ME 230 Kinematics and Dynamics

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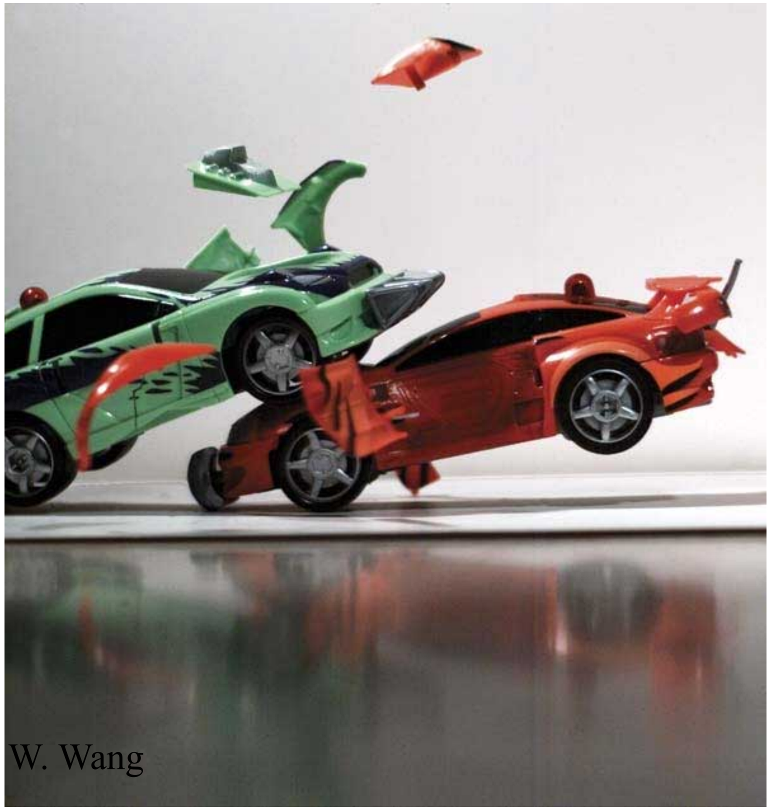
Lecture 8

- **Kinetics of a particle: Work and Energy** (Chapter 14)

- 14.1-14.3



Kinetics of a particle: Work & Energy

Chapter 14

Chapter objectives

• Develop the principle of work and energy and apply it in order to solve problems that involve force, velocity and displacement

• Problems that involve power and efficiency will be studied

• Concept of conservative force will be introduced and application of theorem of conservation of energy, in order to solve kinetic problems, will be described

Material covered

• **Kinetics of a particle: Work & Energy**

-The work of a force

- Principle of Work and Energy

- Principle of Work and Energy for a system of particles

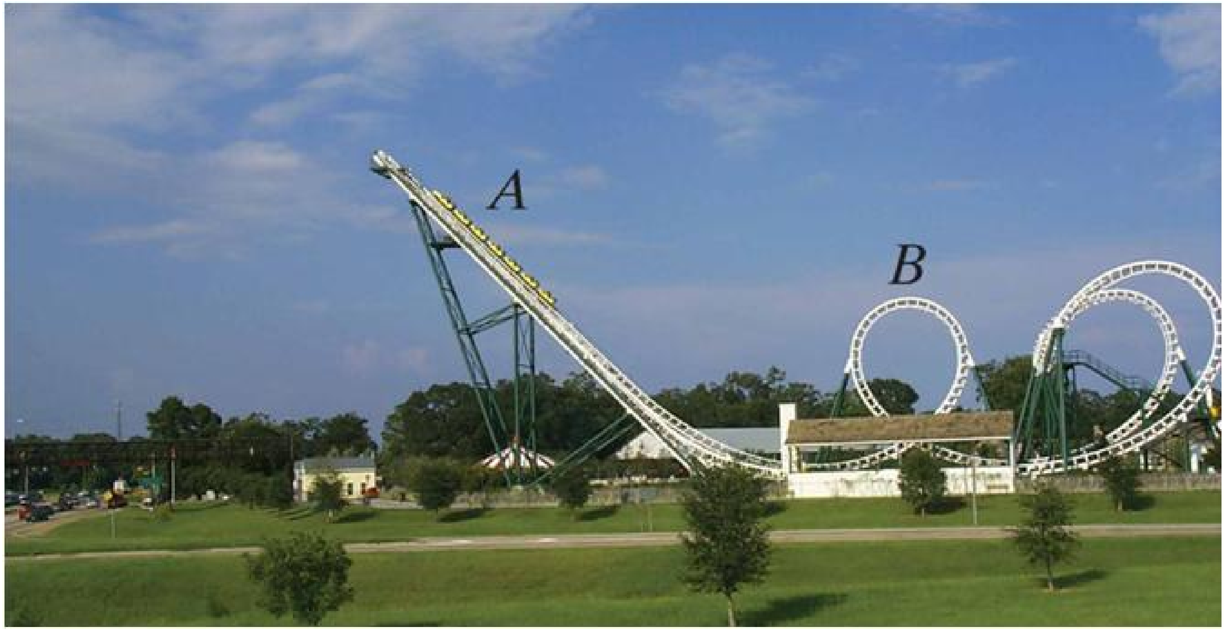
…**Next lecture…**Power and efficiency, conservative forces and potential energy, conservation of energy

Objectives

Students should be able to:

1. Calculate the work of a force
2. Apply the principle of work and energy to a particle or system of particles

Applications I



A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the “valleys” of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, ρ) to control the forces experienced by the passengers?

Applications II

Crash barrels are often used along roadways for crash protection. The barrels absorb the car's kinetic energy by deforming

If we know the typical velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

Work and Energy

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion ($\boldsymbol{F}$ = m$\boldsymbol{a}$) with respect to displacement

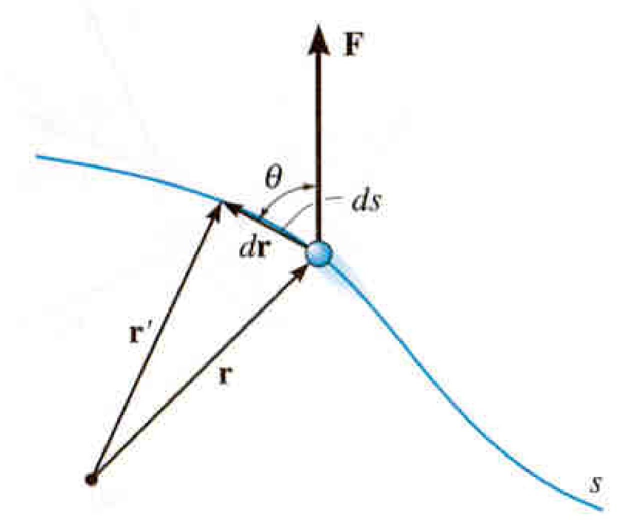
By substituting $a_t = v\,(dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy (F ds = mvdv)

This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power

To use this principle, we must first understand how to calculate the work of a force

Work of a force (14.1)

A force does work on a particle when the particle undergoes a displacement along the line of action of the force



Work is defined as the product of force and displacement components acting in the same direction. So, if the angle between the force and displacement vector is θ, the increment of work dU done by the force is;

$$dU = F\, ds \cos\theta$$

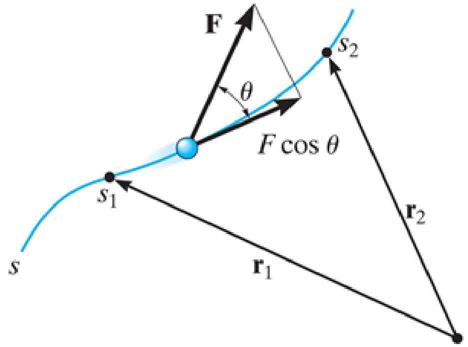
By using the definition of the dot product and integration, the total work can be written as:

$$U_{1\text{-}2} = \int_{r_1}^{r_2} \boldsymbol{F} \bullet d\boldsymbol{r}$$

Work of a force (14.1) continued…

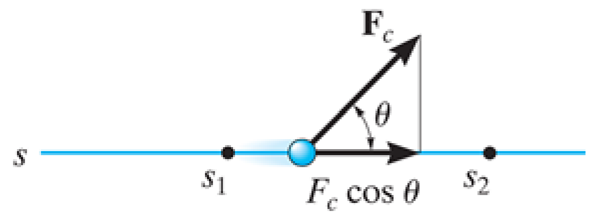
If ***F*** is a function of position (a common case) this becomes

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$



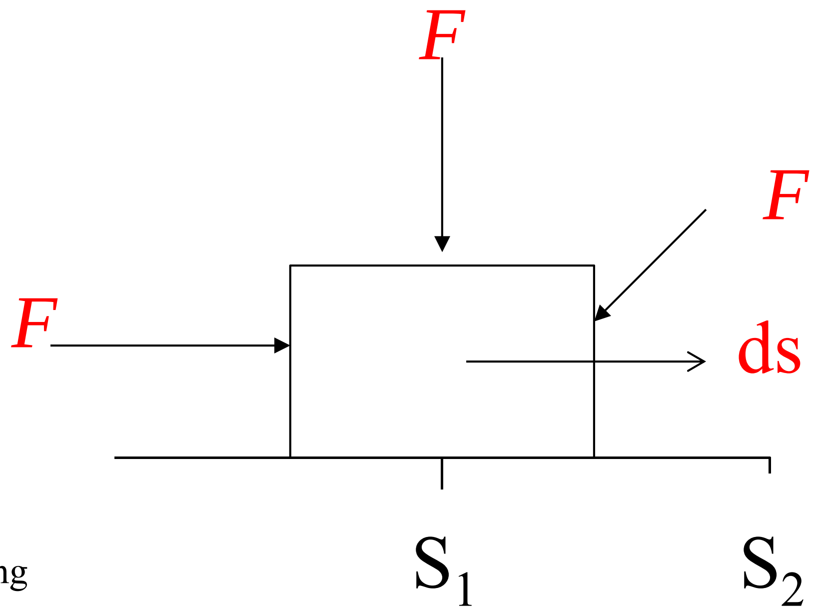
If both F and θ are constant ($F = F_c$), this equation further simplifies to

$$U_{1\text{-}2} = F_c \cos\theta \,(s_2 - s_1)$$



Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero

Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero

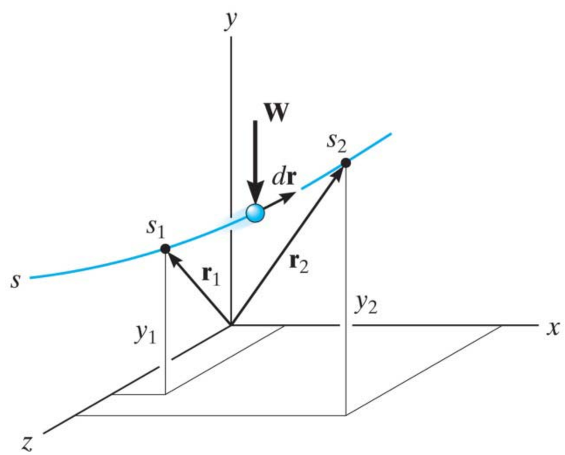


$$U_{1\text{-}2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

Work of a weight (negative work)

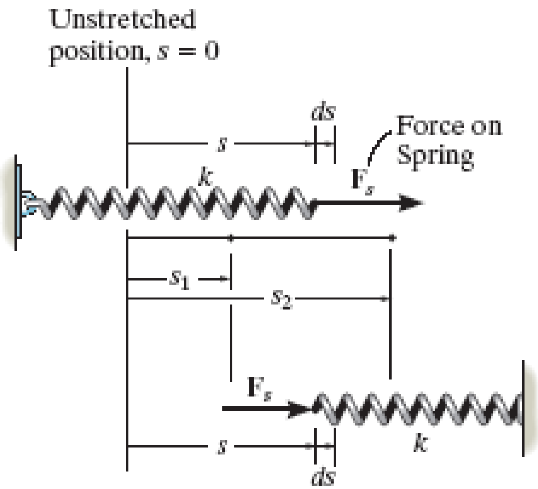
The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using;

$$U_{1\text{-}2} = \int_{y_1}^{y_2} -W\,dy = -W\,(y_2 - y_1) = -W\,\Delta y$$



The work of a weight is the product of the magnitude of the particle's weight and its vertical displacement. If Δy is upward, the work is negative since the weight force always acts downward

Work of a spring force



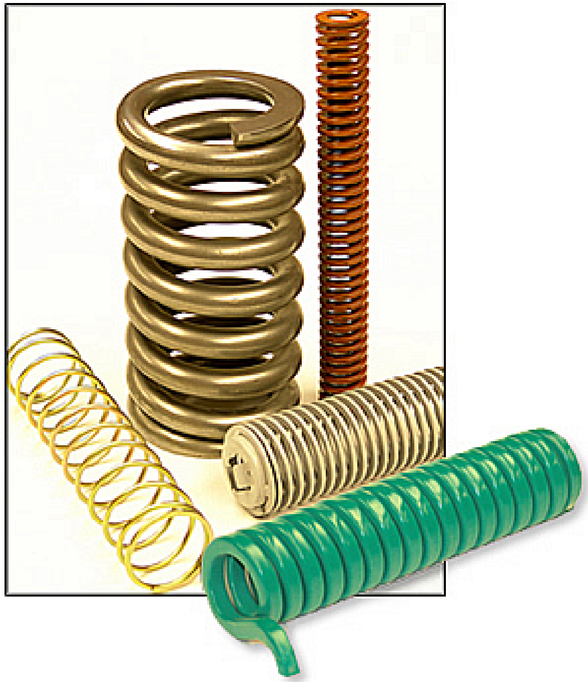
When stretched, a linear elastic spring develops a force of magnitude $F_s = ks$, where k is the spring stiffness and s is the displacement from the unstretched position

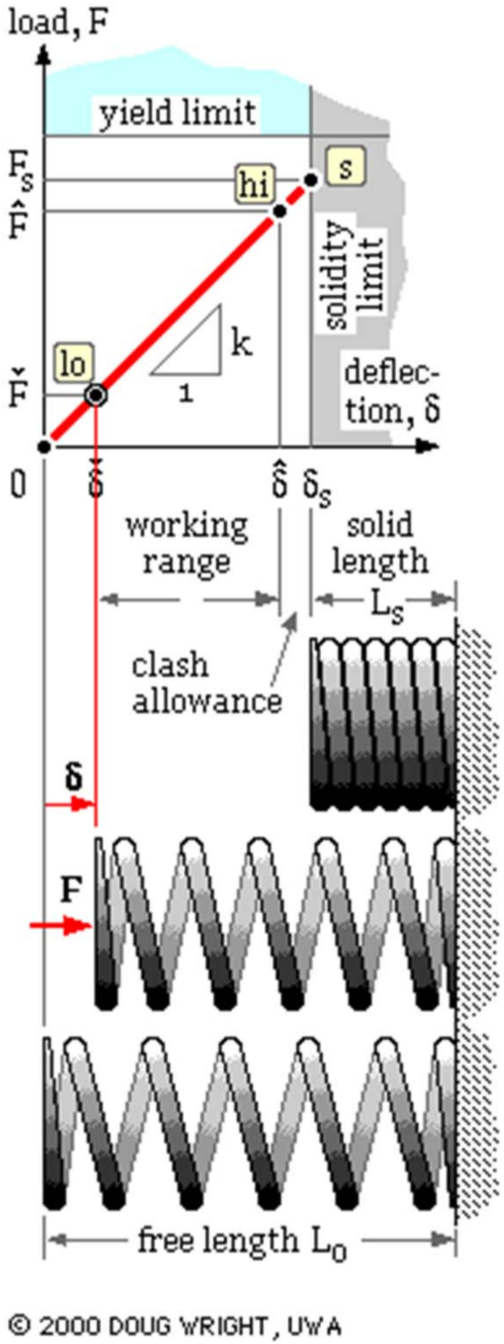
The work of the spring force moving from position s_1 to position s_2 is;

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k\, s\, ds = 0.5k(s_2)^2 - 0.5k(s_1)^2$$

If a particle is attached to the spring, the force F_s exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative or

$$U_{1\text{-}2} = -[\, 0.5k\,(s_2)^2 - 0.5k\,(s_1)^2 \,]$$





http://www.mech.uwa.edu.au/DANotes/springs/intro/springAnimation.gif

Spring forces

It is important to note the following about spring forces:

1. The equations shown are just for linear springs only! Recall that a linear spring develops a force according to $F = ks$ (essentially the equation of a line)
2. The work of a spring is not just spring force times distance at some point, i.e., $(ks_i)(s_i)$. Beware, this is a trap that students often fall into!

   Remember the work of the spring force moving from position s_1 to position s_2 is;

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k\, s\, ds = 0.5k(s_2)^2 - 0.5k(s_1)^2$$

3. Always double check the sign of the spring work after calculating it. It is positive work if the force put on the object by the spring and the movement are in the same direction

Principle of work and energy (14.2 & 14.3)

By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, the principle of work and energy can be written as

$$\sum U_{1\text{-}2} = 1/2m(v_2)^2 - 1/2m(v_1)^2 \quad \text{or} \quad T_1 + \sum U_{1\text{-}2} = T_2$$

$\sum U_{1\text{-}2}$ is the work done by all the forces acting on the particle as it moves from point 1 to point 2. Work can be either a positive or negative scalar

T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively. Thus, $T_1 = 1/2\ m\ (v_1)^2$ and $T_2 = 1/2\ m\ (v_2)^2$. The kinetic energy is always a positive scalar (velocity is squared!)

So, the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy

Principle of work and energy (continued…)

Note that the principle of work and energy ($T_1 + \sum U_{1\text{-}2} = T_2$) is not a vector equation! Each term results in a scalar value

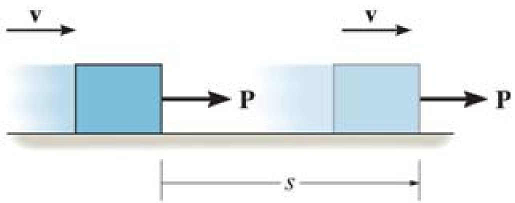
Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb

The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work

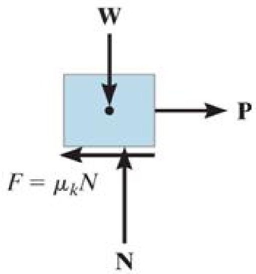
The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system

WORK OF FRICTION CAUSED BY SLIDING

The case of a body sliding over a rough surface merits special consideration.



Consider a block which is moving over a rough surface. If the applied force ***P*** just balances the resultant frictional force μ_k N, a constant velocity v would be maintained.



Kinetic friction because it's sliding

The principle of work and energy would be applied as

$$1/2m\,(v)^2 + P\,s - (\mu_k\,N)\,s = 1/2m\,(v)^2$$

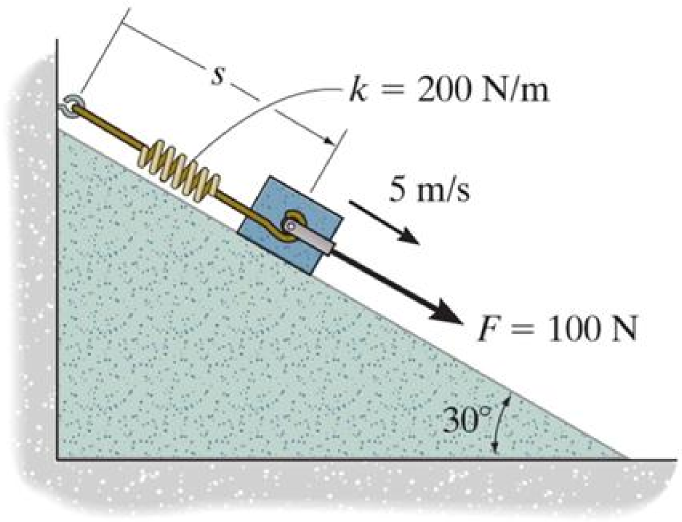
This equation is satisfied if $P = \mu_k N$. However, we know from experience that friction generates heat, a form of energy that does not seem to be accounted for in this equation. It can be shown that the work term $(\mu_k N)s$ represents both the external work of the friction force and the internal work that is converted into heat.

Remember!

Energy equation is useful for solving problems that involve force, velocity, and displacement.

Energy and work is a scalar, but still needs to find forces doing work on the object. Only the forces going in the direction of the motion will contribute any work!

EXAMPLE



Given: When s = 0.6 m, the spring is not stretched or compressed, and the 10 kg block, which is subjected to a force of 100 N, has a speed of 5 m/s down the smooth plane.

Find: The distance s when the block stops.

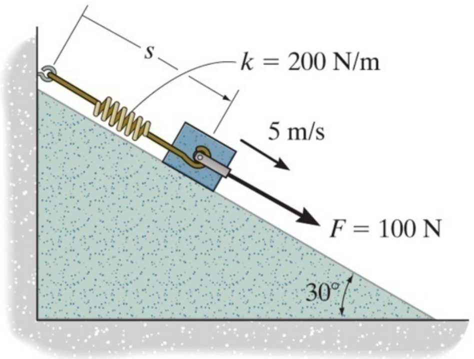
Plan: Since this problem involves forces, velocity and displacement, apply the principle of work and energy to determine s.

EXAMPLE
(continued)

Solution:

Apply the principle of work and energy between position 1 (s_1 = 0.6 m) and position 2 (s_2). Note that the normal force (***N***) does no work since it is always perpendicular to the displacement.

$$T_1 + \sum U_{1\text{-}2} = T_2$$



There is work done by three different forces;

1) work of a the force F =100 N;

$$U_F = 100\,(s_2 - s_1) = 100\,(s_2 - 0.6)$$

2) work of the block weight;

$$U_W = 10\,(9.81)\,(s_2 - s_1)\sin 30° = 49.05\,(s_2 - 0.6)$$

3) and, work of the spring force.

$$U_S = -\,0.5\,(200)\,(s_2 - 0.6)^2 = -100\,(s_2 - 0.6)^2$$

EXAMPLE
(continued)

The work and energy equation will be

$$T_1 + \sum U_{1\text{-}2} = T_2$$

$$0.5\ (10) + 100(s_2 - 0.6) + 49.05(s_2 - 0.6) - 100(s_2 - 0.6)^2 = 0$$

$$\Rightarrow\ 125 + 149.05(s_2 - 0.6) - 100(s_2 - 0.6)^2 = 0$$

Solving for $(s_2 - 0.6)$,

$$(s_2 - 0.6) = \{-149.05 \pm (149.05^2 - 4\times(-100)\times 125)^{0.5}\}\ /\ 2(-100)$$

Selecting the positive root, indicating a positive spring deflection,

$(s_2 - 0.6) = 2.09$ m

Therefore, $s_2 = 2.69$ m

CONCEPT QUIZ

1. A spring with an un-stretched length of 5 in expands from a length of 2 in to a length of 4 in. The work done on the spring is __________ in·lb .

   A) $-[0.5\ k(4\ in)^2 - 0.5\ k(2\ in)^2]$ B) $0.5\ k\ (2\ in)^2$

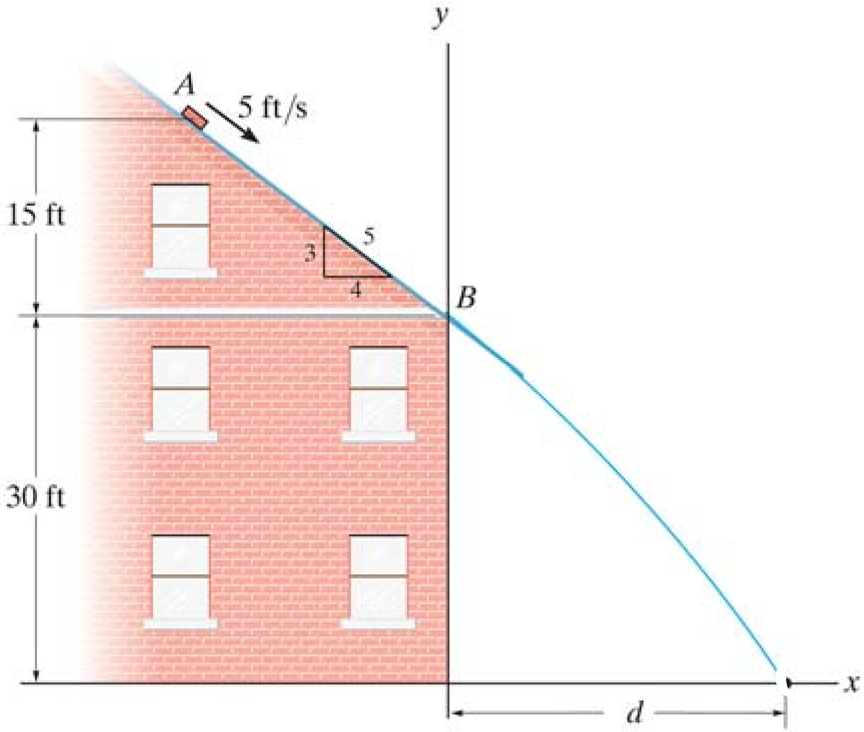
   C) $-[0.5\ k(3\ in)^2 - 0.5\ k(1\ in)^2]$ D) $0.5\ k(3\ in)^2 - 0.5\ k(1\ in)^2$

2. If a spring force is $F = 5\ s^3$ N/m and the spring is compressed by $s = 0.5$ m, the work done on a particle attached to the spring will be

   A) $0.625\ N \cdot m$ B) $-0.625\ N \cdot m$

   C) $0.0781\ N \cdot m$ D) $-0.0781\ N \cdot m$

Example



Given: The 2 lb brick slides down a smooth roof, with v_A=5 ft/s.

Find: The speed at B, the distance d from the wall to where the brick strikes the ground, and its speed at C.

Plan: 1) Apply the principle of work and energy to the brick, and determine the speeds at B and C.
2) Apply the kinematic relations in x and y-directions.

Example (continued)

Solution:

scalar

1) Apply the principle of work and energy

$$\Sigma T_A + \Sigma U_{A\text{-}B} = \Sigma T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)5^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2$$

Solving for the unknown velocity yields $v_B = 31.48$ ft/s

Similarly, apply the work and energy principle between A and C

$$\Sigma T_A + \Sigma U_{A\text{-}C} = \Sigma T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)5^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2$$

$v_C = 54.1$ ft/s

Theory: Projectile Motion (12.6)

- projectile motion is a special case of erratic motion usually modeled using Cartesian vectors
- it is a special case, because projectiles move in the presence of a constant gravitational acceleration in one direction (up/down), and (usually) negligible acceleration in another (horizontally); projectiles are modeled as particles
- we solve the problem using Cartesian coordinates, in two parts

Vertical motion	Horizontal motion
Y direction	X direction
$V_{oy} = V_o \sin\theta$	$V_{ox} = V_o \cos\theta$
$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$	$V_{fx}{}^2 = V_{ox}{}^2$
$V_{fy} = V_{oy} + at$	$V_{fx} = V_{ox}$
$Y = V_{oy}\, t + 1/2\, at^2$	$X = V_{ox}\, t$
$Y = 1/2(V_{fy} + V_{oy})t$	$X = 1/2(V_{fx} + V_{ox})t$

Since we know V_c we can find V_{cy} fron V_{cx} and since V_{Ax} is known then t can be solved

Example (continued)

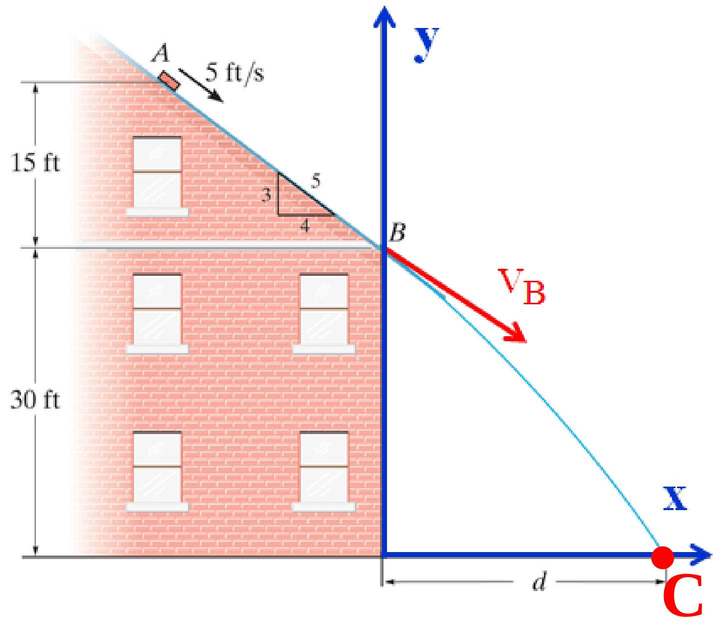
2) Apply the kinematic relations in x and y-directions:

Equation for horizontal motion

$(+\rightarrow)\;\; x_C = x_B + v_{Bx}\, t_{BC}$

$d = 0 + 31.48\,(4/5)\, t_{BC}$

$\Rightarrow d = 6.996\, t_{BC}$



Equation for vertical motion

$(+\uparrow)\;\; y_C = y_B + v_{By}\, t_{BC} - 0.5\, g\, t_{BC}^2$

$\Rightarrow\;\; -30 = 0 + (-31.48)(3/5)\, t_{BC} - 0.5\,(32.2)\, t_{BC}^2$

Solving for the positive t_{BC} yields $t_{BC} = 0.899$ s.

$\Rightarrow d = 6.996\, t_{BC} = 6.996\,(0.899) = 22.6$ ft

Theory: Projectile Motion (12.6)

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Vertical motion

Y direction

$V_{oy} = V_o \sin\theta$

$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$

$V_{fy} = V_{oy} + at$

$Y = V_{oy}\, t + 1/2\, at^2$

$Y = 1/2(V_{fy} + V_{oy})t$

Since we know V_c we can find V_{cy} fron V_{cx} and since V_{Ax} is known then t can be solved

Horizontal motion

X direction

$v_C = 54.1$ ft/s

$V_{ox} = V_o \cos\theta$

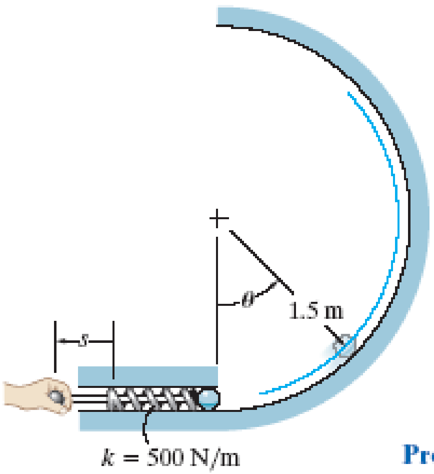
$V_{fx}{}^2 = V_{ox}{}^2$

$V_{fx} = V_{ox}$

$X = V_{ox}\, t$

$X = 1/2(V_{fx} + V_{ox})t$

Example



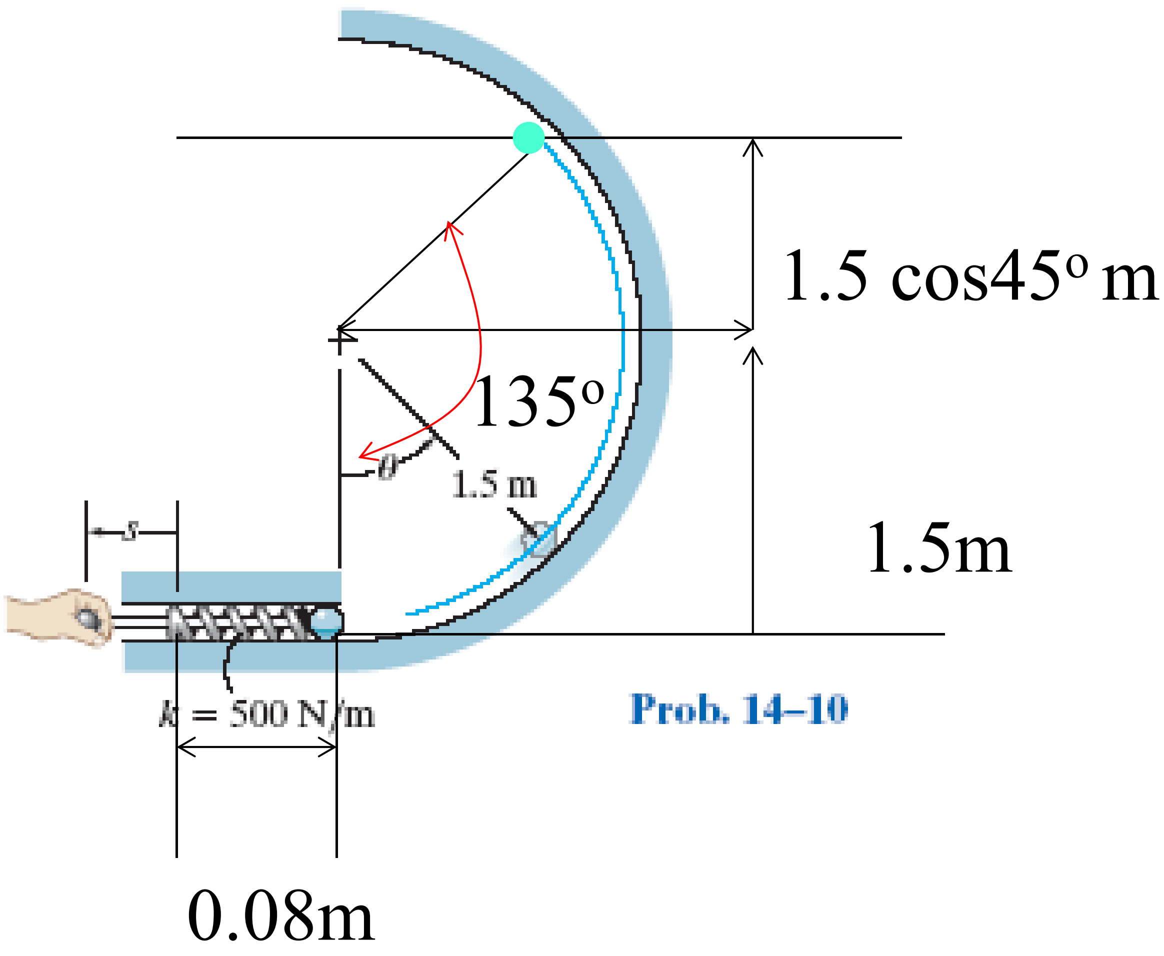
Prob. 14–10

Given: A 0.5 kg ball of negligible size is fired up a vertical track of radius 1.5 m using a spring plunger with k = 500 N/m. The plunger keeps the spring compressed 0.08 m when s = 0

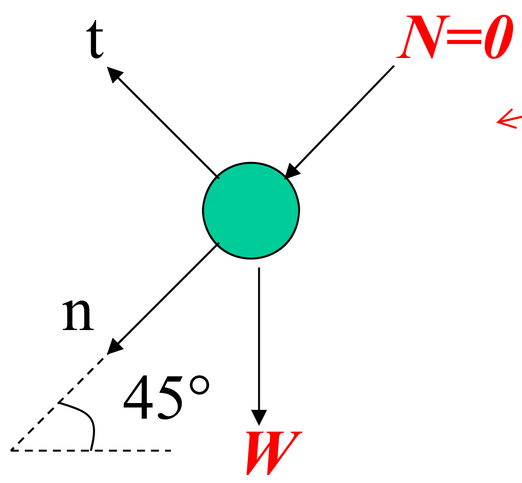
Find: The distance s the plunger must be pulled back and released so the ball will begin to leave the track (N=0) when $\theta = 135°$

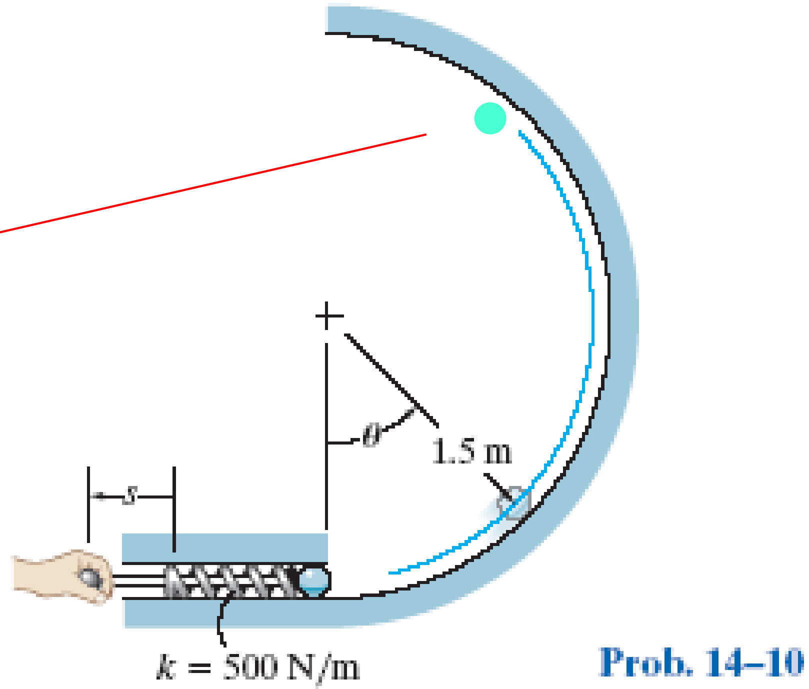
Plan:
1) Draw the FBD of the ball at $\theta = 135°$.
2) Apply the equation of motion in the n-direction to determine the speed of the ball when it leaves the track.
3) Apply the principle of work and energy to determine s

Look at what's given



FBD





Normal tangential coordinate is easiest

What are the unknown?

$$T_1 + \sum U_{1\text{-}2} = T_2$$

$$0.5m\,(v_1)^2 - W\,\Delta y - (0.5k(s_2)^2 - 0.5k\,(s_1)^2) = 0.5m\,(v_2)^2$$

$$s_1 = s + 0.08\text{ m},\ s_2 = 0.08\text{ m}$$

$$\Delta y = 1.5 + 1.5\sin 45° = 2.5607\text{ m}$$

s and V_2 are unknown

How to find V_2?

Equation of motion at θ=135°

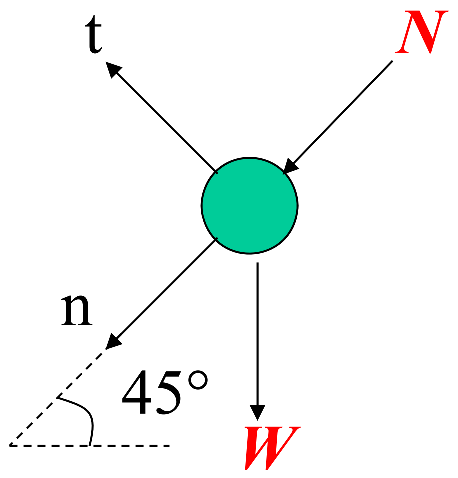
Why?

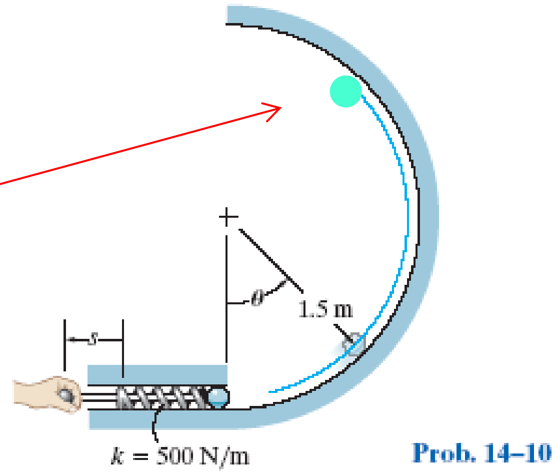
$$ma_n = mv^2/\rho = \sum F_n$$

Example (continued)

Solution:

1) Draw the FBD of the ball at $\theta = 135°$





The weight (***W***) acts downward through the center of the ball. The normal force exerted by the track is perpendicular to the surface. The friction force between the ball and the track has no component in the n-direction

2) Apply the equation of motion in the n-direction. Since the ball leaves the track at $\theta = 135°$, set $N = 0$

$$\Rightarrow \ \swarrow + \ \sum F_n = m a_n = m\,(v^2/\rho) \quad \Rightarrow \quad W \cos 45° = m\,(v^2/\rho)$$

$$\Rightarrow (0.5)(9.81) \cos 45° = (0.5/1.5) v^2 \quad \Rightarrow \quad v = 3.2257 \text{ m/s}$$

Example (continued)

3) Apply the principle of work and energy between position 1 ($\theta = 0$) and position 2 ($\theta = 135°$). Note that the normal force (***N***) does no work since it is always perpendicular to the displacement direction. (Students: Draw a FBD to confirm the work forces)

$$T_1 + \sum U_{1\text{-}2} = T_2$$

$$0.5m\,(v_1)^2 - W\,\Delta y - (0.5k(s_2)^2 - 0.5k\,(s_1)^2) = 0.5m\,(v_2)^2$$

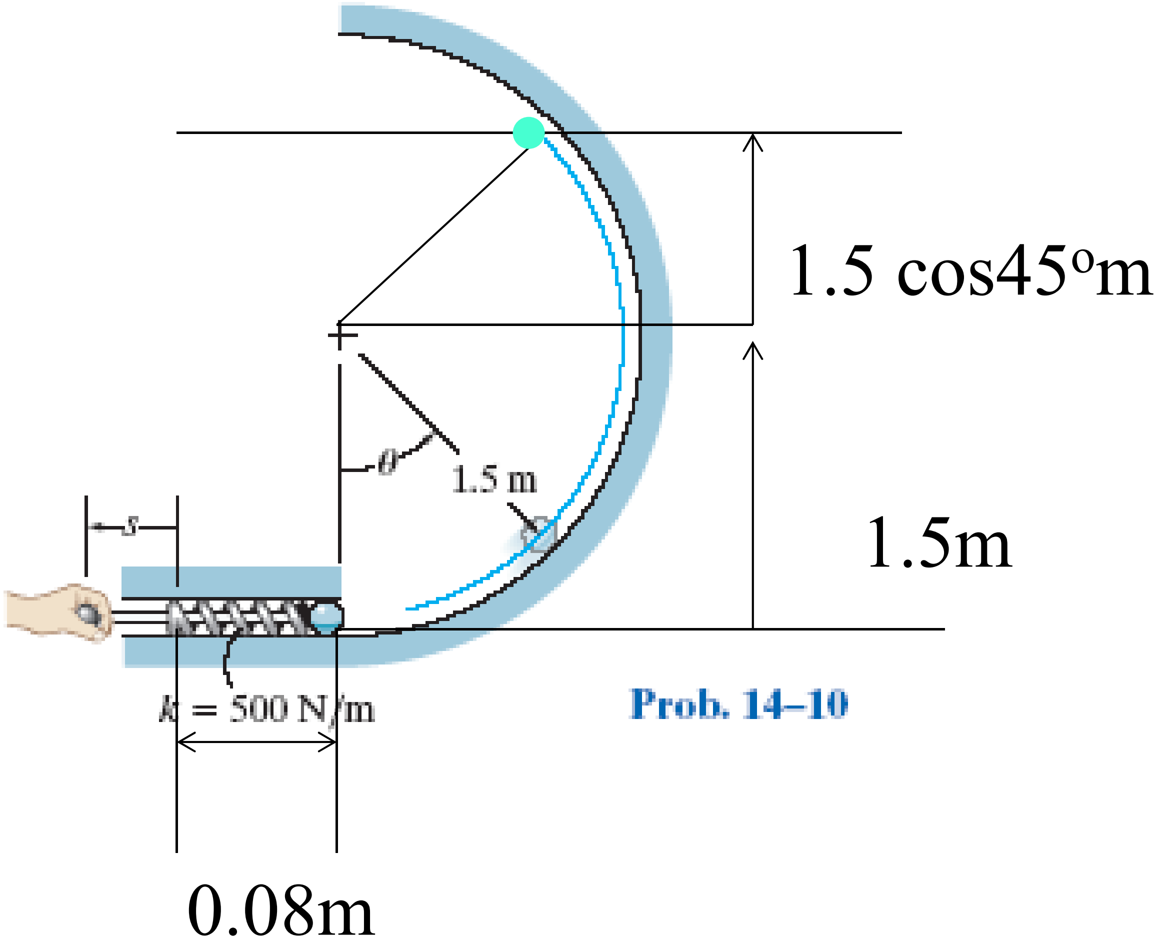
and $\quad v_1 = 0,\ v_2 = 3.2257$ m/s

$s_1 = s + 0.08$ m, $s_2 = 0.08$ m

$\Delta y = 1.5 + 1.5 \sin 45° = 2.5607$ m

$$\Rightarrow 0 - (0.5)(9.81)(2.5607) - [0.5(500)(0.08)^2 - 0.5(500)(s + 0.08)^2] = 0.5(0.5)(3.2257)^2$$

$$\Rightarrow s = 0.179 \text{ m} = 179 \text{ mm}$$


1.5 cos45ºm
1.5 m
1.5m
k = 500 N/m
Prob. 14–10
0.08m

Homework Assignment

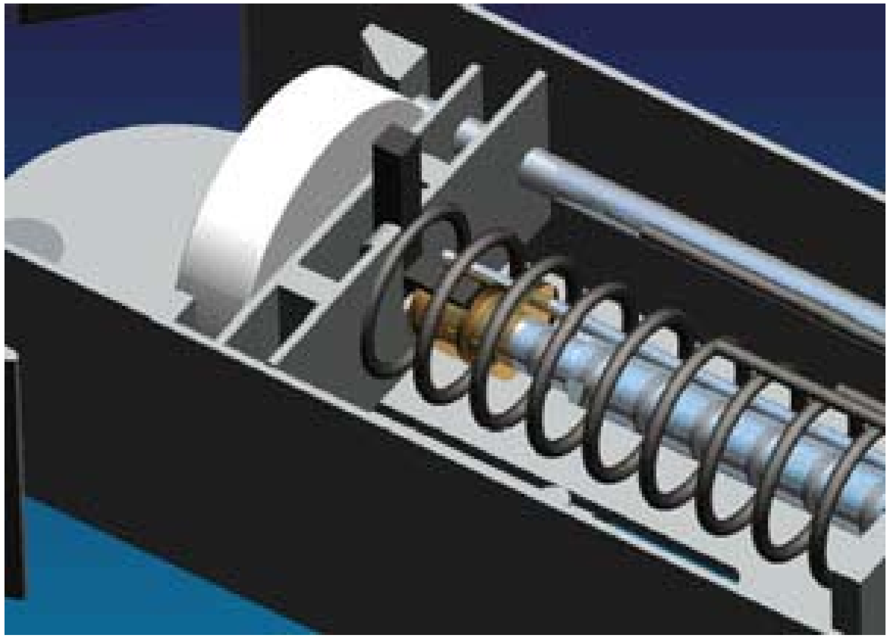
Chapter13-59, 65, 66, 75, 91, 93, 97, 107
Chapter14-3,11,14, 21

Due Wednesday !!!

Lecture 9

- **Kinetics of a particle: Work and Energy** (Chapter 14)

- 14.4-14.6



Kinetics of a particle: Work & Energy

Chapter 14

Chapter objectives

- Develop the principle of work and energy and apply it in order to solve problems that involve force, velocity and displacement
- Problems that involve power and efficiency will be studied
- Concept of conservative force will be introduced and application of theorem of conservation of energy, in order to solve kinetic problems, will be described

Material covered

- **Kinetics of a particle**

- Power and efficiency

- Conservative forces and potential energy

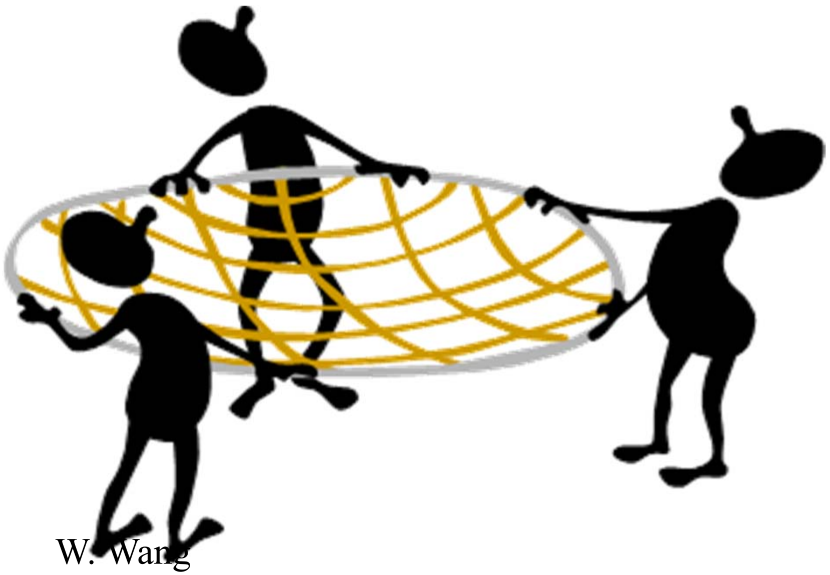
- Conservation of energy

…Next lecture…MIDTERM REVIEW

Today's Objectives

Students should be able to:

1. Determine the power generated by a machine, engine, or motor
2. Calculate the mechanical efficiency of a machine
3. Understand the concept of conservative forces and determine the potential energy of such forces
4. Apply the principle of conservation of energy

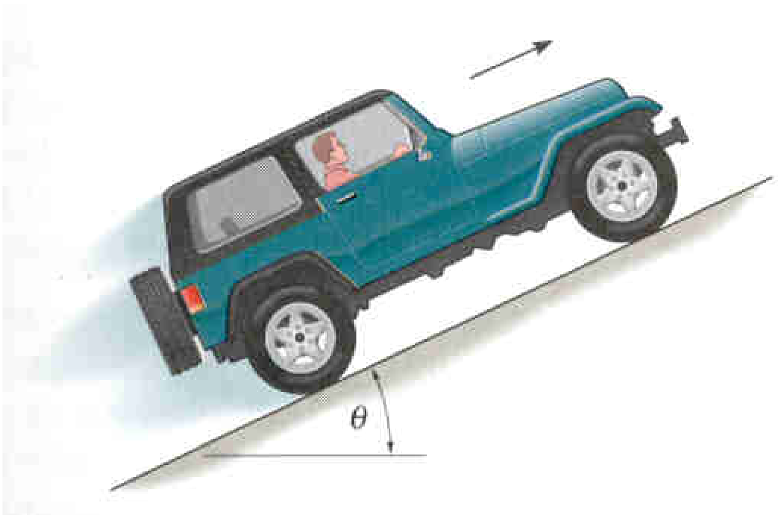


Applications of power and efficiency I

Engines and motors are often rated in terms of their power output. The power requirements of the motor lifting this elevator depend on the vertical force ***F*** that acts on the elevator, causing it to move upwards

Given the desired lift velocity for the elevator, how can we determine the power requirement of the motor?

Applications of power and efficiency II



The speed at which a vehicle can climb a hill depends in part on the power output of the engine and the angle of inclination of the hill

For a given angle, how can we determine the speed of this jeep, knowing the power transmitted by the engine to the wheels?

Power and efficiency (14.4)

Power is defined as the amount of work performed per unit of time

If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as

$$P = dU/dt$$

Since the work can be expressed as $dU = \boldsymbol{F} \cdot d\boldsymbol{r}$, the power can be written

$$P = dU/dt = (\boldsymbol{F} \cdot d\boldsymbol{r})/dt = \boldsymbol{F} \cdot (d\boldsymbol{r}/dt) = \boldsymbol{F} \cdot \boldsymbol{v}$$

Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction

Power

Using scalar notation, power can be written

$$P = \boldsymbol{F} \bullet \boldsymbol{v} = F\, v \cos\theta$$

where θ is the angle between the force and velocity vectors

So if the velocity of a body acted on by a force ***F*** is known, the power can be determined by calculating the dot product or by multiplying force and velocity components

The unit of power in the SI system is the watt (W) where

$$1\ \text{W} = 1\ \text{J/s} = 1\ (\text{N} \cdot \text{m})/\text{s}$$

In the FPS system, power is usually expressed in units of horsepower (hp) where

$$1\ \text{hp} = 550\ (\text{ft} \cdot \text{lb})/\text{s} = 746\ \text{W}$$

Efficiency

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power) or

$$\varepsilon = (\text{power output})/(\text{power input})$$

If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or

$$\varepsilon = (\text{energy output})/(\text{energy input})$$

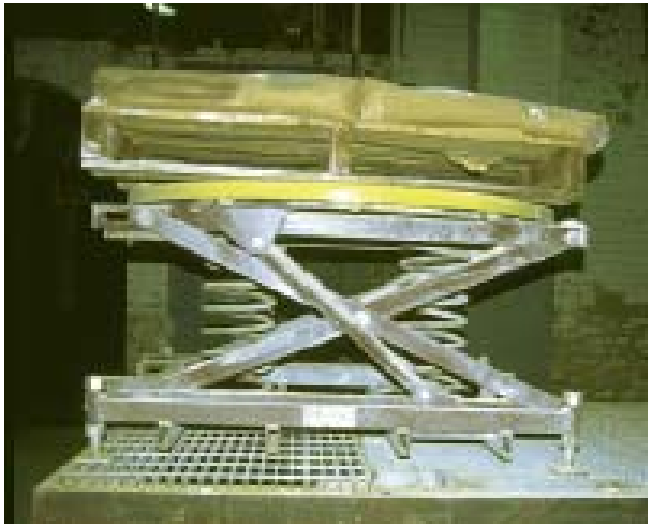
Machines will always have frictional forces. Since frictional forces dissipate energy, additional power will be required to overcome these forces. Consequently, **the efficiency of a machine is always less than 1**

Procedure of analysis

- Find the resultant external force acting on the body causing its motion. It may be necessary to draw a free-body diagram
- Determine the velocity of the point on the body at which the force is applied. Energy methods or the equation of motion and appropriate kinematic relations, may be necessary
- Multiply the force magnitude by the component of velocity acting in the direction of ***F*** to determine the power supplied to the body ($P = F\ v \cos\theta$)
- In some cases, power may be found by calculating the work done per unit of time ($P = dU/dt$)
- If the mechanical efficiency of a machine is known, either the power input or output can be determined

Conservative forces and potential energy

APPLICATIONS



The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

As each sack is removed, the platform will *rise* slightly since some of the potential energy within the springs will be transformed into an increase in gravitational potential energy of the remaining sacks.

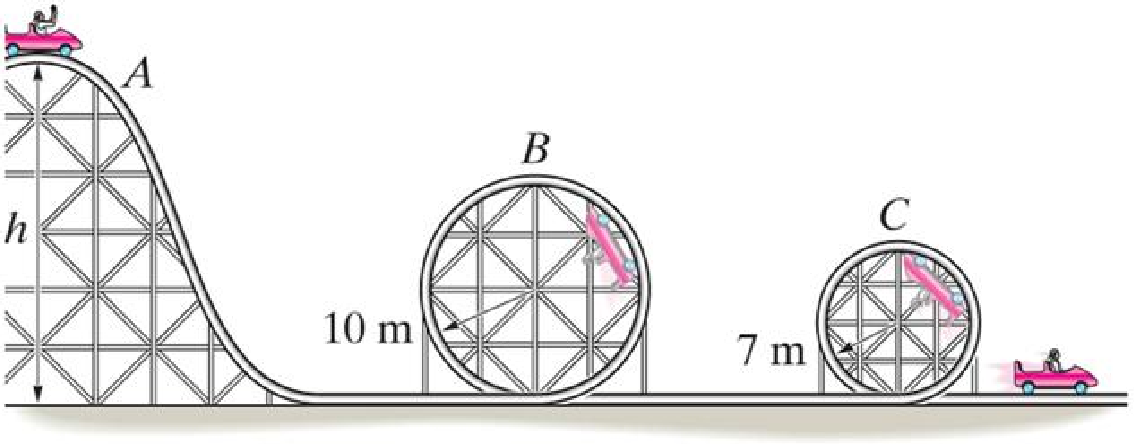
If the sacks weigh 100 lb and the equivalent spring constant is k = 500 lb/ft, what is the energy stored in the springs?

APPLICATIONS (continued)

The young woman pulls the water balloon launcher back, stretching each of the four elastic cords.

If we know the unstretched length and stiffness of each cord, can we estimate the maximum height and the maximum range of the water balloon when it is released from the current position? Would we need to know any other information?

APPLICATIONS (continued)



The roller coaster is released from rest at the top of the hill A. As the coaster moves down the hill, potential energy is transformed into kinetic energy.

What is the velocity of the coaster when it is at B and C?

Also, how can we determine the minimum height of hill A so that the car travels around both inside loops without leaving the track?

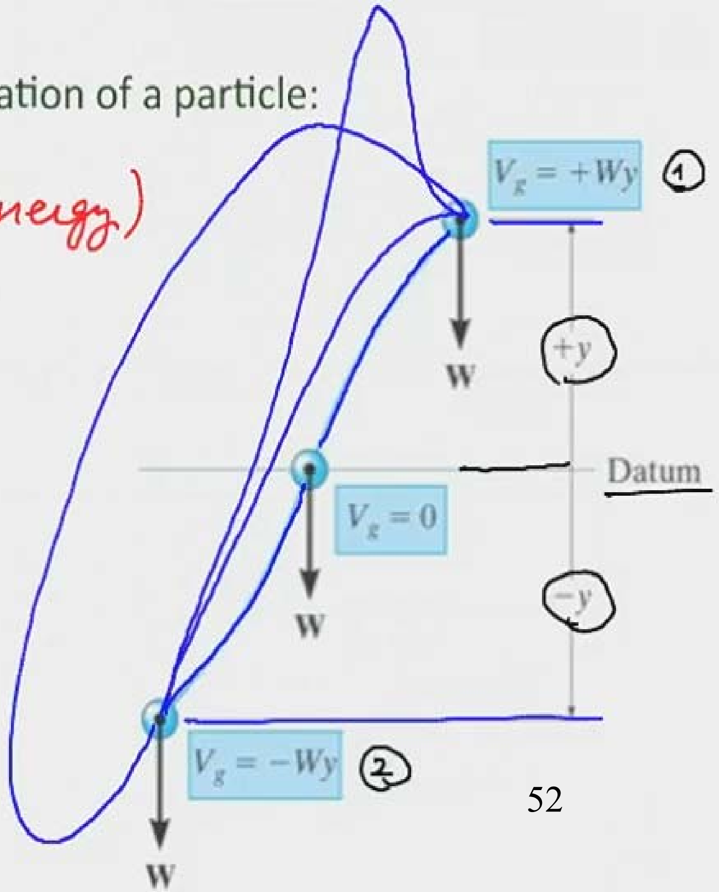
Theory: Conservative Forces and Potential En. (14.5)

- a force is called "conservative" if the work of that force is independent of path and instead depends only upon the starting and ending points on the path
- weight of a particle and spring force are both conservative forces
- potential energy is the amount of work a conservative force can do when it moves from a given position in the datum (the capacity to do work…)
- gravitational potential energy is related to the vertical location of a particle:

$$V_g = Wy \quad (V\text{: potential energy})$$

- elastic potential energy is related to spring deformation:

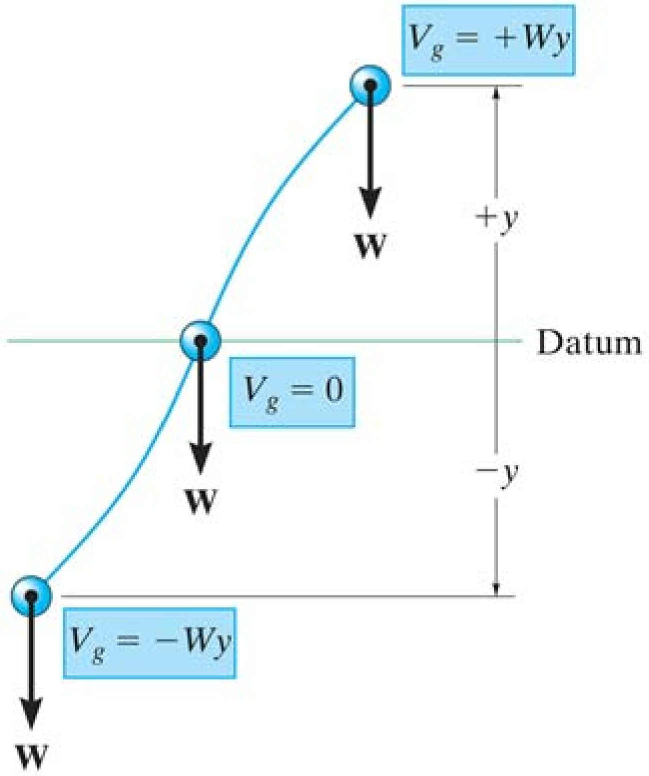
$$V_e = \frac{1}{2} k s^2$$



Frictional force is a non-conservative force because it depends on the path.

Potential energy due to gravity

The potential function (formula) for a gravitational force, e.g., weight ($W = mg$), is the force multiplied by its elevation from a datum (a fixed starting point). The datum can be defined at any convenient location

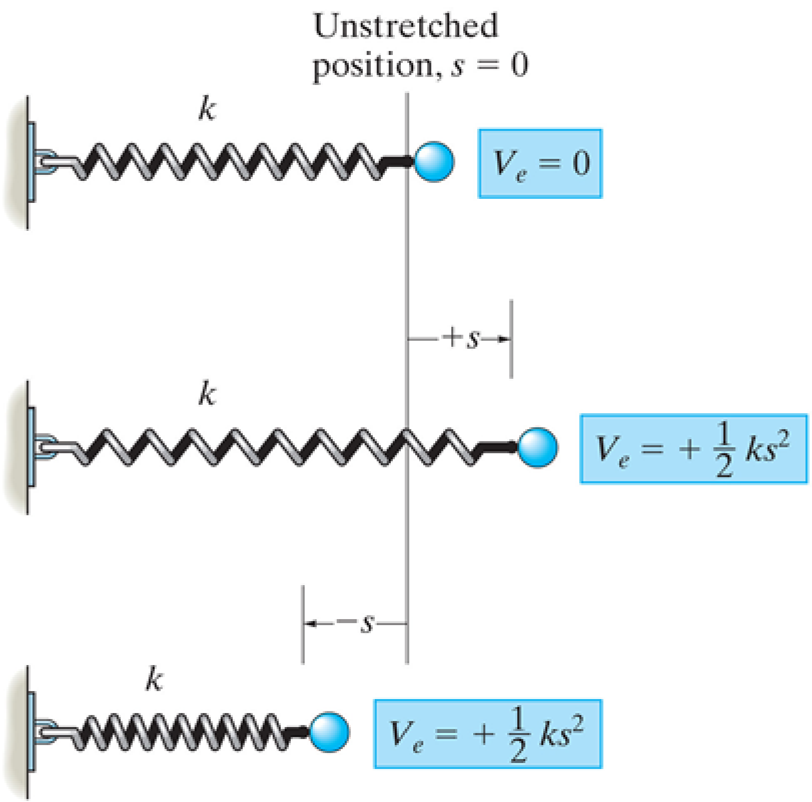


$$V_g = \pm W\, y$$

V_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum

Elastic potential energy

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula



V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2}ks^2$$

Notice that the potential function V_e always yields positive energy

Theory: The Potential Function

- the potential function is the algebraic sum of the potential energies:

$$V = V_g + V_e$$

- work of a conservative force can therefore be characterized as a change in the potential function:

$$\underbrace{U_{1-2}}_{\text{work}} = \underbrace{V_1 - V_2}_{\text{change in potential function}}$$

$$T_1 + U_{1-2} = T_2$$

U_{1-2}^{cons} $\Downarrow$ V_1, V_2

$U_{1-2}^{non-cons}$

$\Rightarrow$ work - energy

Theory: Conservation of Energy (14.6)

- recall that nonconservative forces are those that are, well, not conservative; a complete energy expression would then be: $U_{1-2} = U_{1-2}^{cons} + U_{1-2}^{non-cons}$

  $\to V_1 , V_2$

$$T_1 + V_1 + U_{1-2}^{non-cons} = T_2 + V_2$$

- if only conservative forces (gravity, springs) are present in a problem, then:

  energy is conserved

$$\boxed{T_1 + V_1 = T_2 + V_2}$$

- this equation illustrates the trading of one type of energy for another for a particle moving along a path and acted upon only by conservative forces

Conservation of energy

(Section 14.6)

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 \quad = \text{Constant}$$

T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_2 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = \frac{1}{2} mv^2$

An Aside: The Potential Function and ∇ Operator

- we can derive a more general expression for the potential function:

work

$$\begin{cases} dU = \bar{F}\cdot d\bar{r} & \begin{cases} d\bar{r} = (dx\,\bar{i} + dy\,\bar{j} + dz\,\bar{k}) \\ \bar{F} = (F_x\,\bar{i} + F_y\,\bar{j} + F_z\,\bar{k}) \end{cases} \end{cases}$$

$$dU = F_x\,dx + F_y\,dy + F_z\,dz$$

potential fcn.

$$\underbrace{dU = V_1 - V_2}_{} = V(x,y,z) - V(x+dx,\, y+dy,\, z+dz)$$

$$= -dV(x,y,z)$$

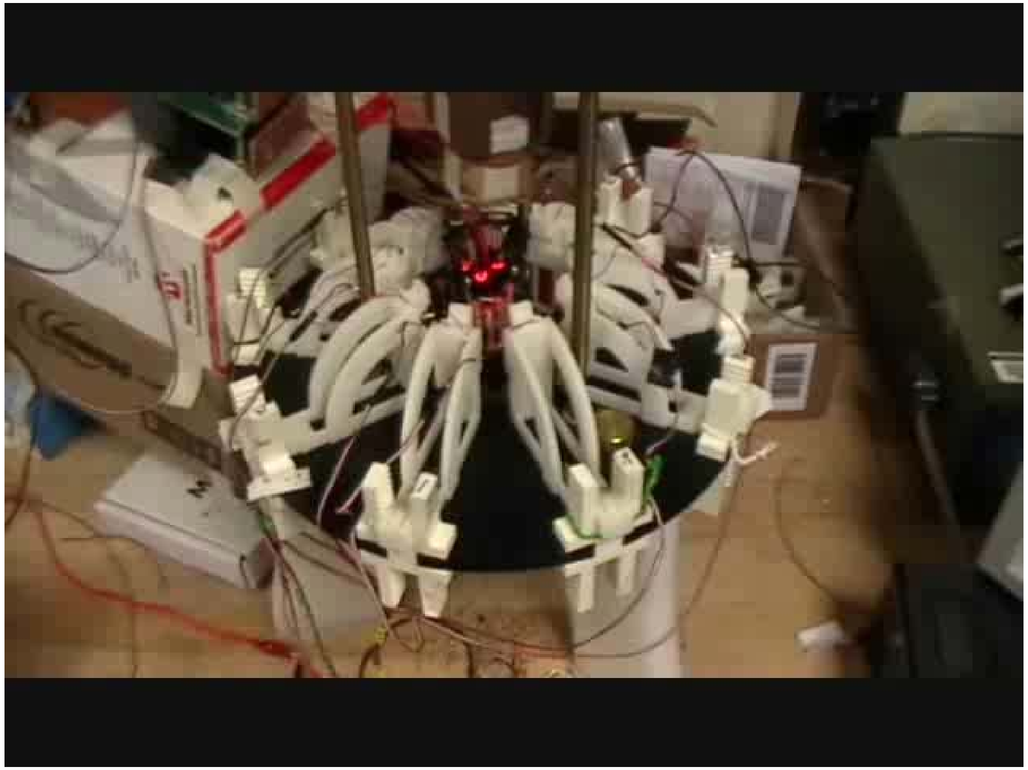
$$= -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$$

~ OR ~ $\boxed{\bar{F} = -\nabla V}$

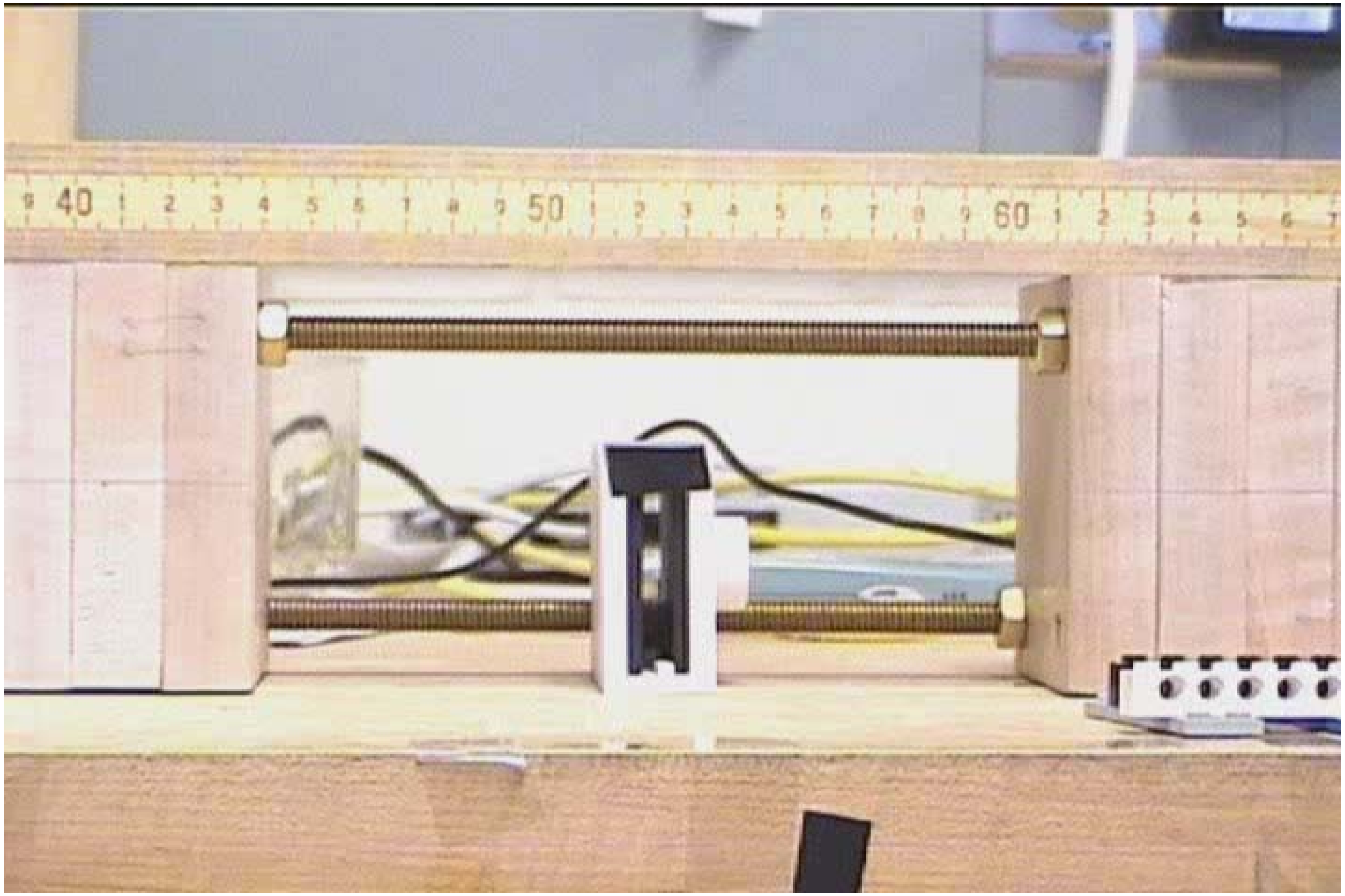
$$\nabla = \left(\frac{\partial}{\partial x}\bar{i} + \frac{\partial}{\partial y}\bar{j} + \frac{\partial}{\partial z}\bar{k}\right)$$

Example: Three Phase Diamagnetic Levitation Motor

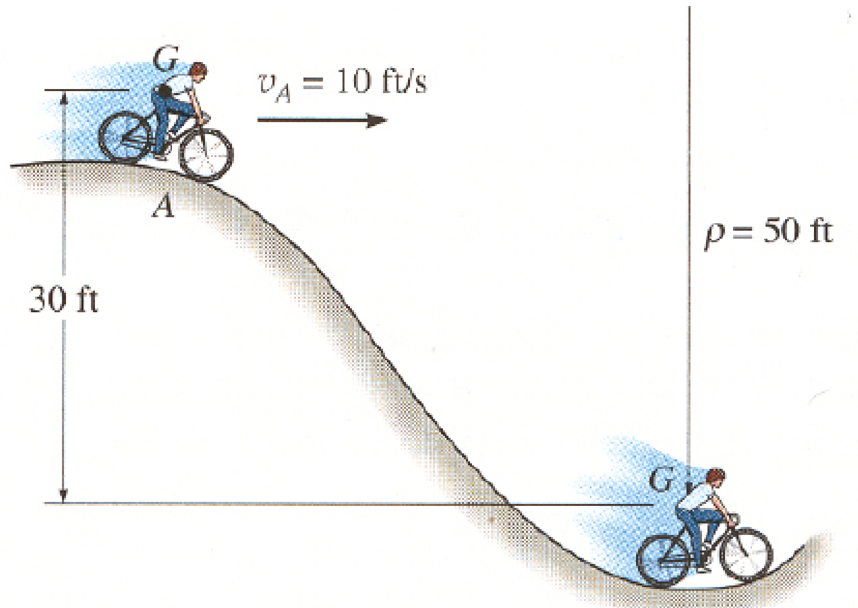


Magnetic restoring force ~ spring

Horizontal levitation



Example



(a)

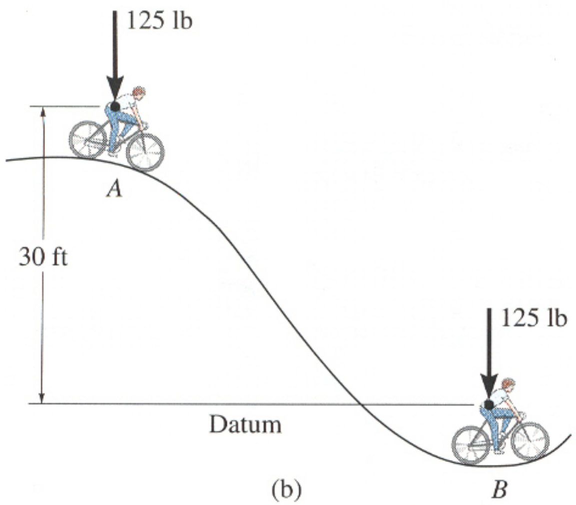
Given: The girl and bicycle weigh 125 lbs. She moves from point A to B.

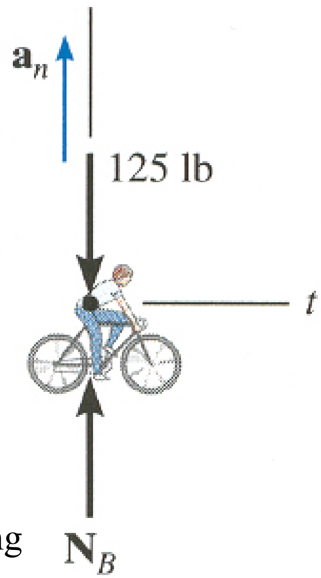
Find: The velocity and the normal force at B if the velocity at A is 10 ft/s and she stops pedaling at A.

Plan: Note that only kinetic energy and potential energy due to gravity (V_g) are involved. Determine the velocity at B using the conservation of energy equation and then apply equilibrium equations to find the normal force.

Example (continued)

Solution:





Placing the datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{125}{32.2}\right)(10)^2 + 125(30) = \frac{1}{2}\left(\frac{125}{32.2}\right)v_B^2$$

$$V_B = 45.1 \ \text{ft}/\text{s}$$

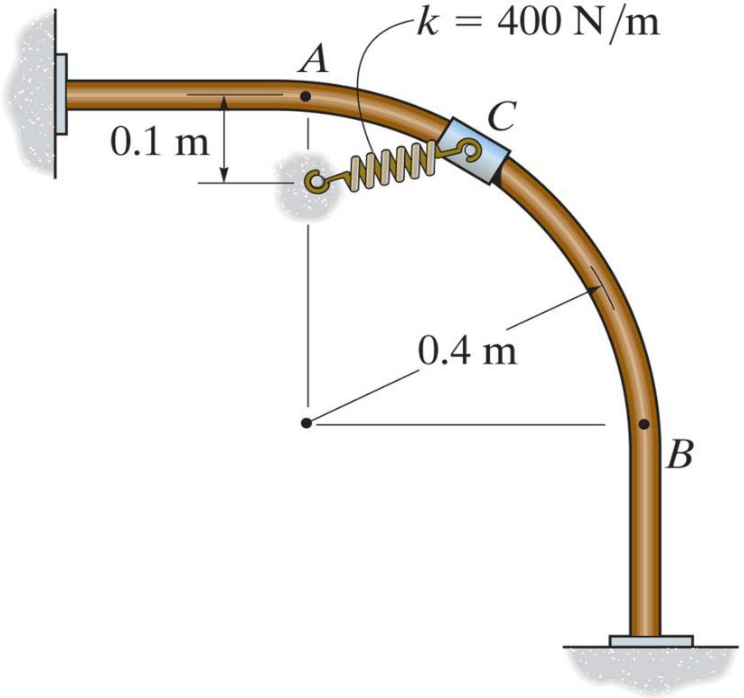
Equation of motion applied at B:

$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

$$N_B - 125 = \frac{125}{32.2}\,\frac{(45.1)^2}{50}$$

$$N_B = 283\,\text{lb}$$

EXAMPLE



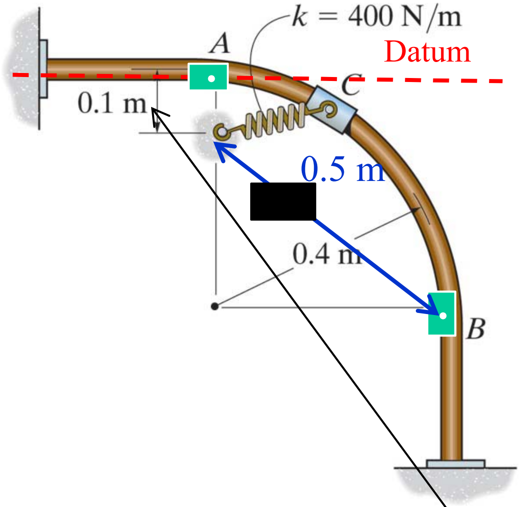
Given: The 4 kg collar, C, has a velocity of 2 m/s at A. The spring constant is 400 N/m. The unstretched length of the spring is 0.2 m.

Find: The velocity of the collar at B.

Plan: Apply the conservation of energy equation between A and B. Set the gravitational potential energy datum at point A or point B (in this example, choose point A—why?).

EXAMPLE (continued)

Solution:



Note that the potential energy at *B* has two parts.

$$V_B = (V_B)_e + (V_B)_g$$

$$V_B = 0.5\,(400)\,(0.5 - 0.2)^2 - 4\,(9.81)\,0.4$$

The kinetic energy at B is

$$T_B = 0.5\,(4)\,v_B^2$$

Similarly, the potential and kinetic energies at A will be

$$V_A = 0.5\,(400)\,(0.1 - 0.2)^2, \quad T_A = 0.5\,(4)\,2^2$$

The energy conservation equation becomes $T_A + V_A = T_B + V_B$.

$$[\,0.5(400)\,(0.5 - 0.2)^2 - 4(9.81)0.4\,] + 0.5\,(4)\,v_B^2$$

$$= [0.5\,(400)\,(0.1 - 0.2)^2\,] + 0.5\,(4)\,2^2$$

$$\Rightarrow v_B = 1.96 \text{ m/s}$$

CONCEPT QUIZ

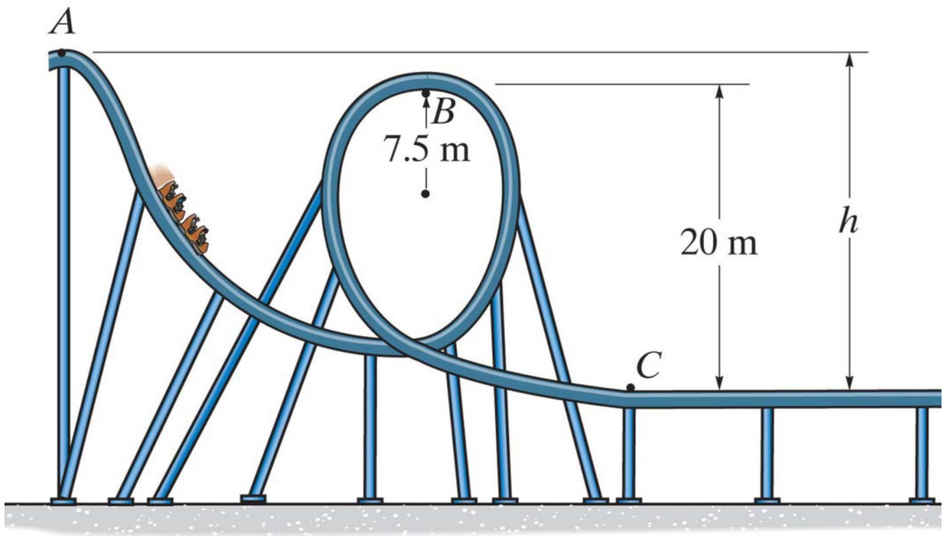
1. If the work done by a conservative force on a particle as it moves between two positions is –10 ft·lb, the change in its potential energy is _______

   A) 0 ft·lb.
   B) -10 ft·lb.
   C) +10 ft·lb.
   D) None of the above.

2. Recall that the work of a spring is $U_{1\text{-}2} = -\frac{1}{2} k(s_2^2 - s_1^2)$ and can be either positive or negative. The potential energy of a spring is $V = \frac{1}{2} ks^2$. Its value is __________

   A) always negative.
   B) either positive or negative.
   C) always positive.
   D) an imaginary number!

Example



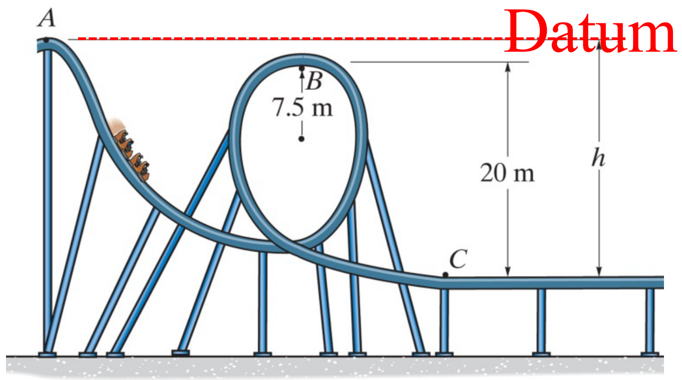
Given: The 800 kg roller coaster car is released from rest at A.

Find: The minimum height, h, of Point A so that the car travels around inside loop at B without leaving the track. Also find the velocity of the car at C for this height, h, of A.

Plan: Note that only kinetic energy and potential energy due to gravity are involved. Determine the velocity at B using the equation of motion and then apply the conservation of energy equation to find minimum height h .

GROUP PROBLEM SOLVING (continued)

Solution:



1) Placing the datum at A:

$$T_A + V_A = T_B + V_B$$

Two unknowns!!

$$\Rightarrow 0.5\ (800)\ 0^2 + 0$$
$$= 0.5\ (800)\ (v_B)^2 - 800(9.81)\ (h - 20) \qquad (1)$$

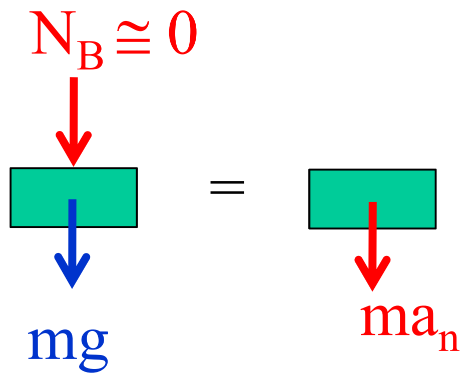
2) Find the required velocity of the coaster at B so it doesn't leave the track.

Equation of motion applied at B:

$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

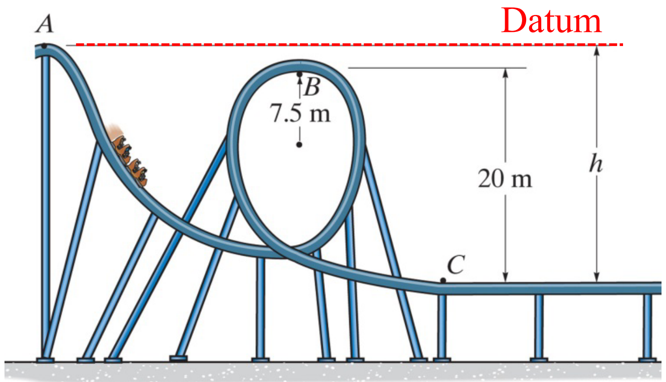
$$800\ (9.81) = 800\frac{(v_B)^2}{7.5}$$

$$\Rightarrow \ v_B = 8.578 \text{ m/s}$$



Example (continued)

Now using the energy conservation, eq. (1), the minimum h can be determined.



$$0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(8.578)^2 - 800(9.81)\,(h - 20)$$

$\Rightarrow$ $h = 23.75$ m

3) Find the velocity at C applying the energy conservation.

$$T_A + V_A = T_C + V_C$$

$$\Rightarrow 0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(v_C)^2 - 800(9.81)\,(23.75)$$

$$\Rightarrow V_C = 21.6 \text{ m/s}$$

Theory: Projectile Motion (12.6)

- projectile motion is a special case of erratic motion usually modeled using Cartesian vectors
- it is a special case, because projectiles move in the presence of a constant gravitational acceleration in one direction (up/down), and (usually) negligible acceleration in another (horizontally); projectiles are modeled as particles
- we solve the problem using Cartesian coordinates, in two parts

Vertical motion

Y direction

$V_{oy} = V_o \sin\theta$

$$\boxed{V_{fy}^{2} = V_{oy}^{2} + 2aY}$$

$V_{fy} = V_{oy} + at$

$Y = V_{oy}\, t + 1/2\, at^2$

$Y = 1/2(V_{fy} + V_{oy})t$

Horizontal motion

X direction

$V_{ox} = V_o \cos\theta$

$V_{fx}^{2} = V_{ox}^{2}$

$V_{fx} = V_{ox}$

$X = V_{ox}\, t$

$X = 1/2(V_{fx} + V_{ox})t$

Homework Assignment

Chapter14-71, 77, 79, 91, 92 (work on these problems <- it will appear in 1st midterm)
Chapter15-6,11, 21,42, 54,57

Due next Wednesday !!!

Chapter reviews

Chapter 12: pages 101-105

Chapter 13: pages 166-167

Chapter 14: pages 217-219

Chapter 15: pages 295-297

Chapter 16: pages 391-393

Chapter 17: pages 452-453

Chapter 18: pages 490-493

Chapter 19: pages 531-533

Book chapter reviews give you a good but brief idea about each chapter…

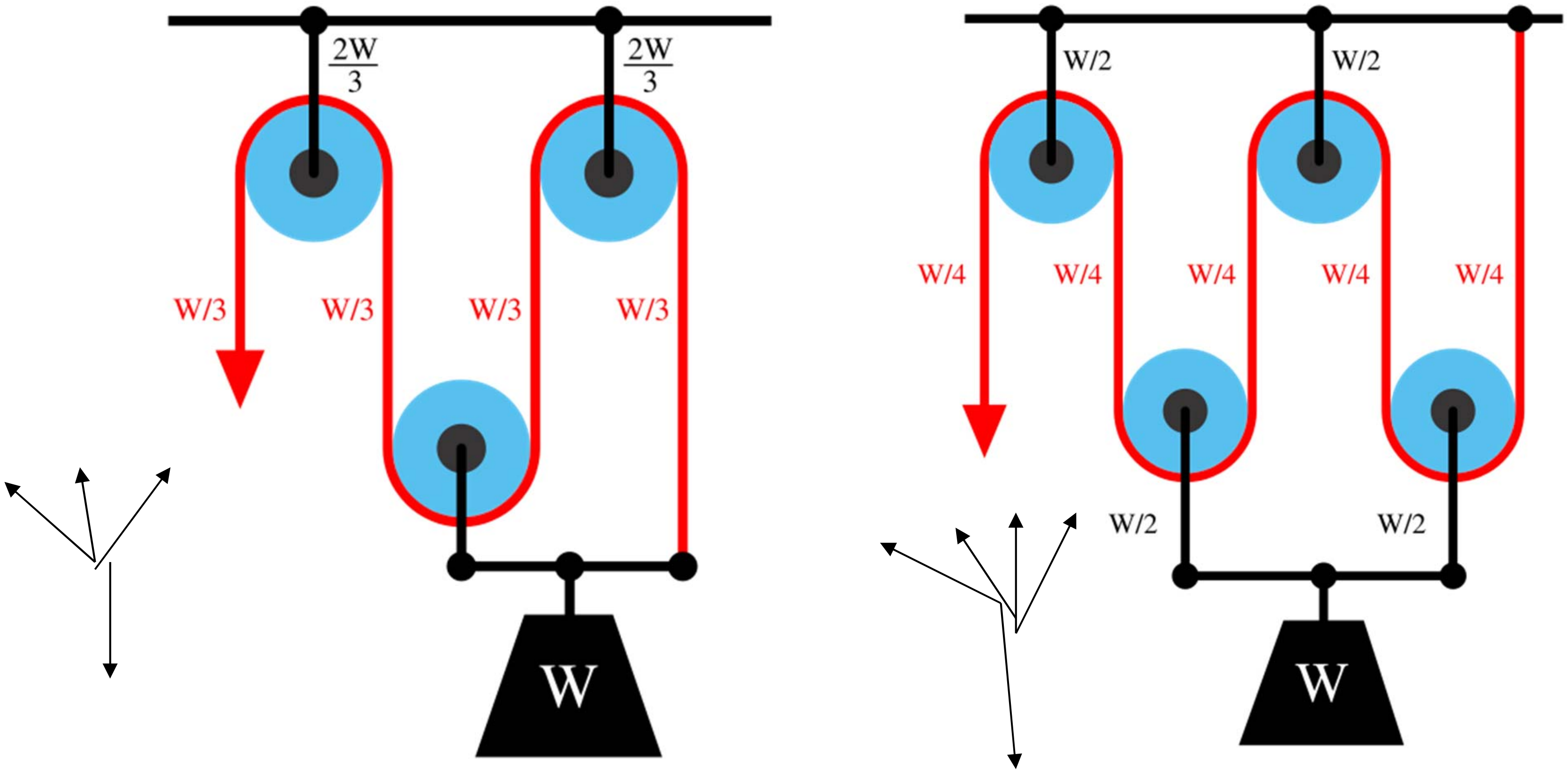
General exam rules

- Midterm exam will consist of 4 questions. **3** questions must be solved. The 4th question will be a bonus question.
- Sub-questions may include statements of theoretical definitions
- Midterm exam counts for **25%** of the total mark
- Come on time. Since the lecture theatre will be used for another class at 1:30, there will be no extra time
- **Calculators with memory are not allowed**

Midterm Exam

- Exam is on Friday!
- Exam will cover materials from Chapter 12, 13 and 14
- No wilress electronics
- Be careful of UNITS – Practice
- Solving equations symbolically
- Free body diagrams – Resolve forces
- Explain why are you doing things
- Full marks will be awarded for FULLY explained solutio
- Do not use random formulae but ONLY the relevant ones
- READ THE QUESTIONS CAREFULLY

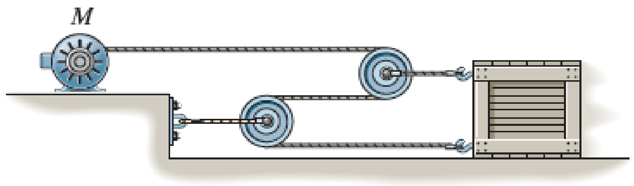
Fix and movable pulleys



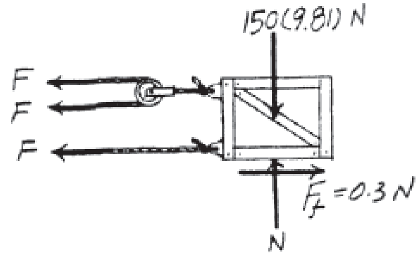
Look at direction of supporting forces elative to load before summing

14–50.

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



SOLUTION



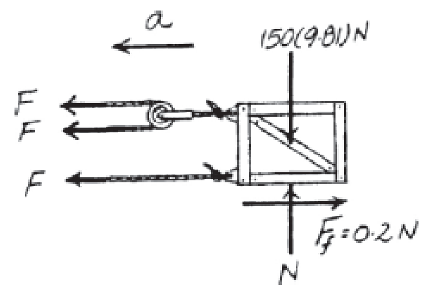
(a)

Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867\text{ s}$$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150\,(0) \qquad N = 1471.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2\,(1471.5) - 3\left(8t^2 + 20\right) = 150\,(-a)$$

$$a = \left(0.160t^2 - 1.562\right)\ \text{m/s}^2$$

Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867\text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

$$v = 1.7045\text{ m/s}$$

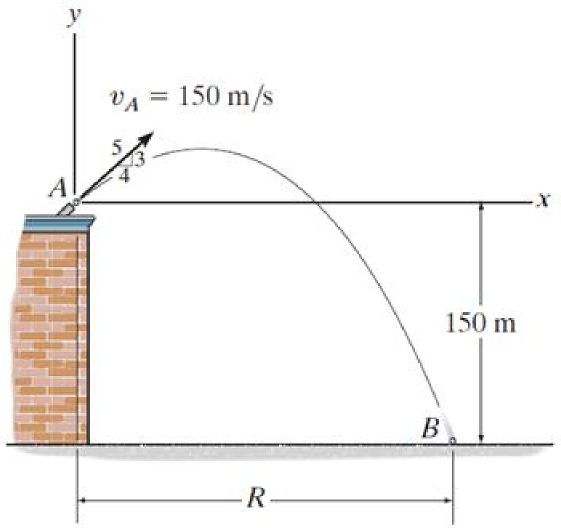
Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 3\,(220)\,(1.7045) = 1124.97\text{ W} = 1.12\text{ kW} \qquad \textbf{Ans.}$$

Static friction for tipping, wheel turning and coefficient used when thing just start to move.

Kinetic friction when thing is sliding.

EXAMPLE II

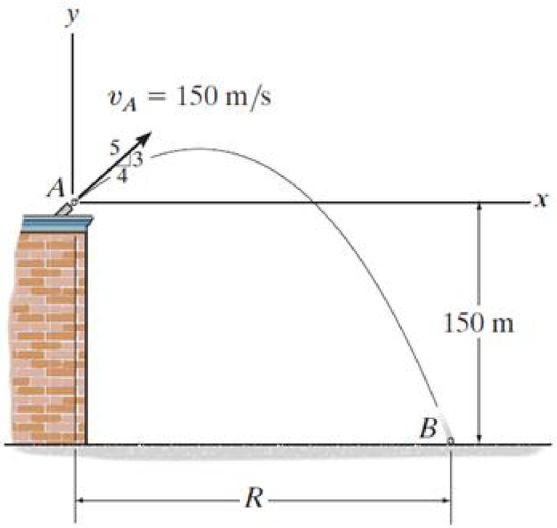


Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: How will you proceed?

EXAMPLE II



Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: Establish a fixed x, y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the kinematic relations in x- and y-directions.

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Vertical

motion

Y direction

$$V_{oy} = V_o \sin\theta$$

$$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$$

$$V_{fy} = V_{oy} + at$$

$$Y = V_{oy}\, t + 1/2\, at^2$$

Horizontal

motion

X direction

$$V_{ox} = V_o \cos\theta$$

$$V_{fx}{}^2 = V_{ox}{}^2$$

$$V_{fx} = V_{ox}$$

$$X = V_{ox}\, t$$

EXAMPLE II (continued)

Solution:

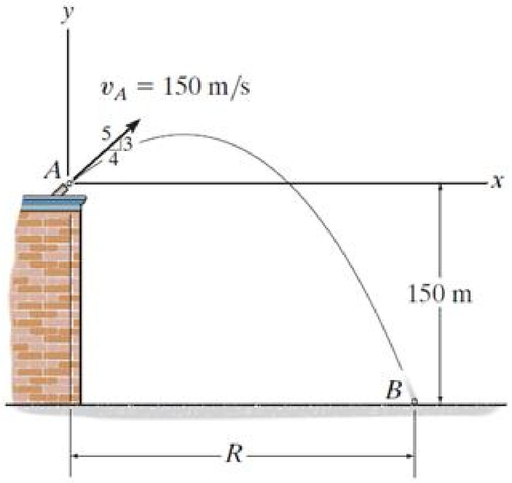
1) Place the coordinate system at point A.

Then, write the equation for horizontal motion

$$+\rightarrow \quad x_B = x_A + v_{Ax}\, t_{AB}$$

where $x_B = R$, $x_A = 0$, $v_{Ax} = 150\ (4/5)$ m/s

Range, R, will be $R = 120\ t_{AB}$



2) Now write a vertical motion equation. Use the distance equation.

$$+\uparrow \quad y_B = y_A + v_{Ay}\, t_{AB} - 0.5\, g\, t_{AB}^2$$

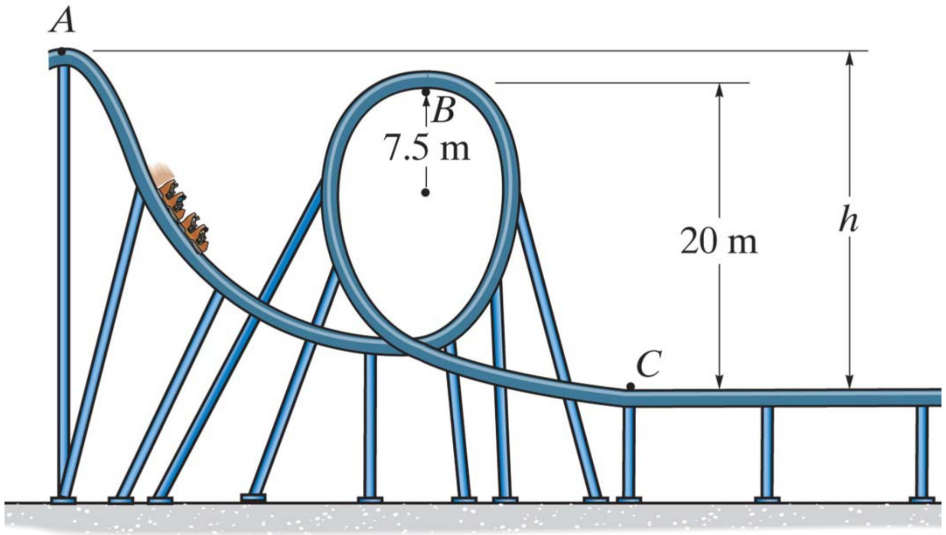
where $y_B = -150$, $y_A = 0$, and $v_{Ay} = 150(3/5)$ m/s

We get the following equation: $-150 = 90\ t_{AB} + 0.5\ (-9.81)\ t_{AB}^2$

Solving for t_{AB} first, $t_{AB} = 19.89$ s.

Then, $R = 120\ t_{AB} = 120\ (19.89) = 2387$ m

Example



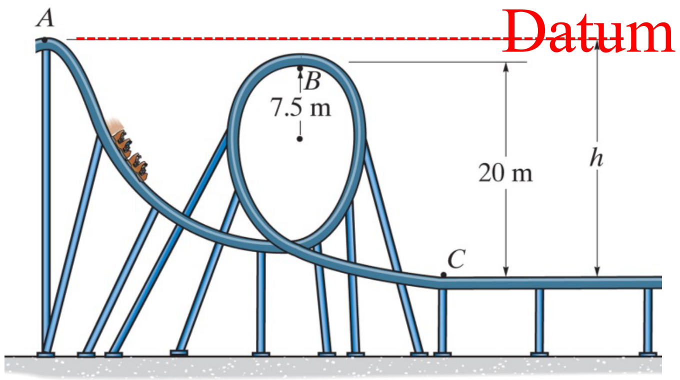
Given: The 800 kg roller coaster car is released from rest at A.

Find: The minimum height, h, of Point A so that the car travels around inside loop at B without leaving the track. Also find the velocity of the car at C for this height, h, of A.

Plan: Note that only kinetic energy and potential energy due to gravity are involved. Determine the velocity at B using the equation of motion and then apply the conservation of energy equation to find minimum height h .

GROUP PROBLEM SOLVING (continued)

Solution:



1) Placing the datum at A:

$$T_A + V_A = T_B + V_B$$

Two unknowns!!

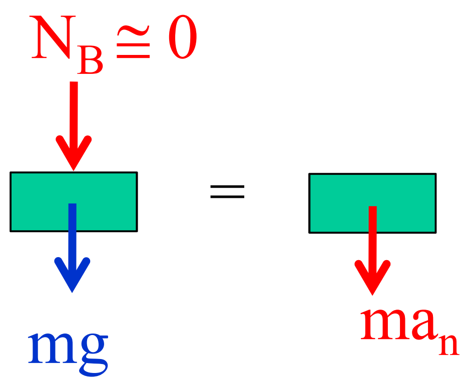
$$\Rightarrow 0.5\ (800)\ 0^2 + 0$$
$$= 0.5\ (800)\ (v_B)^2 - 800(9.81)\ (h - 20) \qquad (1)$$

2) Find the required velocity of the coaster at B so it doesn't leave the track.

Equation of motion applied at B:

$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

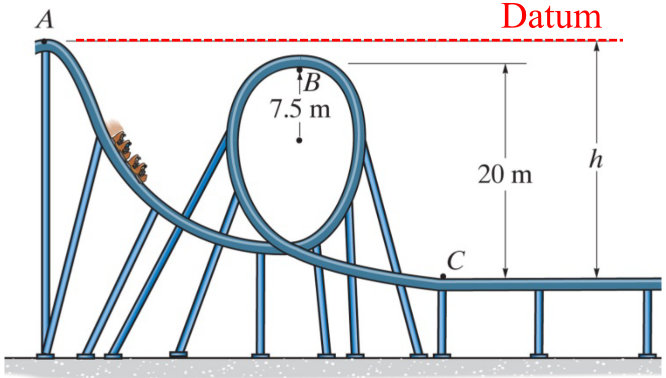
$$800\ (9.81) = 800\frac{(v_B)^2}{7.5}$$



$$\Rightarrow v_B = 8.578 \text{ m/s}$$

Example (continued)

Now using the energy conservation, eq. (1), the minimum h can be determined.



$$0.5\ (800)\ 0^2 + 0 = 0.5\ (800)\ (8.578)^2 - 800(9.81)\ (h - 20)$$

$$\Rightarrow\ h = 23.75\ \text{m}$$

3) Find the velocity at C applying the energy conservation.

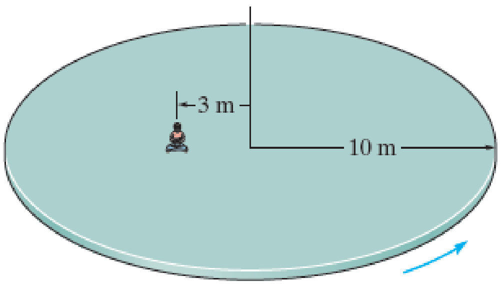
$$T_A + V_A = T_C + V_C$$

$$\Rightarrow 0.5\ (800)\ 0^2 + 0 = 0.5\ (800)\ (v_C)^2 - 800(9.81)\ (23.75)$$

$$\Rightarrow V_C = 21.6\ \text{m/s}$$

13–66.

The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.

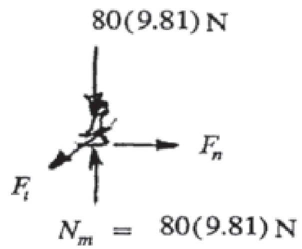


SOLUTION

$\Sigma F_t = m\, a_t;\qquad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\, a_n;\qquad F_n = (80)\dfrac{v^2}{3}$



$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\dfrac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\dfrac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

$a_t = \dfrac{dv}{dt} = 0.4$

$\int_0^v dv = \int_0^t 0.4\, dt$

$v = 0.4\, t$

$2.9575 = 0.4\, t$

$t = 7.39\ \text{s}$ **Ans.**

n-t coordinate system

The tangential component of acceleration is constant, $a_t = (a_t)_c$.

In this case,

$$\sum F_t \boldsymbol{u}_t + \sum F_n \boldsymbol{u}_n = m\boldsymbol{a}_t + m\boldsymbol{a}_n$$

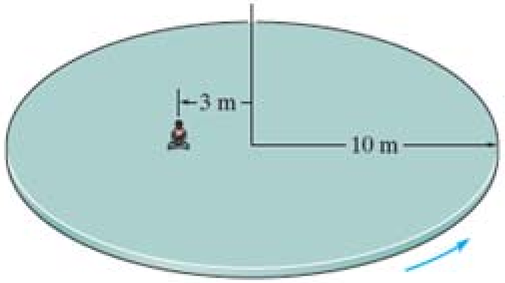
$$s = s_o + v_o t + (1/2)(a_t)_c t^2$$

$$v = v_o + (a_t)_c t$$

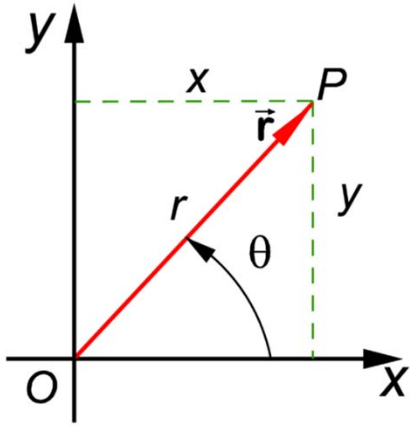
$$v^2 = (v_o)^2 + 2(a_t)_c(s - s_o)$$

$$\boldsymbol{a} = \dot{v}\boldsymbol{u}_t + (v^2/\rho)\boldsymbol{u}_n = a_t\boldsymbol{u}_t + a_n\boldsymbol{u}_n$$

As before, s_o and v_o are the initial position and velocity of the particle at $t = 0$



Solve the problem in polar coordinates:



$$\sum F_r = ma_r = m(\ddot{r} - \dot{r}\dot{\theta}^2)$$

$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Position $\quad \boldsymbol{r} = r\boldsymbol{u}_r$

Velocity: $\quad \boldsymbol{v} = \dot{r}\boldsymbol{u}_r + r\dot{\theta}\boldsymbol{u}_\theta$

Acceleration: $\quad a = \dot{v} = (\ddot{r} - r\dot{\theta}\dot{\theta})\ \boldsymbol{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\ \boldsymbol{u}_\theta$

(I) a= constant (Constant acceleration)

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{0}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{0}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

W. Wang

Remember…

…next time is Exam time