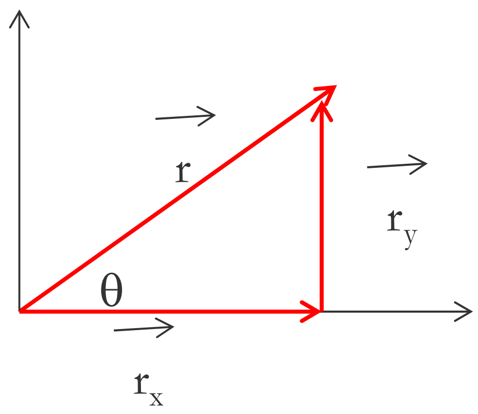
Contents:

- Rectilinear Motion: Moving along a straight line

- Curvilinear Motion: 2-D or 3-D motion
  - (a) rectangular coordinates
  - (b) Normal and tangential coordinates
  - (c) cylindrical (or Polar) coordinates

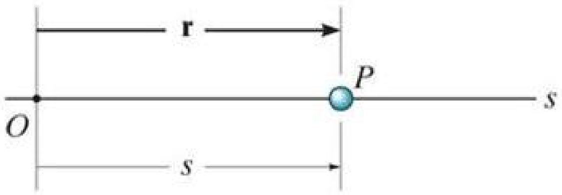
- Relative motion: For complicated motion
  - (a) Translating axes
  - (b) rotating axes

Vector

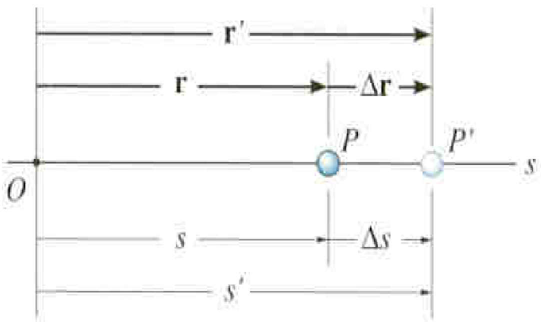


Recall in your high school math, a vector $\vec{r}$ quantity is a quantity that is described by both magnitude $|r| = \sqrt{|r_x|^2 + |r_y|^2}$ and direction θ, where $|r_x| = |r|cos\theta\ and |r_y| = |r|sin\theta$

Rectilinear kinematics: Continuous motion



Position



Displacement

A particle travels along a straight-line path defined by the coordinate axis s

The **POSITION** of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar (magnitude) s. Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m or cm) or feet (ft or inches).

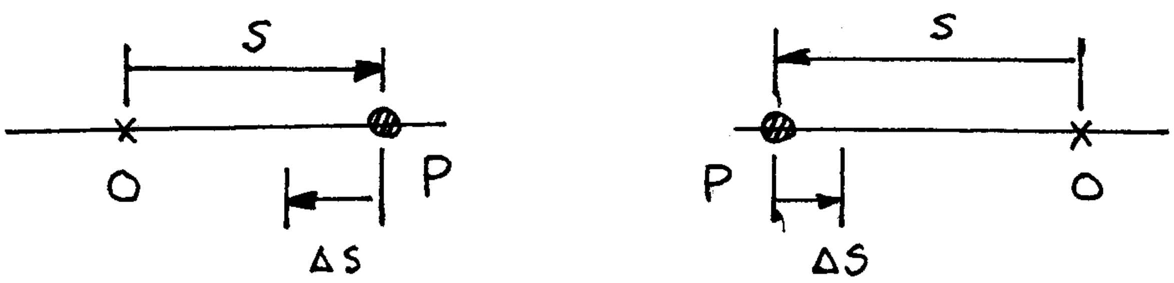
The **displacement** of the particle is defined as its change in position.

Vector form: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

Scalar form: $\Delta s = s' - s$

The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.

Some Remarks on Displacements



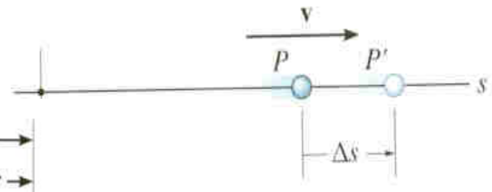
$S > 0$, $\Delta S < 0$ $\qquad$ $S < 0$, $\Delta S > 0$

- Displacement ΔS can be positive or negative
- The signs of S and ΔS are irrelevant.

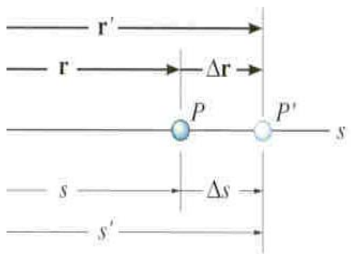
Depends on how you define your origin and positive and negative direction

Velocity

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.



Velocity



Displacement

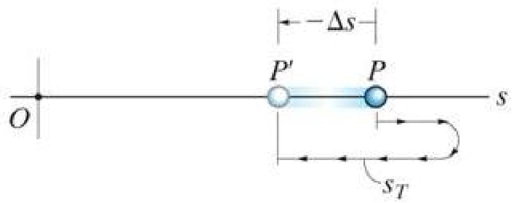
The average velocity of a particle during a time interval Δt is

$$\boldsymbol{v}_{avg} = \Delta\boldsymbol{r}/\Delta t$$

The instantaneous velocity is the time-derivative of position.

$$\boldsymbol{v} = d\boldsymbol{r}/dt \quad \text{at P}$$

Speed is the magnitude of velocity: $v = ds/dt$



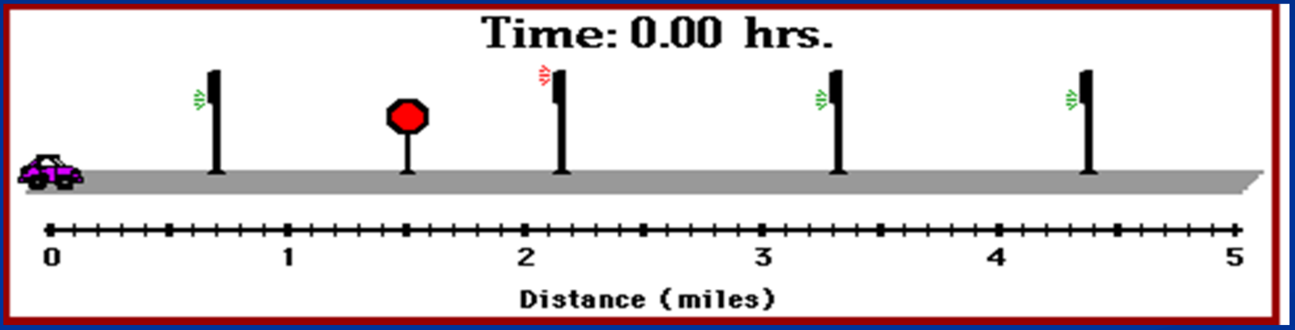
Average velocity and
Average speed

Average speed is the **total distance** traveled divided by elapsed time:

$$(v_{sp})_{avg} = s_T / \Delta t$$

Average vs. Instantaneous Speed

During a typical trip to school, your car will undergo a series of changes in its speed. If you were to inspect the speedometer readings at regular intervals, you would notice that it changes often. The speedometer of a car reveals information about the instantaneous speed of your car. It shows your speed at a particular instant in time.



The instantaneous speed of an object is not to be confused with the average speed. Average speed is a measure of the distance traveled in a given period of time; it is sometimes referred to as the distance *per* time ratio. Suppose that during your trip to school, you traveled a distance of 5 miles and the trip lasted 0.2 hours (12 minutes). The average speed of your car could be determined as

$$\text{Ave. Speed} = \frac{5 \text{ miles}}{0.2 \text{ hours}} = 25 \text{ miles/hour}$$

On the average, your car was moving with a speed of 25 miles per hour. During your trip, there may have been times that you were stopped and other times that your speedometer was reading 50 miles per hour. Yet, on average, you were moving with a speed of 25 miles per hour.

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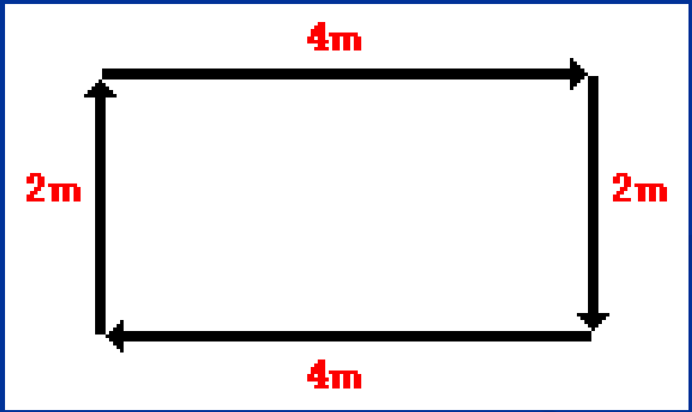
Average Speed and Average Velocity

Now let's consider the motion of that physics teacher again. The physics teacher walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North. The entire motion lasted for 24 seconds. Determine the average speed and the average velocity.

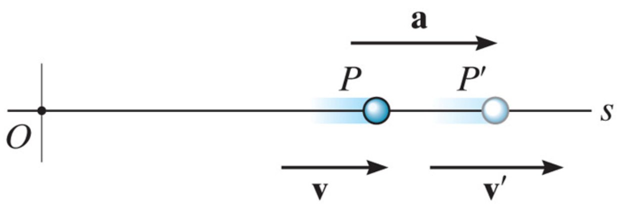


The physics teacher walked a distance of 12 meters in 24 seconds; thus, her average speed was 0.50 m/s. However, since her displacement is 0 meters, her average velocity is 0 m/s. Remember that the displacement refers to the change in position and the velocity is based upon this position change. In this case of the teacher's motion, there is a position change of 0 meters and thus an average velocity of 0 m/s.

Acceleration

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s^2 or ft/s^2.

The instantaneous acceleration is the time derivative of velocity.

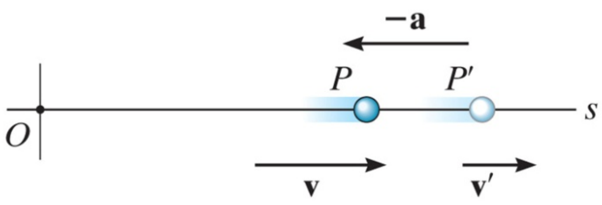


Acceleration

(e)

Vector form: $\boldsymbol{a} = d\boldsymbol{v}/dt$

Scalar form: $a = dv/dt = d^2s/dt^2$



Deceleration

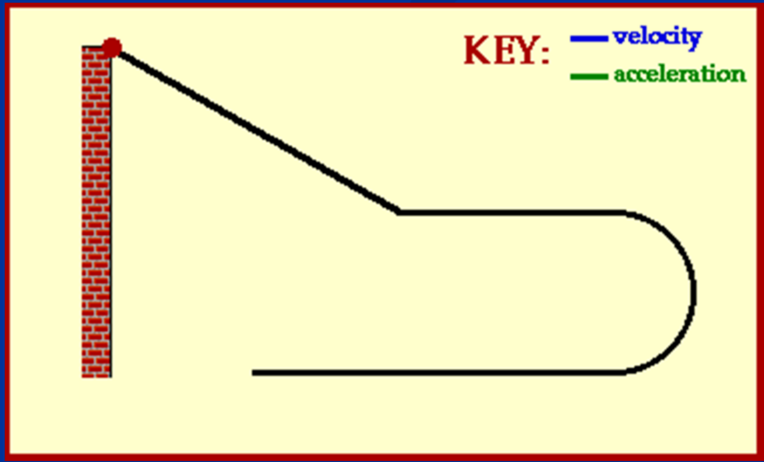
(f)

Acceleration can be positive (speed increasing) or negative (speed decreasing).

As the book indicates, the derivative equations for velocity and acceleration can be manipulated to get $a\,ds = v\,dv$

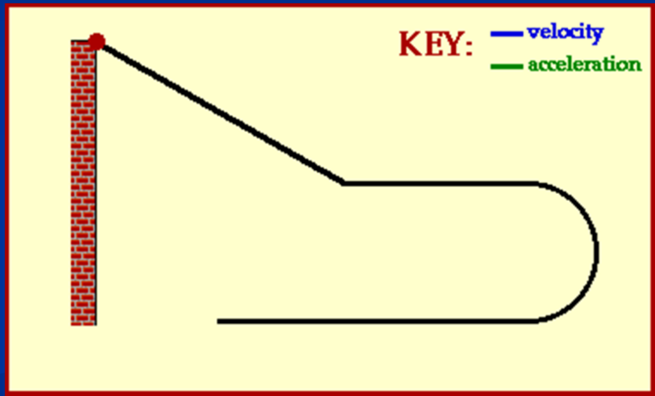
Direction of Acceleration and Velocity

Consider the motion of a Hot Wheels car down an incline, across a level and straight section of track, around a 180-degree curve, and finally along a final straight section of track. Such a motion is depicted in the animation below. The car gains speed while moving down the incline - that is, it accelerates. Along the straight sections of track, the car slows down slightly (due to air resistance forces). Again the car could be described as having an acceleration. Finally, along the 180-degree curve, the car is changing its direction; once more the car is said to have an acceleration due to the change in the direction. Accelerating objects have a changing velocity - either due to a speed change (speeding up or slowing down) or a direction change.



This simple animation above depicts some additional information about the car's motion. The velocity and acceleration of the car are depicted by vector arrows. The direction of these arrows are representative of the direction of the velocity and acceleration vectors. Note that the velocity vector is always directed in the same direction which the car is moving. A car moving eastward would be described as having an eastward velocity. And a car moving westward would be described as having a westward velocity.

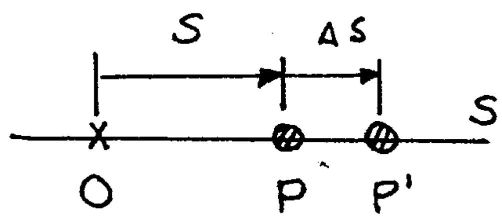
The direction of the acceleration vector is not so easily determined. As shown in the animation, an eastward heading car can have a westward directed acceleration vector. And a westward heading car can have an eastward directed acceleration vector. So how can the direction of the acceleration vector be determined? A simple *rule of thumb* for determining the direction of the acceleration is that an object which is slowing down will have an acceleration directed in the direction opposite of its motion. Applying this *rule of thumb* would lead us to conclude that an eastward heading car can have a westward directed acceleration vector if the car is slowing down. Be careful when discussing the direction of the acceleration of an object; slow down, apply some thought and use the *rule of thumb*.



Definitions

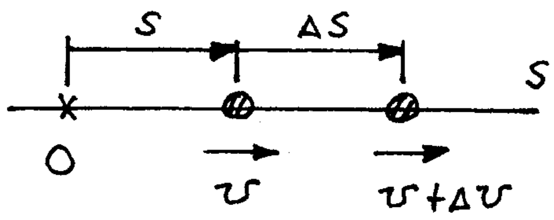
(1) Average Velocity

$$v_{AV} = \frac{\Delta s}{\Delta t}$$



(2) Instantaneous Velocity:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



(3) Average Acceleration:

$$a_{AV} = \frac{\Delta v}{\Delta t}$$

(4) Instantaneous Acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\, dt \quad \text{or} \quad \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at t = 0.

Four types of Acceleration

(I) $a=$ constant (constant acceleration)

e.g. gravitational acceleration

(II) $a= a(t)$

e.g. acceleration of a rocket with a constant thrust

(III) $a=a(v)$

e.g. deceleration from air drag

(IV) $a= a(s)$

e.g. acceleration from a spring load

(I) a= constant (Constant acceleration)

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

Type II: $a = f(t)$

e.g., Acceleration of a rocket.

(1) $$a(t) = \frac{dv}{dt} \Rightarrow dv = a(t)\,dt$$

$$\Rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)\,dt$$

$$\Rightarrow v(t) = v_0 + \underbrace{\int_{t_0}^{t} a(t)\,dt}_{\text{Can be integrated explicitly}}$$

(2)

$$v(t) = \frac{ds}{dt} \Rightarrow ds = v(t)\,dt$$

$$\Rightarrow \int_{s_0}^{s} ds = \int_{t_0}^{t} v(t)\,dt$$

$$\Rightarrow s = s_0 + \underbrace{\int_{t_0}^{t} v(t)\,dt}_{\text{can be integrated explicitly}}$$

(3)

$$a\,ds = v\,dv \Rightarrow a(t)\,ds = v\,dv$$

$$\Rightarrow \underbrace{\int_{s_0}^{s} a(t)\,ds}_{} = \int_{v_0}^{v} v\,dv$$

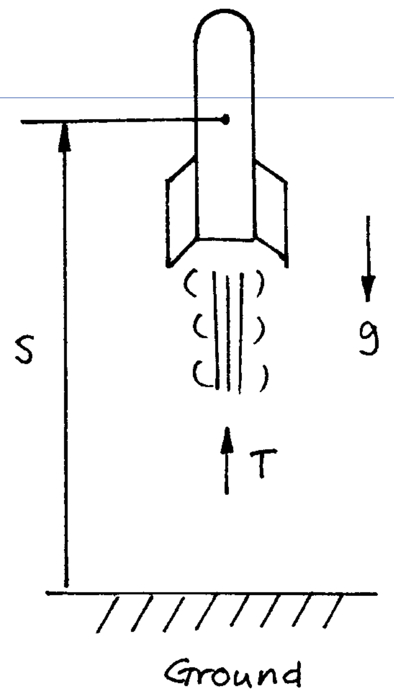
Cannot be integrated. Why?

Independent variable: s

Integrand: function of t

Useless When $a = f(t)$

Example



Mass of Rocket = M (kg)

Total Mass of Fuel = m (kg)

Fuel Consumption Rate = μ (kg/s)

Thrust of the Rocket = T (N)

Initially at Rest on Ground

$$s(0) = v(0) = 0$$

Mass of Rocket & Fuel = $M + m - \mu t$

$$a(t) = \frac{T}{M+m-\mu t} - g$$

Find: $v(t)$, $s(t)$, $0 < t < \frac{m}{\mu}$

Type III: $a = a(v)$

e.g., deceleration by air drag.

(1) $$a(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a(v)}$$

$$\Rightarrow \int_{t_0}^{t} dt = \int_{v_0}^{v} \frac{dv}{a(v)}$$

$$t = t_0 + \int_{v_0}^{v} \frac{dv}{a(v)} = \text{function of } v$$

Solve v in terms of t so that $v = v(t)$

Function of v not t

Don't do this:

$$a(v)\,dt = dv \Rightarrow \underbrace{\int_{t_0}^{t} a(v)\,dt = \int_{v_0}^{v} dv}$$

Cannot Integrate. Why?

(2) $v(t) = \frac{ds}{dt} \Rightarrow ds = v(t)\,dt$

$$\int_{s_0}^{s} ds = \int_{t_0}^{t} v(t)\,dt \Rightarrow s = s_0 + \int_{t_0}^{t} v(t)\,dt$$

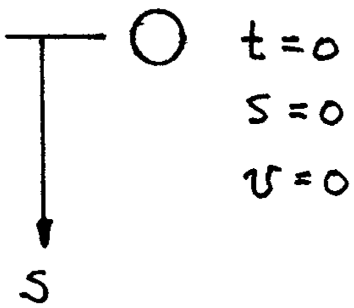
(3) $\underline{a\,ds = v\,dv} \Rightarrow a(v)\,ds = v\,dv \Rightarrow ds = \frac{v}{a(v)}\,dv$

$$\Rightarrow \int_{s_0}^{s} ds = \int_{v_0}^{v} \frac{v}{a(v)}\,dv$$

$$\Rightarrow s = s_0 + \int_{v_0}^{v} \frac{v}{a(v)}\,dv = \text{function of } v$$

Solve v in terms of s, so that $v = v(s)$

Example



A small ball is released from rest in air. The acceleration, taking into acount the air drag, is

$$a(v) = g - kv^2$$

Find: $v(t)$, $v(s)$, $s(t)$

Type IV: $a = a(s)$

e.g., acceleration under spring load $a = -ks$

(1) $a(s) = \frac{dv}{dt} \Rightarrow \underbrace{\int_{t_0}^{t} a(s)\,dt = \int_{v_0}^{v} dv}_{\text{Cannot Integrate}}$ } Useless when $a = a(s)$

(2) $\underline{a\,ds = v\,dv} \Rightarrow a(s)\,ds = v\,dv$

$$\Rightarrow \int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow \int_{s_0}^{s} a(s)\,ds = \frac{1}{2}(v^2 - v_0^2)$$

$$\Rightarrow v = v(s) = \pm\sqrt{v_0^2 + 2\int_{s_0}^{s} a(s)\,ds} = \text{function of } s$$

(3) $v(s) = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{v(s)}$

$$\Rightarrow \int_{t_0}^{t} dt = \int_{s_0}^{s} \frac{ds}{v(s)}$$

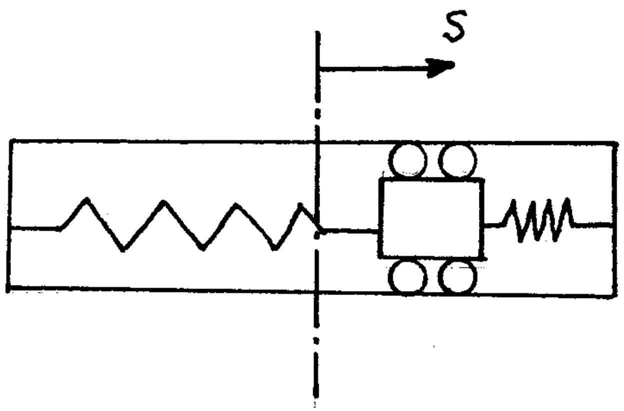
$$\Rightarrow t = t_0 + \int_{s_0}^{s} \frac{ds}{v(s)} = \text{function of } s$$

Solve for s in terms of t to obtain $s = s(t)$.

(4) Velocity is obtained

$$v(t) = \frac{ds(t)}{dt} = \text{function of time } t$$

Example



Unstretched position

$t=0, \; s=0, \; v=v_0$

Initially

- Springs are unstretched
- Cart has velocity v_0

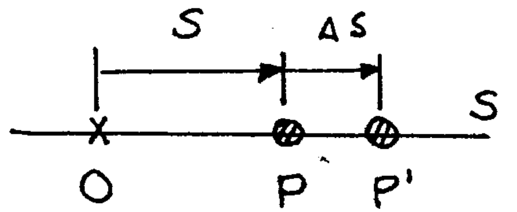
Acceleration

$$a(s) = -k^2 s$$

Find: $v(s)$, $s(t)$, $v(t)$

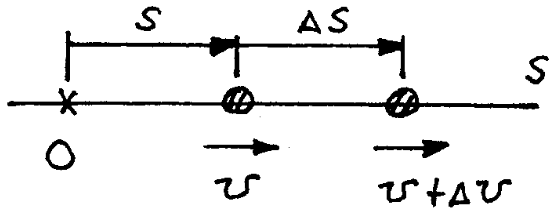
Definitions

(1) Average Velocity $v_{AV} = \frac{\Delta s}{\Delta t}$



(2) Instantaneous Velocity:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



(3) Average Acceleration:

$$a_{AV} = \frac{\Delta v}{\Delta t}$$

(4) Instantaneous Acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\, dt \quad \text{or} \quad \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at t = 0.

EXAMPLE

Given: A particle travels along a straight line to the right with a velocity of $v = (4t - 3t^2)$ m/s where t is in seconds. Also, s = 0 when t = 0.

Find: The position and acceleration of the particle when t = 4 s.

Plan: Establish the positive coordinate, s, in the direction the particle is traveling. Since the velocity is given as a function of time, take a derivative of it to calculate the acceleration. Conversely, integrate the velocity function to calculate the position.

EXAMPLE (continued)

Solution:

1) Take a derivative of the velocity to determine the acceleration.

$$a = dv / dt = d(4t - 3t^2) / dt = 4 - 6t$$

$\Rightarrow$ $a = -20 \text{ m/s}^2$ (decelerating in -> direction) when $t = 4$ s

Hard to tell unless you solve "S" → Originally shown as $a=-20\text{m/s}^2$ (or in the ← direction)

2) Calculate the distance traveled in 4s by integrating the velocity using $s_o = 0$:

$$v = ds / dt \Rightarrow ds = v\,dt \Rightarrow \int_{s_o}^{s} ds = \int_{0}^{t} (4t - 3t^2)\,dt$$

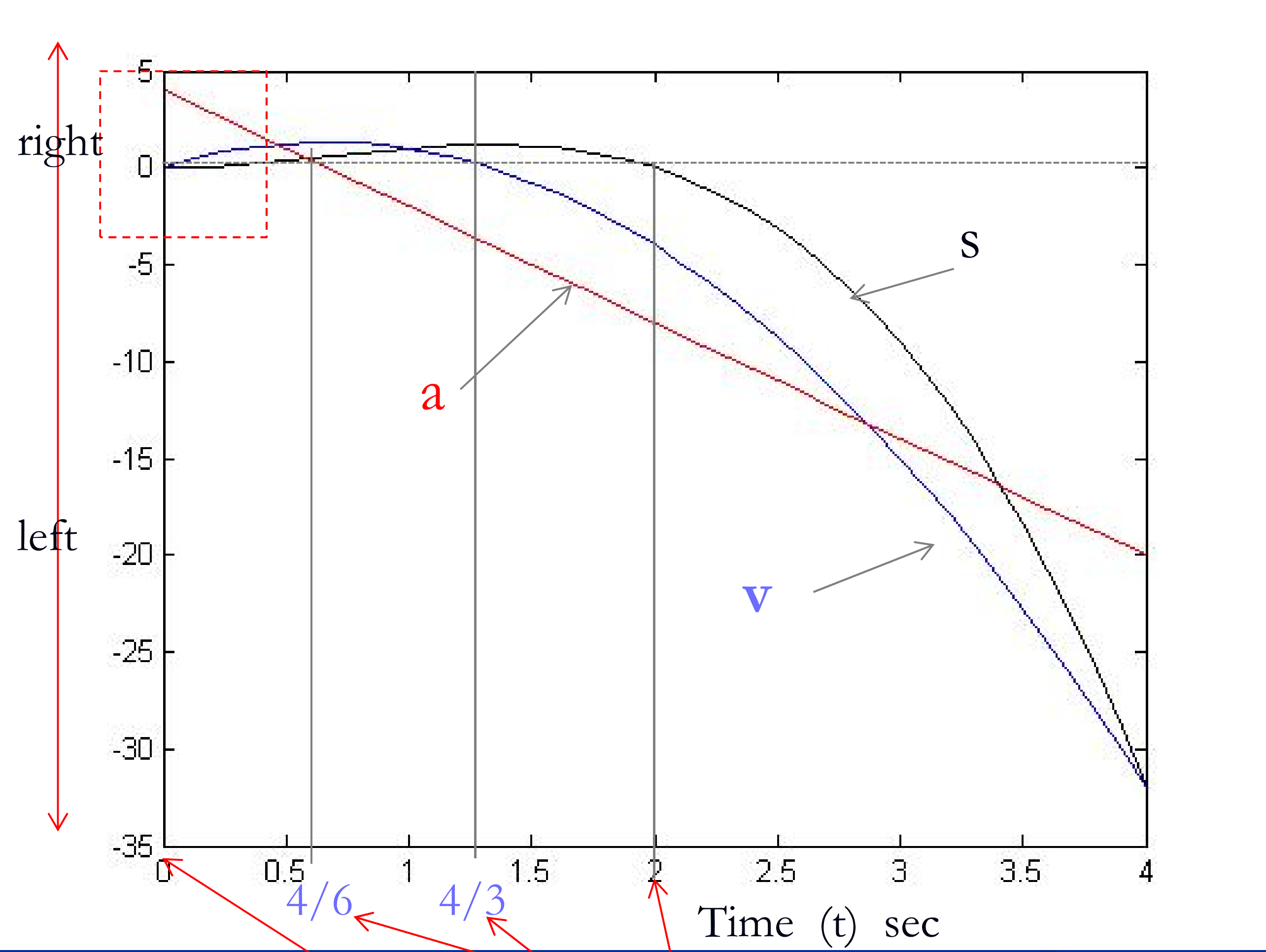
$$\Rightarrow s - s_o = 2t^2 - t^3$$

$$\Rightarrow s - 0 = 2(4)^2 - (4)^3 \Rightarrow s = -32 \text{ m}$$ (**or 32m going in ← direction**)

Originally shown as $s = -32$ m (or ←)

Sign Convention

A simple *rule of thumb* for determining the direction of the acceleration is that an object which is slowing down will have an acceleration directed in the direction opposite of its motion. Applying this *rule of thumb* would lead us to conclude that an eastward heading car can have a westward directed acceleration vector if the car is slowing down. Be careful when discussing the direction of the acceleration of an object; slow down, apply some thought and use the *rule of thumb*.



$$a = dv / dt = 4 - 6t$$

$$v = 4t - 3t^2$$

$$s - s_o = 2t^2 - t^3$$

a = 0 means constant velocity

v = 0 means stopping possible changing direction

s = 0 means at original position

Do this

- First define which direction is your positive direction.
- Just remember slowing down (deceleration) is negative and speed up (acceleration) is positive, **but hard to tell which direction it's going unless you know position.** So just reference to your defined positive reference in acceleration answer.
- Velocity or position negative means going opposite direction of the direction you define as positive and Positive velocity or position means you are going in the same direction as you define as positive direction. Again indicate your reference direction.

Example

1. A particle has an initial velocity of 3 ft/s to the left at $s_0 = 0$ ft. Determine its position when t = 3 s if the acceleration is 2 ft/s^2 to the right.

A) 0.0 ft
B) 6.0 ft ←
C) 18.0 ft →
D) 9.0 ft →

2. A particle is moving with an initial velocity of v = 12 ft/s and constant acceleration of 3.78 ft/s^2 in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 ft/s.

A) 50 ft
B) 100 ft
C) 150 ft
D) 200 ft

Solution for 1

$$\int_{v_o}^{v} dv = \int_{0}^{t} a_c \, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{0}^{t} v \, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v \, dv = \int_{s_o}^{s} a_c \, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

$S = 3 \text{ (m/s) x3 (sec)} + (1/2)\ (-2\text{m}/t^2)\ \text{ x } 3 \text{ (sec)} = 0$

Conservation of Energy equation

Solution for 2

$$\int_{v_o}^{v} dv = \int_{0}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{0}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

Conservation of
Energy equation

Analysing problems in dynamics

Coordinate system

- Establish a position coordinate S along the path and specify its fixed origin and positive direction
- Motion is along a straight line and therefore s, v and α can be represented as algebraic scalars
- Use an arrow alongside each kinematic equation in order to indicate positive sense of each scalar

Kinematic equations

- If any two of α, v, s and t are related, then a third variable can be obtained using one of the kinematic equations (one equation can only solve one unknown)
- When performing integration, position and velocity must be known at a given instant (…so the constants or limits can be evaluated)
- Some equations must be used only when *a is constant*

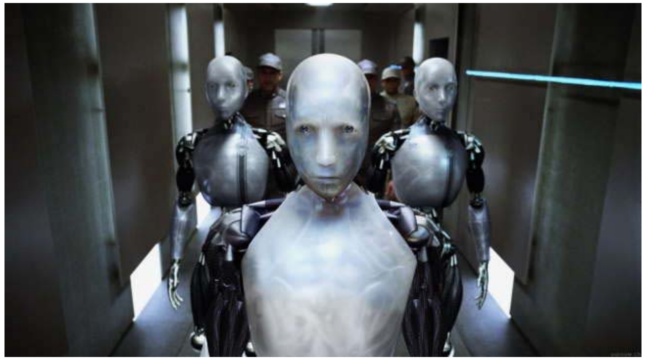
Problem solving MUSTS

1. Read the problem carefully (and read it again)
2. Physical situation and theory link
3. Draw diagrams and tabulate problem data
4. Coordinate system!!!
5. Solve equations and be careful with units
6. Be critical. A mass of an aeroplane can not be 50 g
7. Read the problem carefully

Important points

- Dynamics: Accelerated motion of bodies
- Kinematics: Geometry of motion
- Average speed ≠ average velocity
- Rectilinear kinematics or straight-line motion

- Acceleration is negative when particle is slowing down!!
- α ds = v dv; relation of acceleration, velocity, displacement



Homework Assignment

Chapter 12: 10, 22, 24, 26, 28, 32, 37, 62, 71, 92, 98, 112, 120, 122, 144, 163, 175, 179

Due Next Wednesday !!!

Additional Information

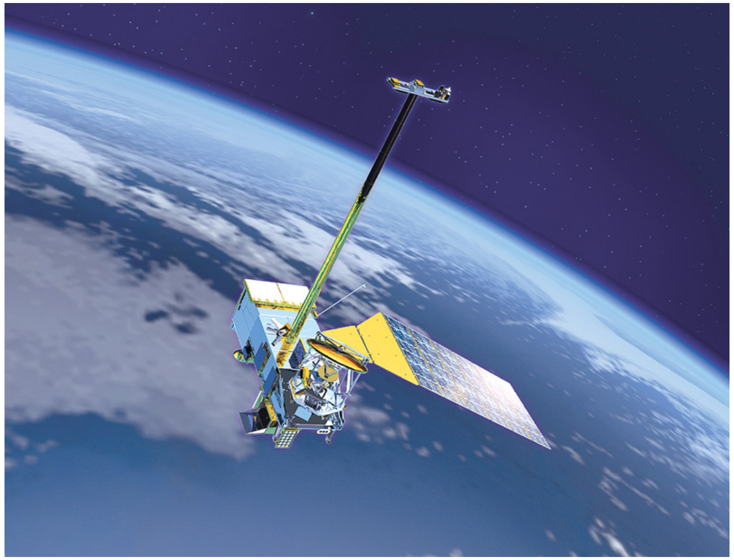
- Lecture notes are online:

http://courses.washington.edu/engr100/me230

Lecture 1: Particle Kinematic

- **Kinematics of a particle** (Chapter 12)

- 12.3



Kinematics of a particle: Objectives

- Concepts such as position, displacement, velocity and acceleration are introduced
- Study the motion of particles along a straight line. Graphical representation
- Investigation of a particle motion along a curved path. Use of different coordinate systems
- Analysis of dependent motion of two particles
- Principles of relative motion of two particles. Use of translating axis

Material covered

- **Kinematics of a particle**

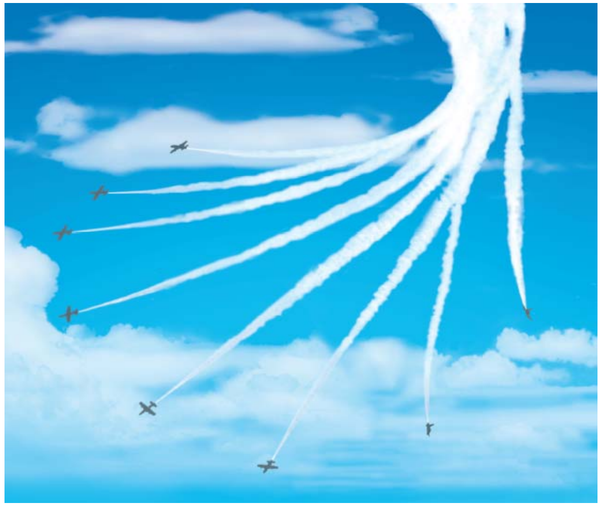
-Rectilinear kinematics: Erratic motion

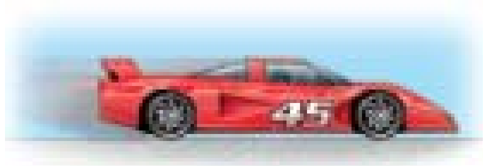
-Next lecture; General curvilinear motion, rectangular components and motion of a projectile

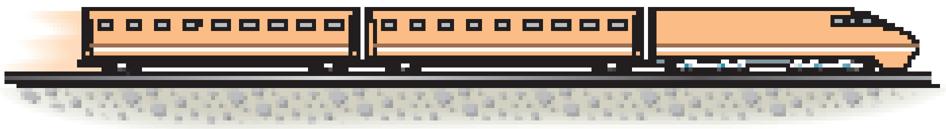
Objectives

Students should be able to:

1. Determine position, velocity, and acceleration of a particle using graphs (12.3)







Erratic (discontinuous) motion

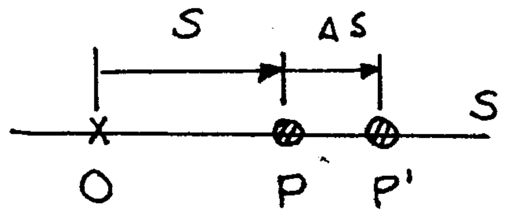
Graphing provides a good way to handle complex motions that would be difficult to describe with formulas. Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics

The approach builds on the facts that slope and differentiation are linked and that integration can be thought of as finding the area under a curve

Definitions

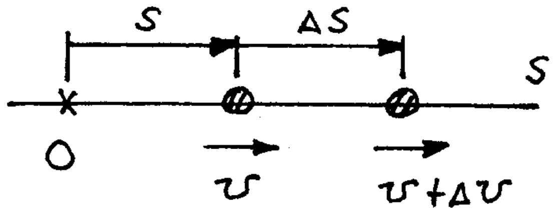
(1) Average Velocity

$$v_{AV} = \frac{\Delta s}{\Delta t}$$



(2) Instantaneous Velocity:

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



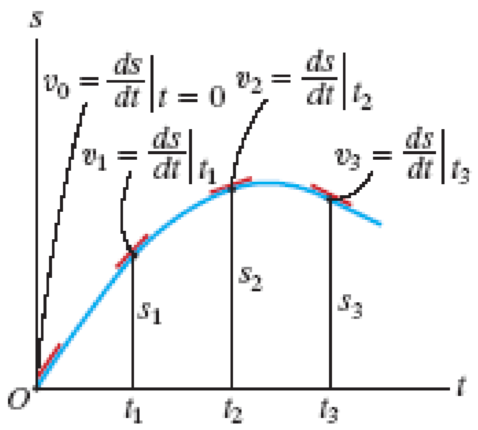
(3) Average Acceleration:

$$a_{AV} = \frac{\Delta v}{\Delta t}$$

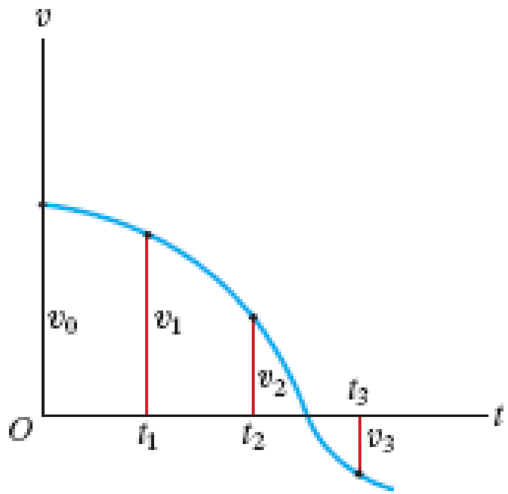
(4) Instantaneous Acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

s-t graph → construct v-t

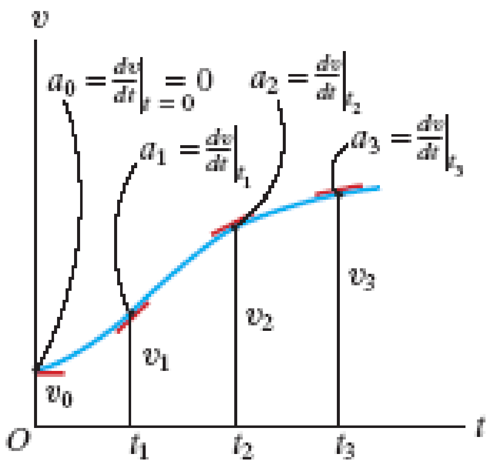


Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or **v = ds/dt**)



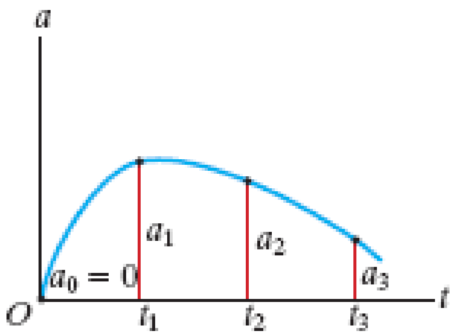
Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph

v-t graph ➡ construct a-t



Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point (or **a = dv/dt**)

Therefore, the a-t graph can be constructed by finding the slope at various points along the v-t graph



Also, the distance moved (displacement) of the particle is the area under the v-t graph during time Δt

Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a \; dt \quad \text{or} \quad \int_{v_o}^{v} v \, dv = \int_{s_o}^{s} a \; ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v \, dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at t = 0.

a-t graph → construct v-t

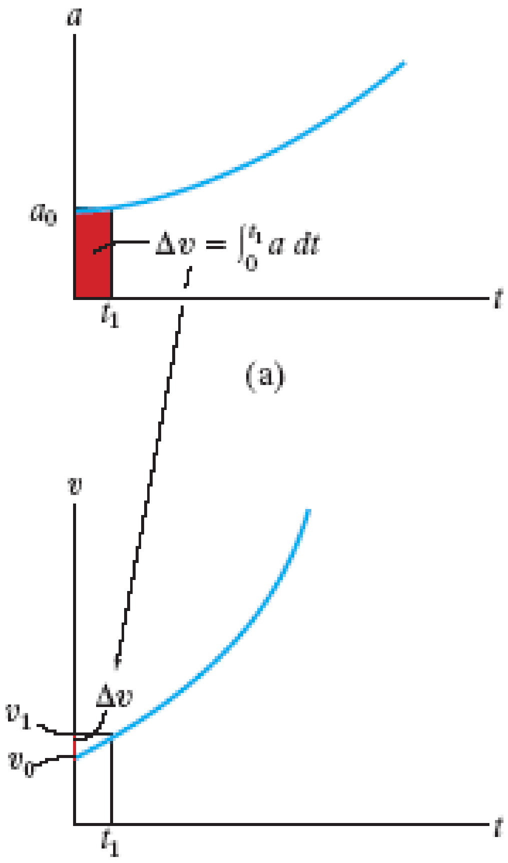


Fig. 12–10

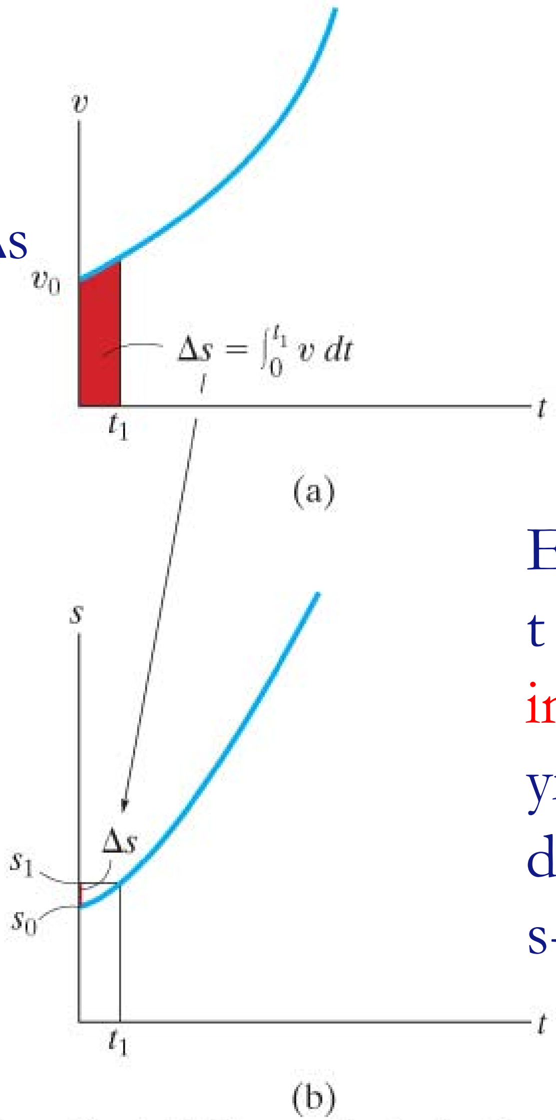
Given the a-t curve, the change in velocity (**Δv**) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle

v-t graph → construct s-t

We begin with initial position S_0 and add algebraically increments Δs determined from the

v-t graph



Equations described by v-t graphs may be integrated in order to yield equations that describe segments of the s-t graph

Please remember the link!!!

handle complex motions

⬇

graphing

⬇

visual description of motion

⬇

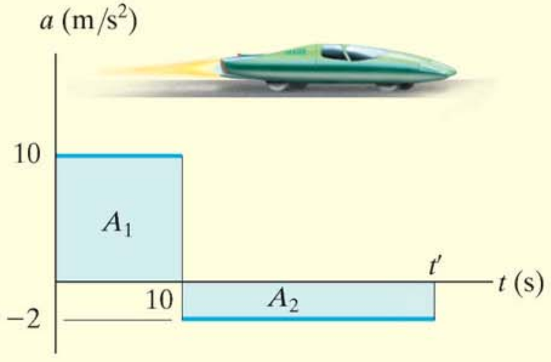
differentiation and integration

⬇

slope and area under curve

Explanation of Example 12.7 (A)

EXAMPLE 12.7



(a)

The test car in Fig. 12–12*a* starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 s and then decelerates at a constant rate. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?

SOLUTION

v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

$$0 \le t < 10\text{ s};\qquad a = 10;\qquad \int_0^v dv = \int_0^t 10\,dt,\qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$$10\text{ s} < t \le t';\qquad a = -2;\qquad \int_{100}^v dv = \int_{10}^t -2\,dt,\qquad v = -2t + 120$$

When $t = t'$ we require $v = 0$. This yields, Fig. 12–12*b*,

$$t' = 60\text{ s} \qquad \textit{Ans.}$$

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–12*a*. Thus

$$0 = 10\text{ m/s}^2(10\text{ s}) + (-2\text{ m/s}^2)(t' - 10\text{ s})$$

$$t' = 60\text{ s} \qquad \textit{Ans.}$$

Explanation of Example 12.7 (B)

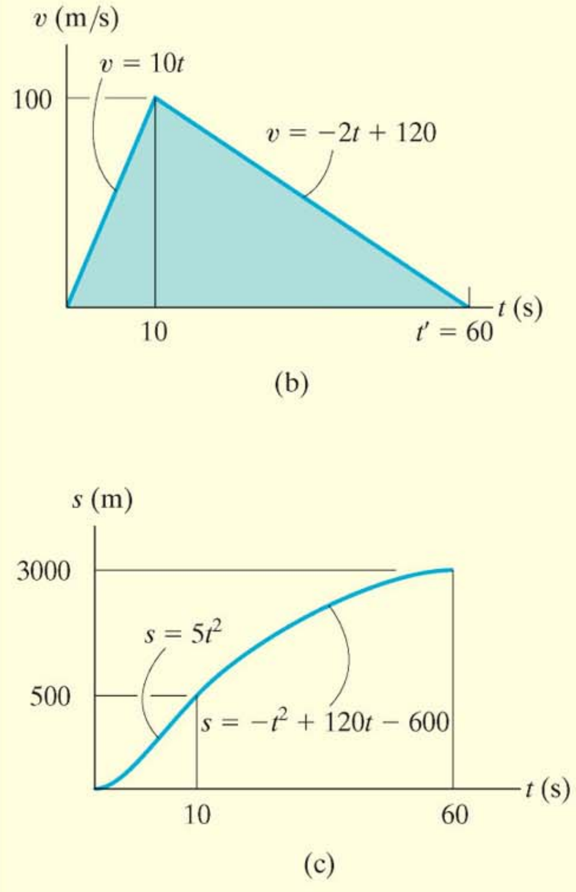


Fig. 12–12

s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10\text{ s};\qquad v = 10t;\qquad \int_0^s ds = \int_0^t 10t\,dt,\qquad s = 5t^2$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

$$10\text{ s} \le t \le 60\text{ s};\quad v = -2t + 120;\quad \int_{500}^s ds = \int_{10}^t (-2t + 120)\,dt$$

$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = -t^2 + 120t - 600$$

When $t' = 60$ s, the position is

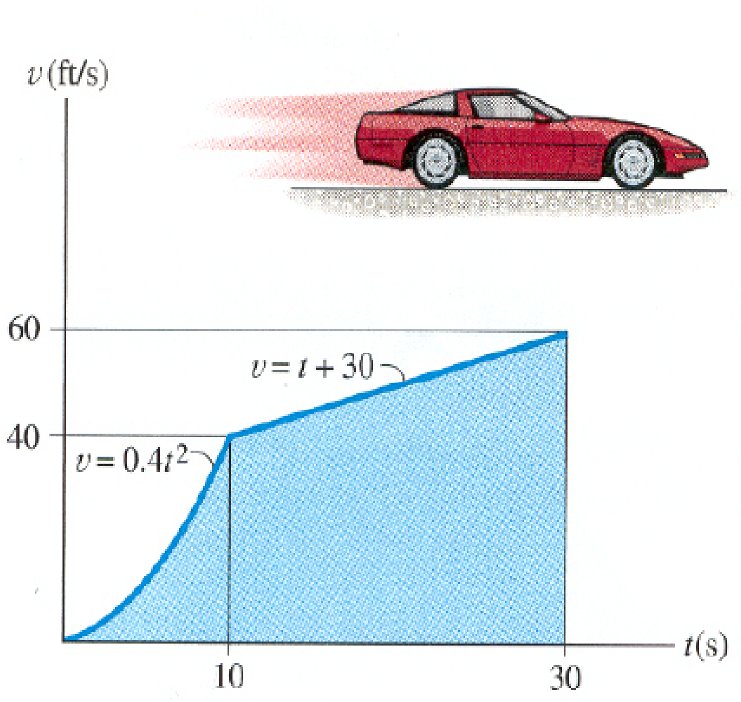
$$s = -(60)^2 + 120(60) - 600 = 3000\text{ m} \qquad \textit{Ans.}$$

The s–t graph is shown in Fig. 12–12*c*.

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60)(100) = 3000\text{ m} \qquad \textit{Ans.}$$

Example



Given: The v-t graph shown

Find: The a-t graph, average speed, and distance traveled for the 30 s interval

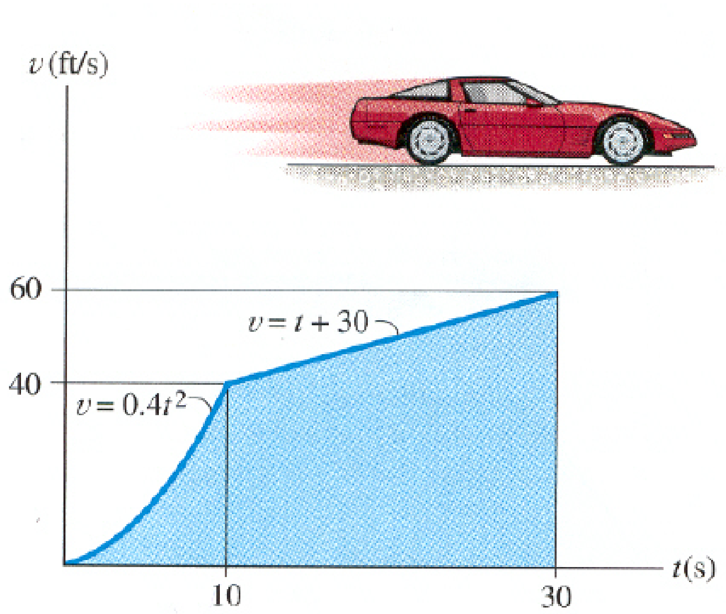
Hint

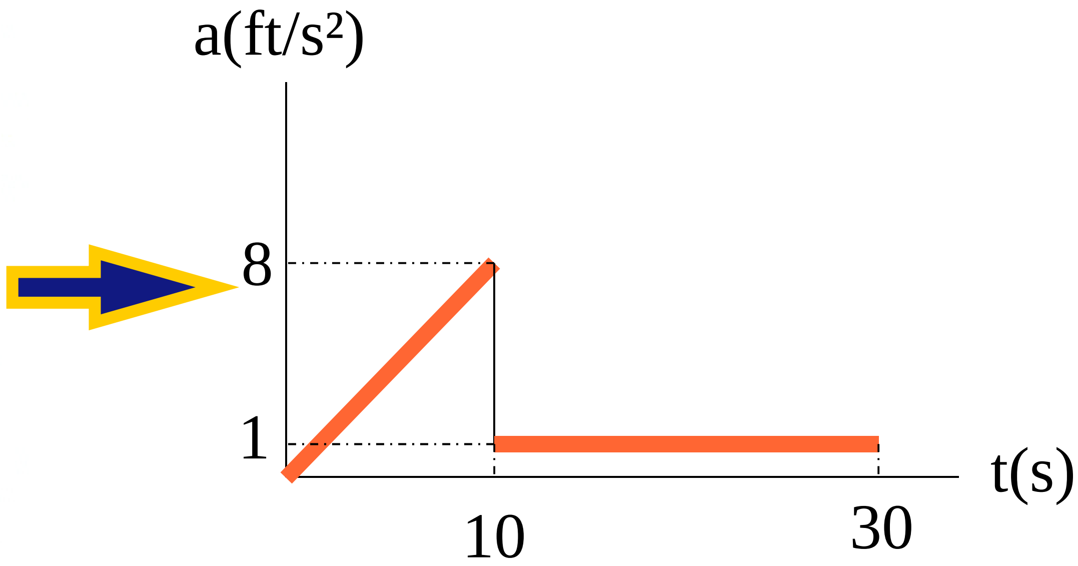
Find slopes of the curves and draw the a-t graph.
Find the area under the curve--that is the distance traveled.
Finally, calculate average speed (using basic definitions!)

Example

For $0 \le t \le 10$ $a = dv/dt = 0.8\,t$ ft/s²

For $10 \le t \le 30$ $a = dv/dt = 1$ ft/s²





Example

$$\Delta s_{0\text{-}10} = \int v\,dt = (1/3)\,(0.4)(10)^3 = 400/3 \text{ ft}$$

$$\Delta s_{10\text{-}30} = \int v\,dt = (0.5)(30)^2 + 30(30) - 0.5(10)^2 - 30(10)$$

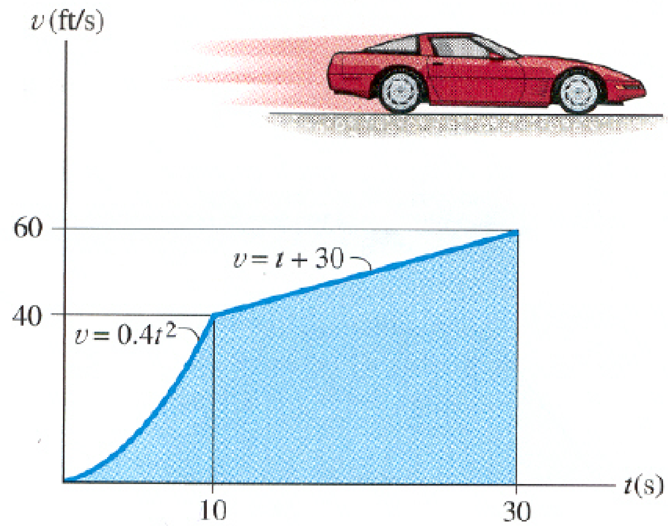
$$= 1000 \text{ ft}$$

$$s_{0\text{-}30} = 1000 + 400/3 = 1133.3 \text{ ft}$$

$$v_{avg(0\text{-}30)} = \text{total distance / time}$$

$$= 1133.3/30$$

$$= 37.78 \text{ ft/s}$$



A couple of cases more…

A couple of cases that are a bit more **…COMPLEX…** and therefore need more attention!!!

Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a \, dt \quad \text{or} \quad \int_{v_o}^{v} v \, dv = \int_{s_o}^{s} a \, ds$$

Position:

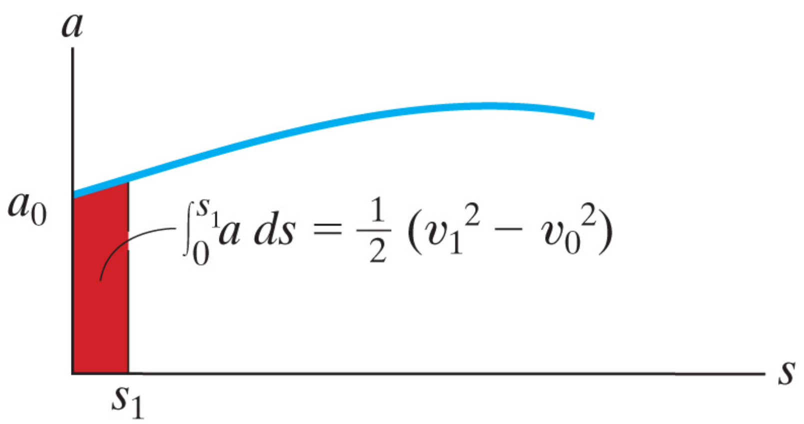
$$\int_{s_o}^{s} ds = \int_{o}^{t} v \, dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at t = 0.

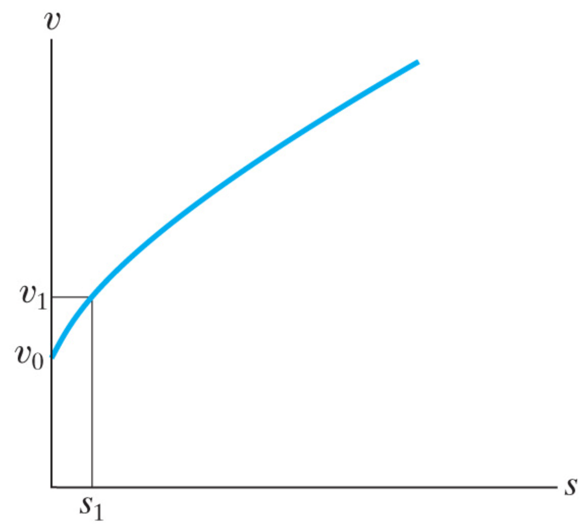
a-s graph → construct v-s

A more complex case is presented by the a-s graph. The area under the acceleration versus position curve represents the change in velocity

(recall $\int$ a ds = $\int$ v dv)



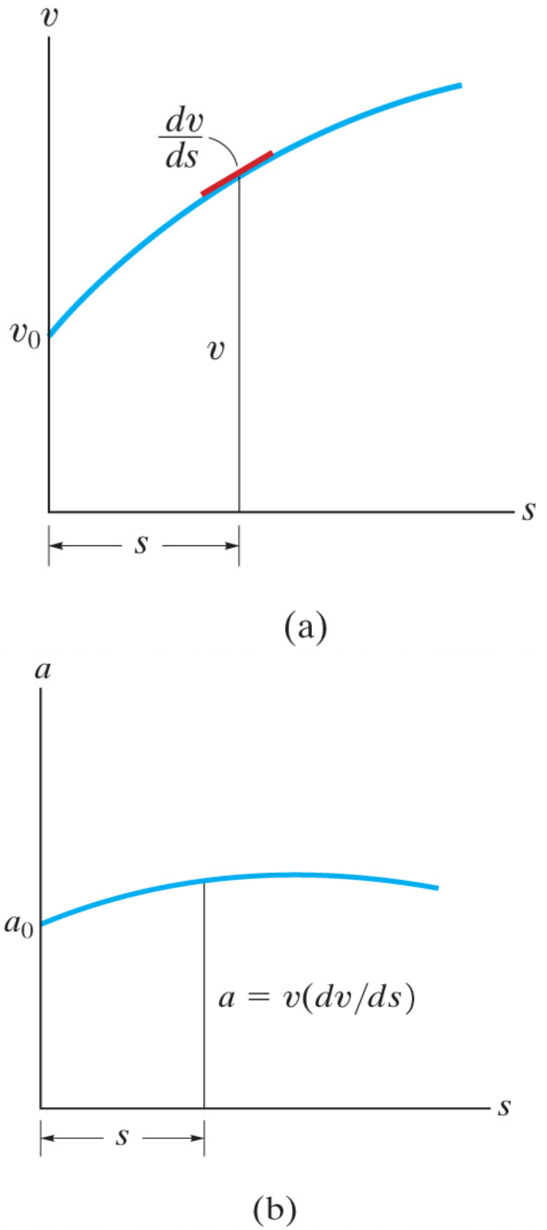
(a)



(b)

This equation can be solved for v_1, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance

v-s graph ➔ construct a-s



Another complex case is presented by the v-s graph. By reading the velocity v at a point on the curve and multiplying it by the slope of the curve (dv/ds) at this same point, we can obtain the acceleration at that point.

a = v (dv/ds)

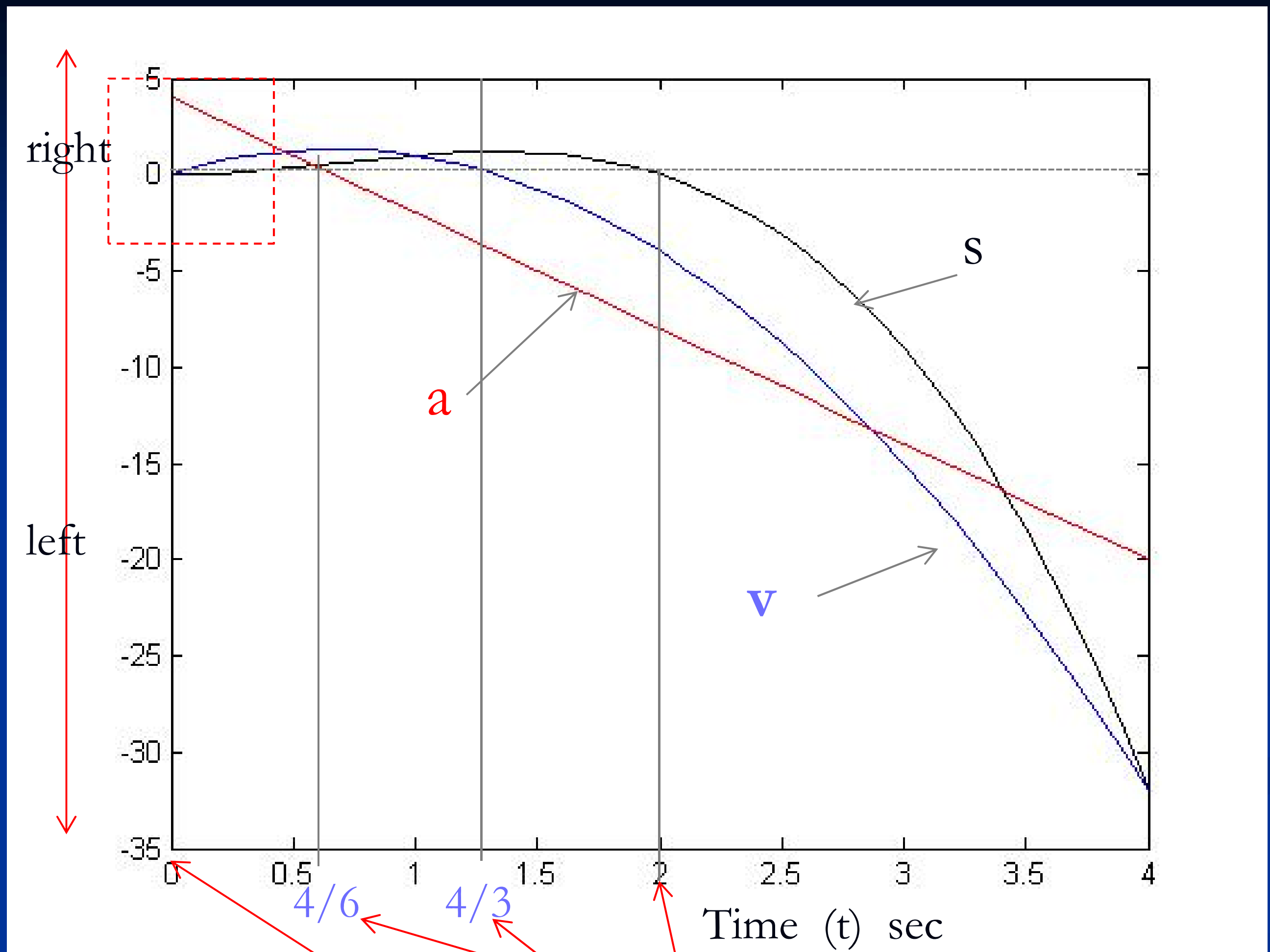
Thus, we can obtain a plot of a vs. s from the v-s curve

Determine position, velocity, and acceleration of a particle using graphs

- Experimental data (very complicate motion)
- Nonlinear motion
- Find a function has the closely match curve or break it up and analyze it section by section.
- Allow one to quickly analyze the changes in direction, velocity acceleration.

Please think about it

If a particle in rectilinear motion has zero speed at some instant in time, is the acceleration necessarily zero at the same instant ? NO!



$a = dv\ /\ dt = 4 - 6\ t$

$v = 4\ t - 3\ t^2$

$s - s_o = 2\ t^2 - t^3$

a = 0 means constant velocity
v = 0 means stopping possible changing direction
s = 0 means at original position

Homework Assignment

Chapter 12: 10, 22, 24, 26, 28, 32, 37, 62, 71, 92, 98, 112, 120, 122, 144, 163, 175, 179

Due Next Wednesday !!!

Water Calculator

Mechanical System

- Conveyor belt system

  - Kinematic equations(position, velocity and acceleration) make sure it stop at the right position and not moving and stop and an appropriate speed to prevent water tipping over.

  - Kinetic equations (gear system that provide enough torque to move the belt, enough friction on the wheels to catch the belt and rotate the belt)

Water Calculator Powerpoint

- Faculty.washington.edu/abong

Chapter 12: Kinematics of a Particle

- Chapter 12 introduces the kinematics of a particle
  - kinematics: the study of the geometry of motion (regardless of the forces which cause that motion)
  - particle: a body which can be modeled as having no physical dimensions
- Chapter 12 unfolds by gradually increasing the complexity of our view of this topic, considering different kinds of motion in different coordinate systems

translational motion

- ☑ rectilinear motion: motion in a straight line (12.2, 12.3)
- ☑ curvilinear motion: motion along a curve (12.4-12.8)
- ☑ continuous motion: motion in one direction (12.2, 12.3)
- ☑ erratic motion: motion in different directions (12.3)
- ☑ rectangular components: using Cartesian coordinates (12.2, 12.3, 12.5, 12.6)
- ☑ normal and tangential components: using a "path" coordinate systems (12.7)
- ☑ cylindrical components: using cylindrical/polar coordinates (12.8)
- ☑ absolute motion: using a global/inertial coordinate system (12.9)
- ☑ relative motion: using a local/relative coordinate system (12.10)

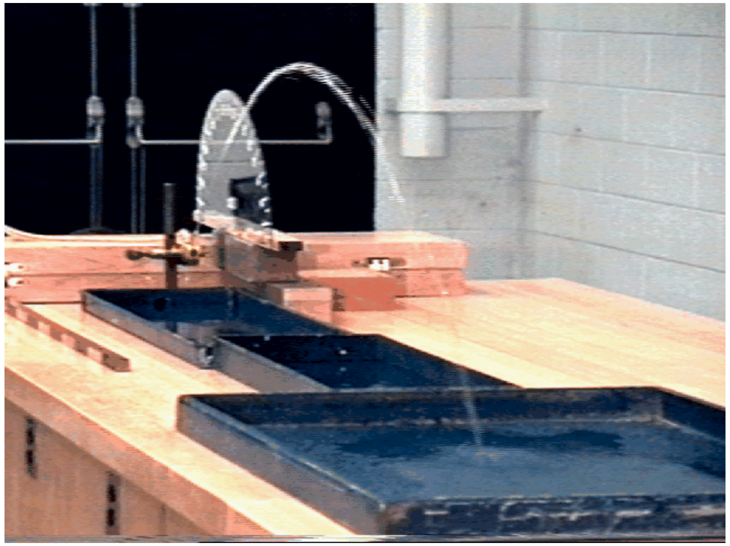
Coordinates

Two classes of motion

Lecture 2: Particle Kinematic

- **Kinematics of a particle** (Chapter 12)

- 12.4-12.6



Kinematics of a particle: Objectives

- Concepts such as position, displacement, velocity and acceleration are introduced
- Study the motion of particles along a straight line. Graphical representation
- Investigation of a particle motion along a curved path. Use of different coordinate systems
- Analysis of dependent motion of two particles
- Principles of relative motion of two particles. Use of translating axis

Material covered

- **Kinematics of a particle**

- General curvilinear motion

- Curvilinear motion: Rectangular components (Cartesian coordinate)

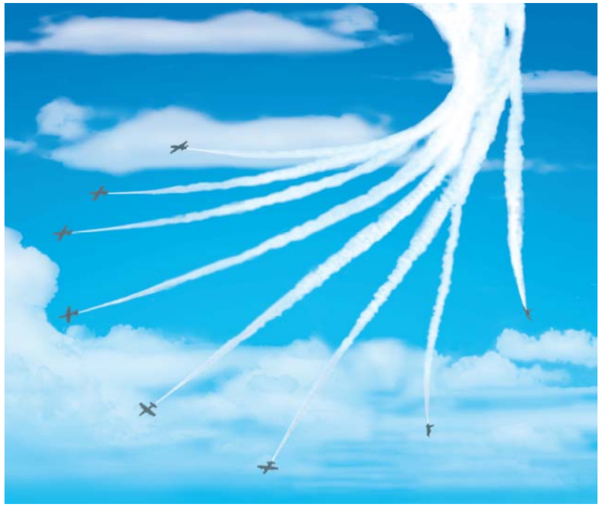
- Motion of a projectile

-Next lecture; Curvilinear motion: Normal & tangential components and cylindrical components

Objectives

Students should be able to:

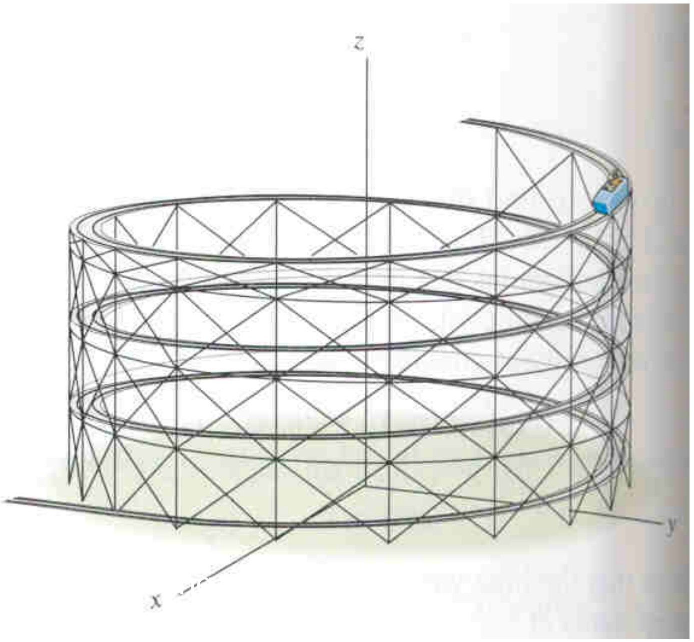
1. Describe the motion of a particle traveling along a curved path
2. Relate kinematic quantities in terms of the rectangular components of the vectors
3. Analyze the free-flight motion of a projectile



Related applications

The path of motion of each plane in this formation can be tracked with radar and their x, y, and z coordinates (relative to a point on earth) recorded as a function of time

How can we determine the velocity or acceleration at any instant?

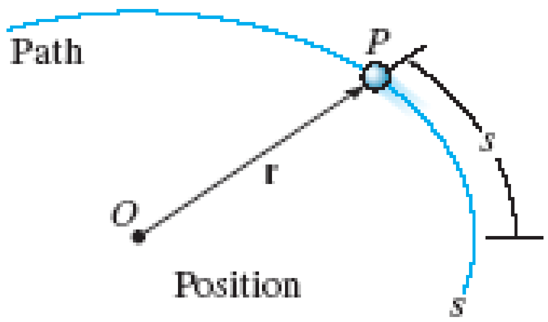


A roller coaster car travels down a fixed, helical path at a constant speed

If you are designing the track, why is it important to be able to predict the acceleration of the car?

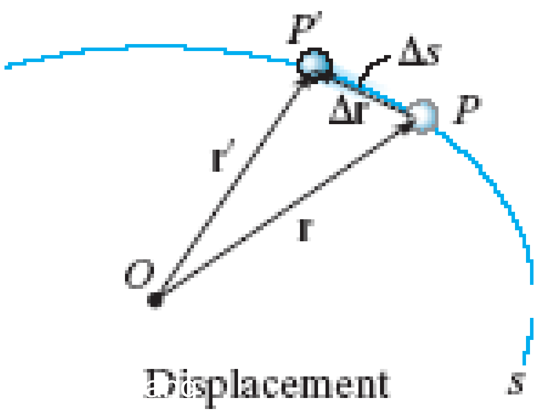
General curvilinear motion

A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, vectors are used to describe the motion



A particle moves along a curve defined by the path function, s

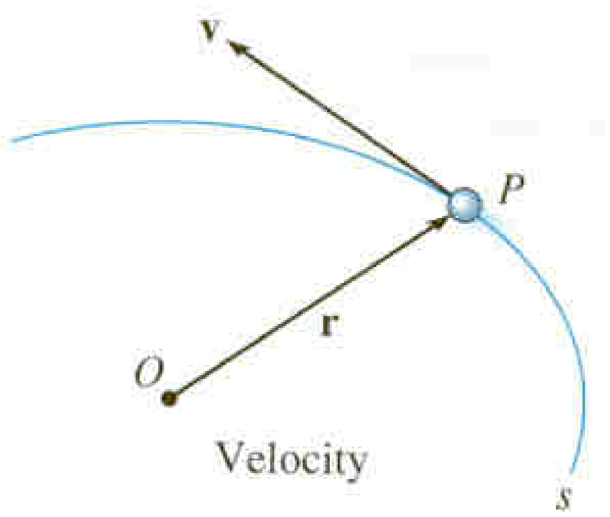
The position of the particle at any instant is designated by the vector $\boldsymbol{r} = \boldsymbol{r}(t)$. Both the magnitude and direction of $\boldsymbol{r}$ may vary with time



If the particle moves a distance Δs along the curve during time interval Δt, the displacement is determined by vector subtraction: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

Velocity

Velocity represents the rate of change in the position of a particle



The average velocity of the particle during the time increment Δt is

$\boldsymbol{v}_{avg} = \Delta\boldsymbol{r}/\Delta t$

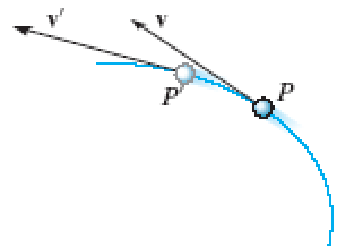
The instantaneous velocity is the time-derivative of position

$\boldsymbol{v} = d\boldsymbol{r}/dt$

The velocity vector, $\boldsymbol{v}$, is always tangent to the path of motion

The magnitude of $\boldsymbol{v}$ is called the speed. Since the arc length Δs approaches the magnitude of $\Delta\boldsymbol{r}$ as $t \to 0$, the speed can be obtained by differentiating the path function **($v = ds/dt$).** Note that this is not a vector!

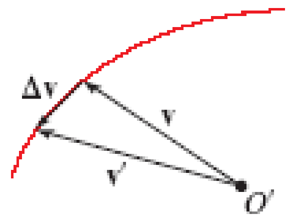
Acceleration



Acceleration represents the rate of change in the velocity of a particle

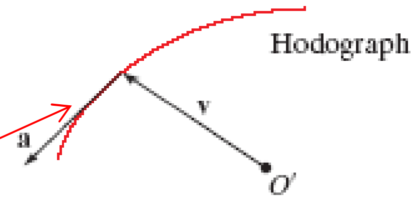
If a particle's velocity changes from $\boldsymbol{v}$ to $\boldsymbol{v'}$ over a time increment Δt, the average acceleration during that increment is:

$$\boldsymbol{a}_{avg} = \Delta \boldsymbol{v}/\Delta t = (\boldsymbol{v} - \boldsymbol{v'})/\Delta t$$

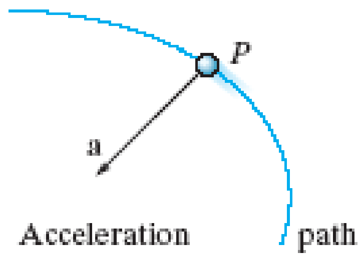


The instantaneous acceleration is the time-derivative of velocity:

$$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2$$



A plot of the locus of points defined by the arrowhead of the velocity vector is called a hodograph. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function



So pretty much the same set of equations we were describing in the rectilinear motion applies to curvilinear motion except in acceleration where due to the fact that when it is moving around a curve, in addition to the magnitude change along the direction of the path, there is a velocity direction change as well. When taking a time derivative of velocity for acceleration, it actually produce an additional acceleration that is not considered in the rectilinear motion.

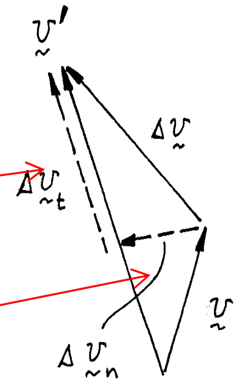
Acceleaton

$$\underset{\sim}{a} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}}{\Delta t} = \frac{d\underset{\sim}{v}}{dt} = \ddot{\underset{\sim}{r}}$$

Remarks

1. Acceleration is a vector.

2. Acceleration results from
   - change of velocity direction (circular motion)
   - Change of velocity magnitude (Rectilinear Motion)



3. How to visualize?

$$\Delta \underset{\sim}{v} = \Delta \underset{\sim}{v}_t + \Delta \underset{\sim}{v}_n$$

change in magnitude ($\Delta \underset{\sim}{v}_t$); change in direction ($\Delta \underset{\sim}{v}_n$)

Curvilinear motion: Rectangular components

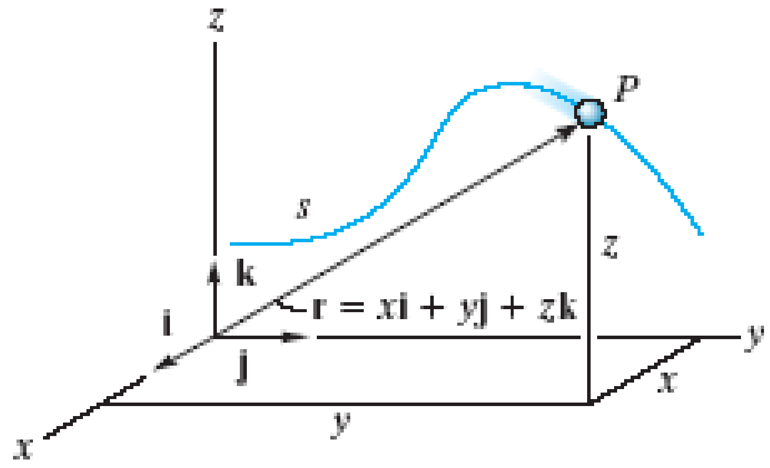
It is often convenient to describe the motion of a particle in terms of its x, y, z or rectangular components, relative to a fixed frame of reference

The position of the particle can be defined at any instant by the position vector

$$\boldsymbol{r} = \boldsymbol{x}\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

The x, y, z components may all be functions of time, i.e.,

$x = x(t)$, $y = y(t)$, and $z = z(t)$

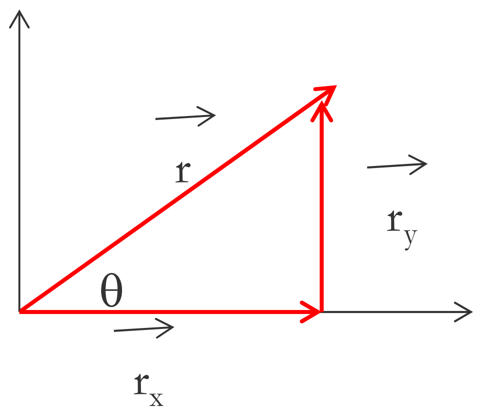


Position

The magnitude of the position vector is: $r = (x^2 + y^2 + z^2)^{0.5}$

The direction of $\boldsymbol{r}$ is defined by the unit vector: $\boldsymbol{u_r} = (1/r)\boldsymbol{r}$

Vector



Recall in your high school math, a vector $\vec{r}$ quantity is a quantity that is described by both magnitude $|r| = \sqrt{|r_x|^2 + |r_y|^2}$ and direction θ, where $|r_x| = |r|cos\theta\ and |r_y| = |r|sin\theta$

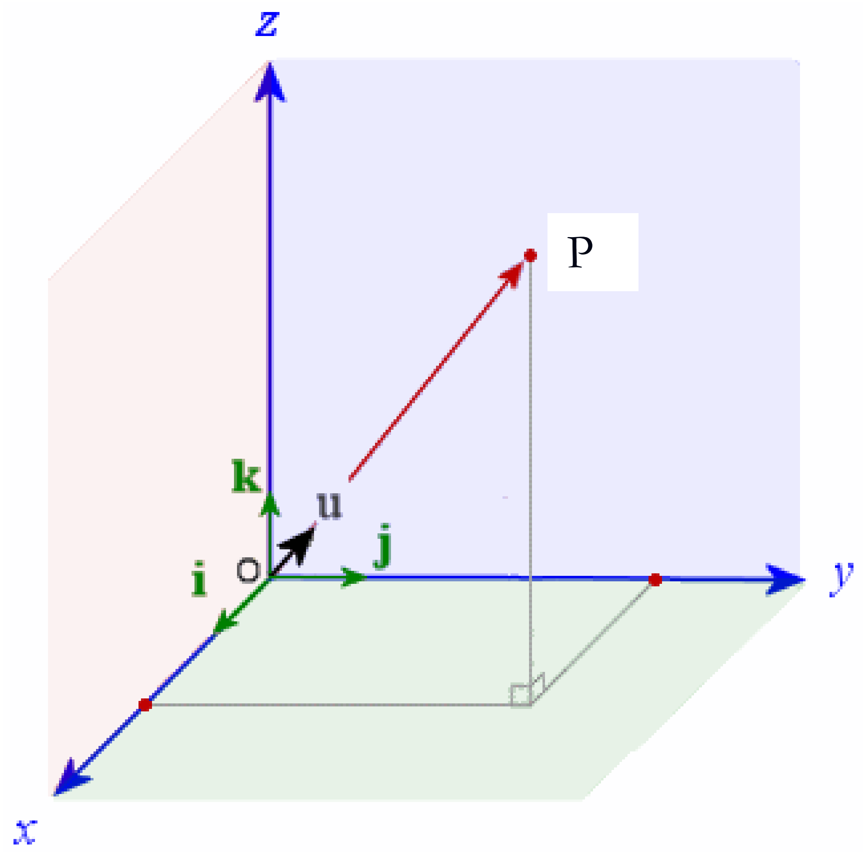
Unit Vector

Basically it is projection of the unit vector to x,y,z coordinates.

Let our unit vector be:

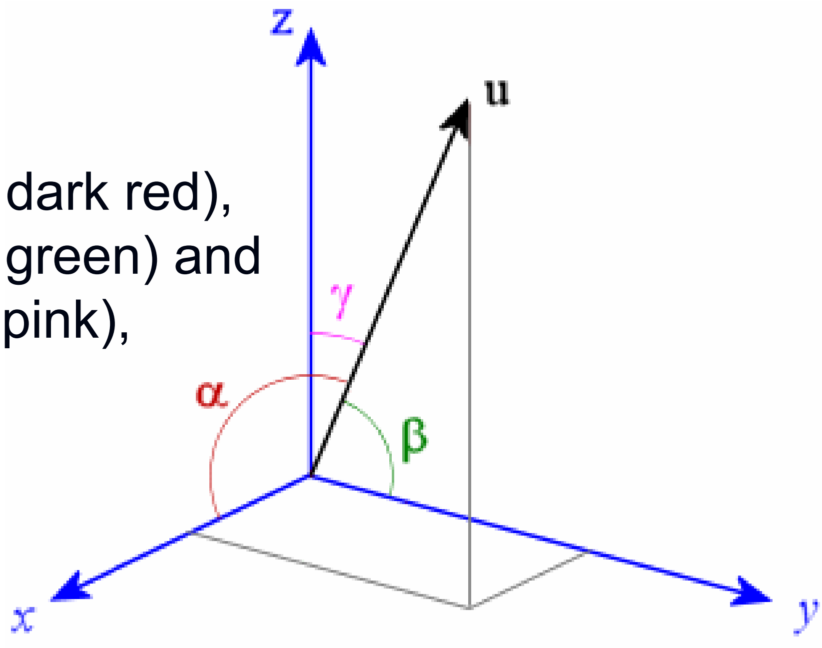
$\mathbf{u_r} = u_1\,\mathbf{i} + u_2\,\mathbf{j} + u_3\,\mathbf{k}$

On the graph, **u** is the unit vector (in black) pointing in the same direction as vector **OP**, and **i**, **j**, and **k** (the unit vectors in the *x*-, *y*- and *z*-directions respectively) are marked in green.



We now zoom in on the vector u, and change orientation slightly, as follows:

Now, if in the diagram in the last page,
α is the angle between **u** and the *x*-axis (in dark red),
β is the angle between **u** and the *y*-axis (in green) and
γ is the angle between **u** and the *z*-axis (in pink),



$$u_1 = \mathrm{u} \cdot \mathbf{i} = 1 \times 1 \times \cos\alpha = \cos\alpha$$
$$u_2 = \mathrm{u} \cdot \mathbf{j} = 1 \times 1 \times \cos\beta = \cos\beta$$
$$u_3 = \mathrm{u} \cdot \mathbf{k} = 1 \times 1 \times \cos\gamma = \cos\gamma$$

So the direction of ***r*** is defined by the unit vector:
$\boldsymbol{u}_r = (1/\mathrm{r})\boldsymbol{r}$

$$\mathbf{u}_r = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

$$\cos\alpha = x/\mathrm{sqrt}(x^2+y^2+z^2)$$
$$\cos\beta = y/\mathrm{sqrt}(x^2+y^2+z^2)$$
$$\cos\gamma = z/\mathrm{sqrt}(x^2+y^2+z^2)$$

$$\mathbf{u}_r = u_1\,\mathbf{i} + u_2\,\mathbf{j} + u_3\,\mathbf{k}$$

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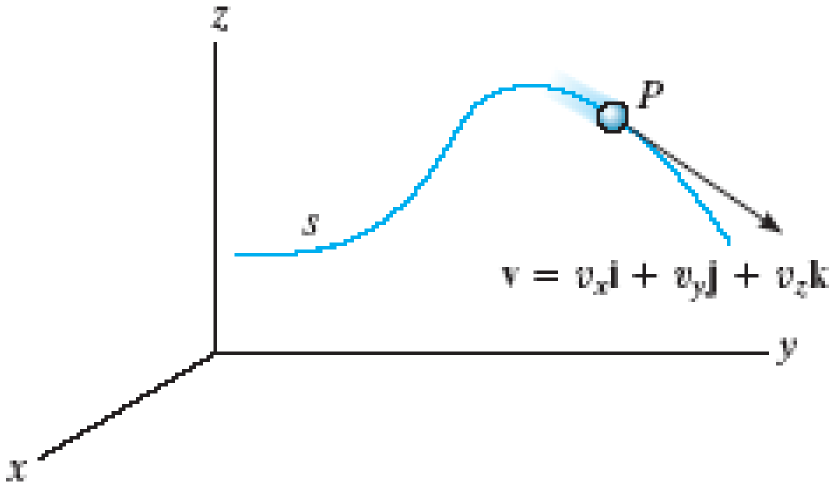
Rectangular components: Velocity

The velocity vector is the time derivative of the position vector:

$$\boldsymbol{v} = d\boldsymbol{r}/dt = d(x\boldsymbol{i})/dt + d(y\boldsymbol{j})/dt + d(z\boldsymbol{k})/dt$$

Since the unit vectors ***i***, ***j***, ***k*** are constant in magnitude and direction, this equation reduces to $v = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$

Where; $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$



Velocity

The magnitude of the velocity vector is

$$v = [(v_x)^2 + (v_y)^2 + (v_z)^2]^{0.5}$$

The direction of $\boldsymbol{v}$ is tangent to the path of motion.

Rectangular components: Acceleration

The acceleration vector is the time derivative of the velocity vector (second derivative of the position vector):

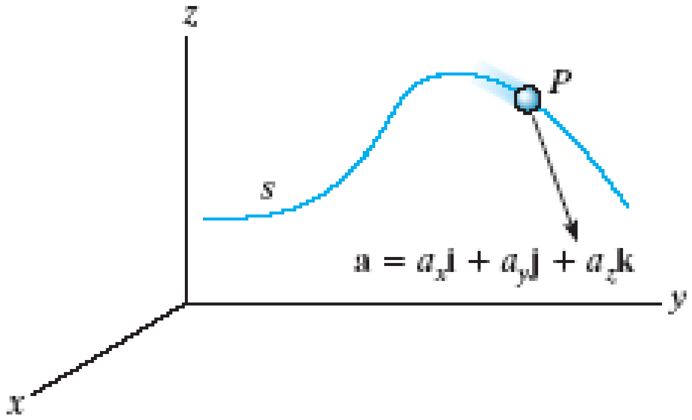
$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2 = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$

where $a_x = \dot{v}_x = \ddot{x} = dv_x/dt$, $a_y = \dot{v}_y = \ddot{y} = dv_y/dt$,

$a_z = \dot{v}_z = \ddot{z} = dv_z/dt$

The magnitude of the acceleration vector is

$a = [(a_x)^2 + (a_y)^2 + (a_z)^2]^{0.5}$

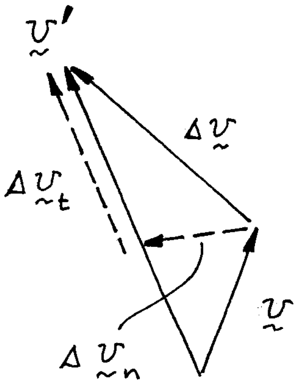


Acceleration

The direction of $\boldsymbol{a}$ is usually not tangent to the path of the particle

Acceleation

$$\underset{\sim}{a} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}}{\Delta t} = \frac{d\underset{\sim}{v}}{dt} = \ddot{\underset{\sim}{r}}$$



Remarks

1. Acceleration is a vector.

2. Acceleration results from
   - change of velocity direction

     (circular motion)
   - change of velocity magnitude

     (Rectilinear Motion)

3. How to visualize ?

$$\Delta \underset{\sim}{v} = \Delta \underset{\sim}{v}_t + \Delta \underset{\sim}{v}_n$$

($\Delta \underset{\sim}{v}_t$ ↑) change in magnitude

($\Delta \underset{\sim}{v}_n$ ↑) change in direction

EXAMPLE

Given: The box slides down the slope described by the equation $y = (0.05x^2)$ m, where x is in meters.

$v_x = -3$ m/s, $a_x = -1.5$ m/s^2 at $x = 5$ m.

Find: The y components of the velocity and the acceleration of the box at at $x = 5$ m.

Plan: Note that the particle's velocity can be related by taking the first time derivative of the path's equation. And the acceleration can be related by taking the second time derivative of the path's equation.

Take a derivative of the position to find the component of the velocity and the acceleration.

EXAMPLE (continued)

Solution:

Find the y-component of velocity by taking a time derivative of the position $y = (0.05x^2)$

$$\Rightarrow \quad \dot{y} = 2\,(0.05)\,x\,\dot{x} = 0.1\,x\,\dot{x}$$

Find the acceleration component by taking a time derivative of the velocity $\dot{y}$

$$\Rightarrow \quad \ddot{y} = 0.1\,\dot{x}\,\dot{x} + 0.1\,x\,\ddot{x}$$

Substituting the x-component of the acceleration, velocity at x=5 into $\dot{y}$ and $\ddot{y}$.

EXAMPLE (continued)

Since $\dot{x} = v_x = -3$ m/s, $\ddot{x} = a_x = -1.5$ m/s^2 at x = 5 m

$$\Rightarrow \quad \dot{y} = 0.1\, x\, \dot{x} = 0.1\,(5)\,(-3) = -1.5 \text{ m/s}$$

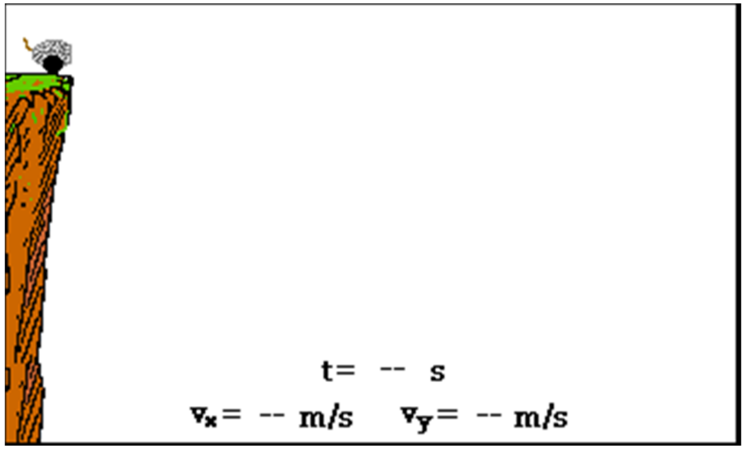
$$\begin{aligned} \Rightarrow \quad \ddot{y} &= 0.1\, \dot{x}\, \dot{x} + 0.1\, x\, \ddot{x} \\ &= 0.1\,(-3)^2 + 0.1\,(5)\,(-1.5) \\ &= 0.9 - 0.75 \\ &= 0.15 \text{ m/s}^2 \end{aligned}$$

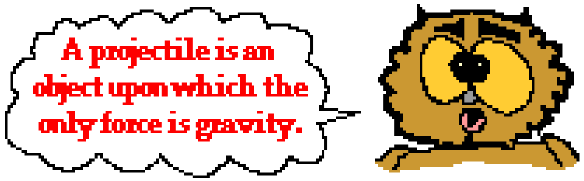
At x = 5 m

$v_y = -1.5$ m/s $= 1.5$ m/s $\downarrow$

$a_y = 0.15$ m/s^2 $\uparrow$

Projectile motion…

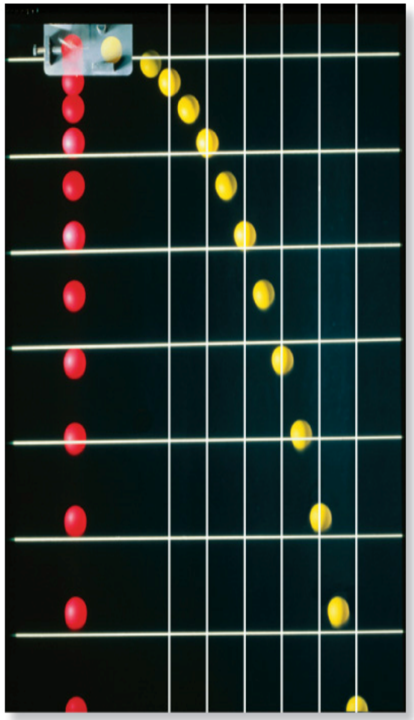




www.glenbrook.k12.il.us/gbssci/phys/mmedia/index.html

Motion of a projectile

Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity)



For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant

Theory: Projectile Motion (12.6)

- projectile motion is a special case of erratic motion usually modeled using Cartesian vectors
- it is a special case, because projectiles move in the presence of a constant gravitational acceleration in one direction (up/down), and (usually) negligible acceleration in another (horizontally); projectiles are modeled as particles
- we solve the problem using Cartesian coordinates, in two parts

Vertical motion	Horizontal motion
Y direction	X direction
$V_{oy} = V_o \sin\theta$	$V_{ox} = V_o \cos\theta$
$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$	$V_{fx}{}^2 = V_{ox}{}^2$
$V_{fy} = V_{oy} + at$	$V_{fx} = V_{ox}$
$Y = V_{oy} t + 1/2\, at^2$	$X = V_{ox} t$
$Y = 1/2(V_{fy} + V_{oy})t$	$X = 1/2(V_{fx} + V_{ox})t$

Kinematic equations: Horizontal &Vertical motion

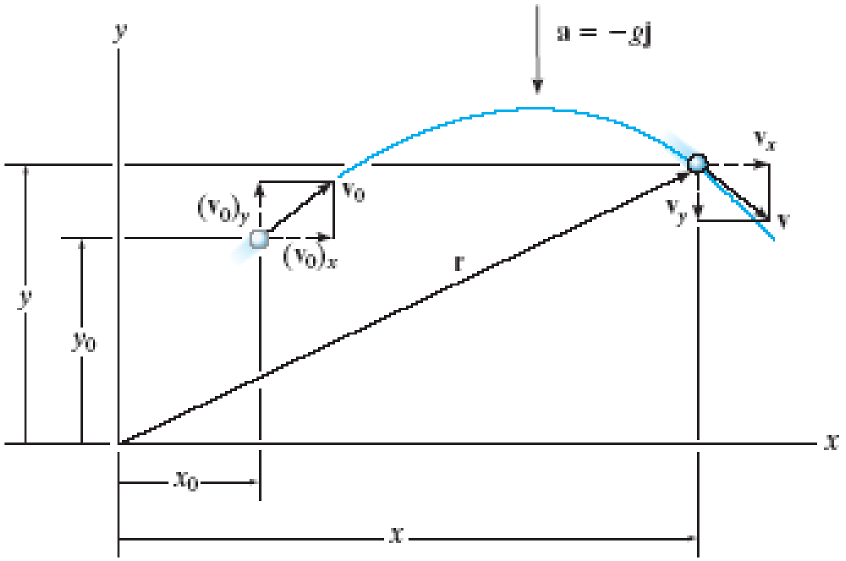


Fig. 12–20

Since $a_x = 0$, the velocity in the horizontal direction remains constant **($v_x = v_{ox}$)** and the position in the x direction can be determined by:

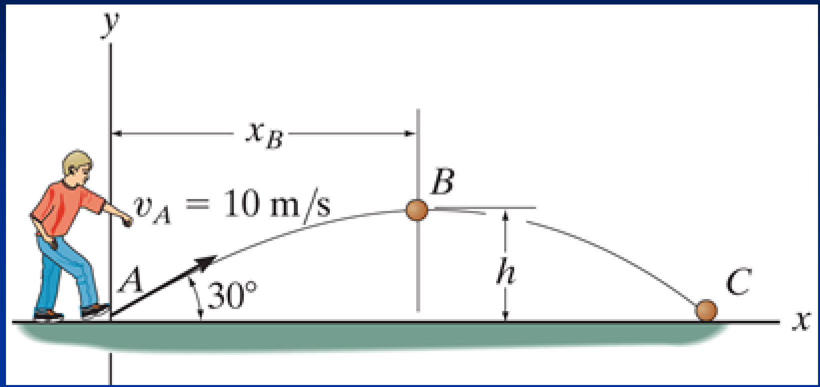
$$x = x_o + (v_{ox})(t)$$

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

$$v_y = v_{oy} - g(t)$$
$$y = y_o + (v_{oy})(t) - \tfrac{1}{2}g(t)^2$$

$$v_y^2 = v_{oy}^2 - 2g(y - y_o)$$

EXAMPLE I



Given: v_A and θ

Find: Horizontal distance it travels and v_C.

Plan: Apply the kinematic relations in x- and y-directions.

Solution: Using $v_{Ax} = 10 \cos 30^o$ and $v_{Ay} = 10 \sin 30^o$

We can write $v_x = 10 \cos 30^o$

$v_y = 10 \sin 30^o - (9.81)\, t$

$x = (10 \cos 30^o)\, t$

$y = (10 \sin 30^o)\, t - ½\, (9.81)\, t^2$

Since y = 0 at C

$0 = (10 \sin 30^o)\, t - ½\, (9.81)\, t^2 \Rightarrow t = 0,$ 1.019 s

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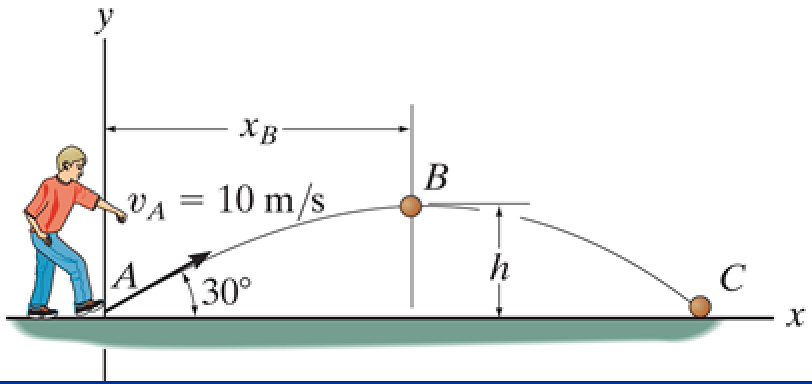
EXAMPLE I (continued)



Velocity components at C are;

$v_{Cx} = 10 \cos 30^o$

$= 8.66$ m/s →

$v_{Cy} = 10 \sin 30^o – (9.81)\,(1.019)$

$= -5$ m/s $= 5$ m/s ↓

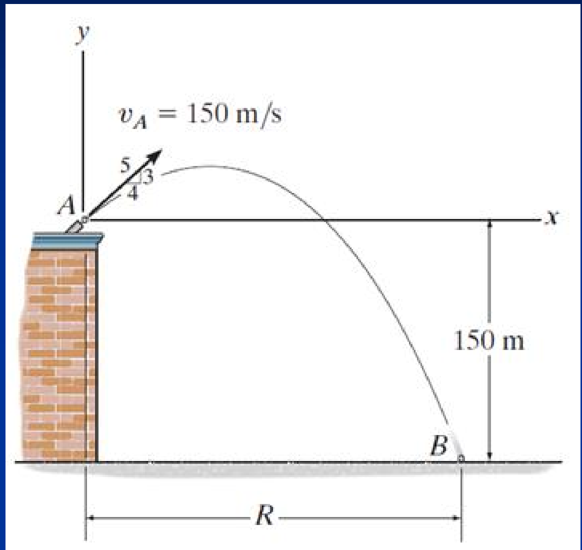
$$v_C = \sqrt{8.66^2 + (-5)^2} = 10 \text{ m/s}$$

Horizontal distance the ball travels is;

$x = (10 \cos 30^o)\ t$

$x = (10 \cos 30^o)\ 1.019 = 8.83$ m

EXAMPLE II

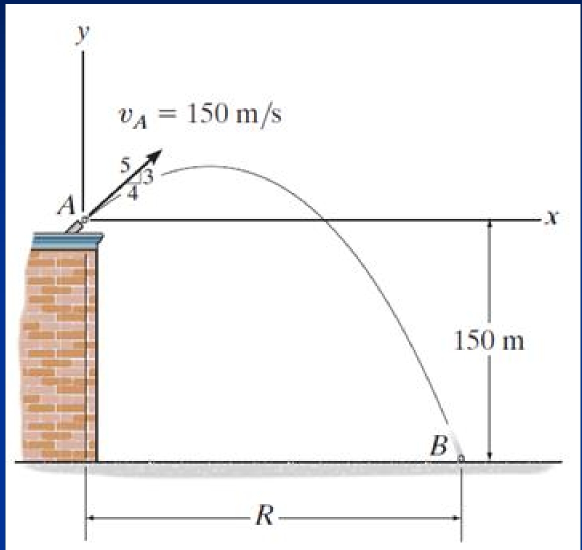


Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: How will you proceed?

EXAMPLE II



Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: Establish a fixed x, y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the kinematic relations in x- and y-directions.

EXAMPLE II (continued)

Solution:

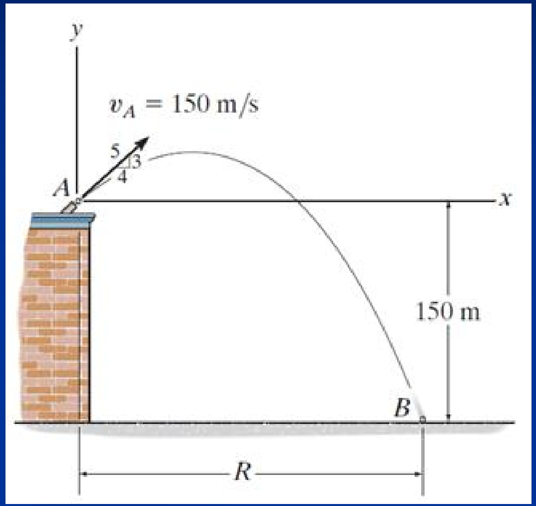
1) Place the coordinate system at point A.

Then, write the equation for horizontal motion.

$+\rightarrow \quad x_B = x_A + v_{Ax}\, t_{AB}$

where $x_B = R$, $x_A = 0$, $v_{Ax} = 150\ (4/5)$ m/s

Range, R, will be $R = 120\ t_{AB}$



2) Now write a vertical motion equation. Use the distance equation.

$+\uparrow \quad y_B = y_A + v_{Ay}\, t_{AB} - 0.5\, g\, t_{AB}^2$

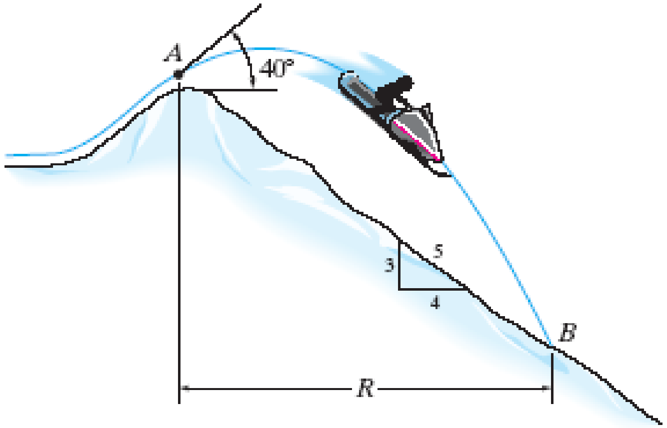
where $y_B = -150$, $y_A = 0$, and $v_{Ay} = 150(3/5)$ m/s

We get the following equation: $-150 = 90\ t_{AB} + 0.5\ (-9.81)\ t_{AB}^2$

Solving for t_{AB} first, $t_{AB} = 19.89$ s.

Then, $R = 120\ t_{AB} = 120\ (19.89) = 2387$ m

Example



Given: Snowmobile is going 15 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Solution:

First, place the coordinate system at point A. Then write the equation for horizontal motion.

$\xrightarrow{}+\quad x_B = x_A + v_{Ax} t_{AB}$ and $v_{Ax} = 15 \cos 40^\circ$ m/s

Now write a vertical motion equation. Use the distance equation.

$\uparrow+\quad y_B = y_A + v_{Ay} t_{AB} - 0.5 g_c t_{AB}^2 \quad v_{Ay} = 15 \sin 40^\circ$ m/s

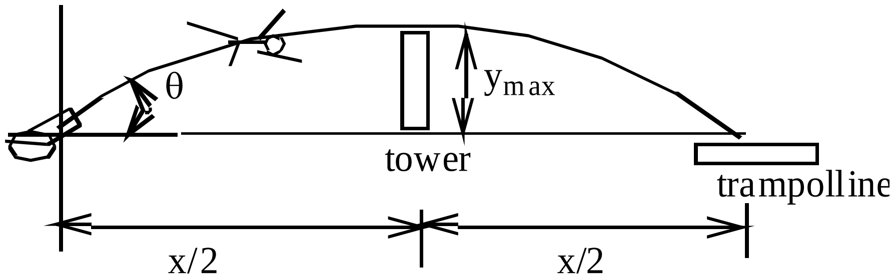
Note that $x_B = R$, $x_A = 0$, $y_B = -(3/4)R$, and $y_A = 0$.

Solving the two equations together (two unknowns) yields

$R = 19.0$ m $\quad t_{AB} = 2.48$ s

Example of Final Project

Engineering with Circus - The Human canon ball
In-Class Team Competition:
The circus is in town! A recently lay-off Boeing engineer is trying out to become a member of the human canon ball team in the circus. The first test he is asked to do is to figure out how to fly over a newly constructed water tower and land safely on a trampoline without injuring himself. Before he actually does the stunt, he decides to make a scale model to test and see if he will be able to make the jump. Please help him !
Problem:
Find the angle, the height and the distance the engineer need to travel to land safely on the trampoline.



Given:

-The distance between the tower and the trampoline is same as the distance between tower to canon.
-The height of the tower is 28cm.
-Initial velocity is 3.885m/s
-Gravitational acceleration is 9.8m/s

W. Wang

Additional Information:

Y direction	X direction
$V_{oy} = V_o \sin\theta$	$V_{ox} = V_o \cos\theta$
$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$	$V_{fx}{}^2 = V_{ox}{}^2$
$V_{fy} = V_{oy} + at$	$V_{fx} = V_{ox}$
$Y = V_{oy}\, t + 1/2\, at^2$	$X = V_{ox}\, t$
$Y = 1/2(V_{fy} + V_{oy})t$	$X = 1/2(V_{fx} + V_{ox})t$

Additional Problem Constraints:

- Can you think of any other problems that are not being addressed in the model?

Judging:

We will see who made the jump. The winners get 1% extra credit points. Turn in the correct calculation get another 2% extra credit.

Homework Assignment

Chapter 12: 10, 22, 24, 26, 28, 32, 37, 62, 71, 92, 98, 112, 120, 122, 144, 163, 175, 179

Due Next Wednesday !!!