ME 230 Kinematics and Dynamics

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Planar kinematics of a rigid body

Chapter 16

Chapter objectives

- To classify the various types of rigid-body planar motion
- To investigate rigid body translation and analyze it
- Study planar motion
- Relative motion analysis using translatin frame of reference
- Find instantaneous center of zero velocit
- Relative motion analysis using rotating frame of reference

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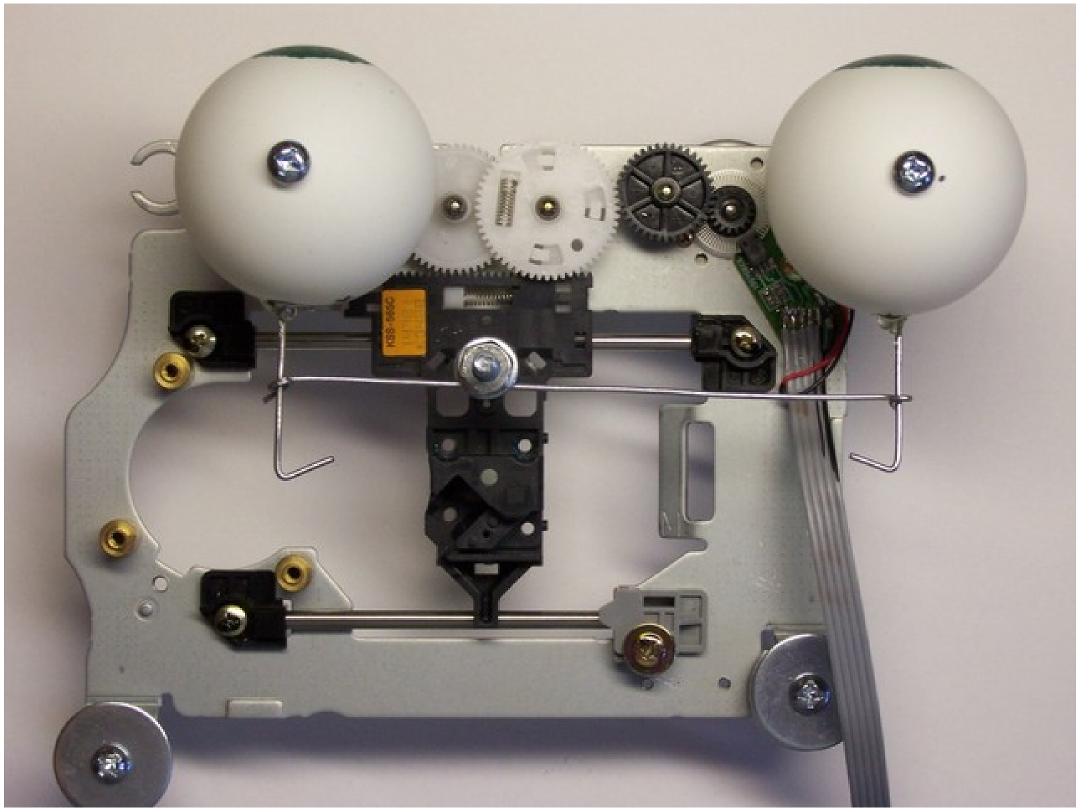
Lecture 13

- **Planar kinematics of a rigid body:**

Rigid body motion, Translation, Rotation about a fixed axis

- 16.1-16.3



Material covered

•**Planar kinematics of a rigid body :**

• Rigid body motion

• Translation

• Rotation about a fixed axis

…Next lecture…continue with Chapter 16

Today’s Objectives

Students should be able to:

1. Analyze the kinematics of a rigid body undergoing planar translation or rotation about a fixed axis

Applications

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but always remains upright.

If the angular motion of the rotating arms is known, how can we determine the velocity and acceleration experienced by the passengers?

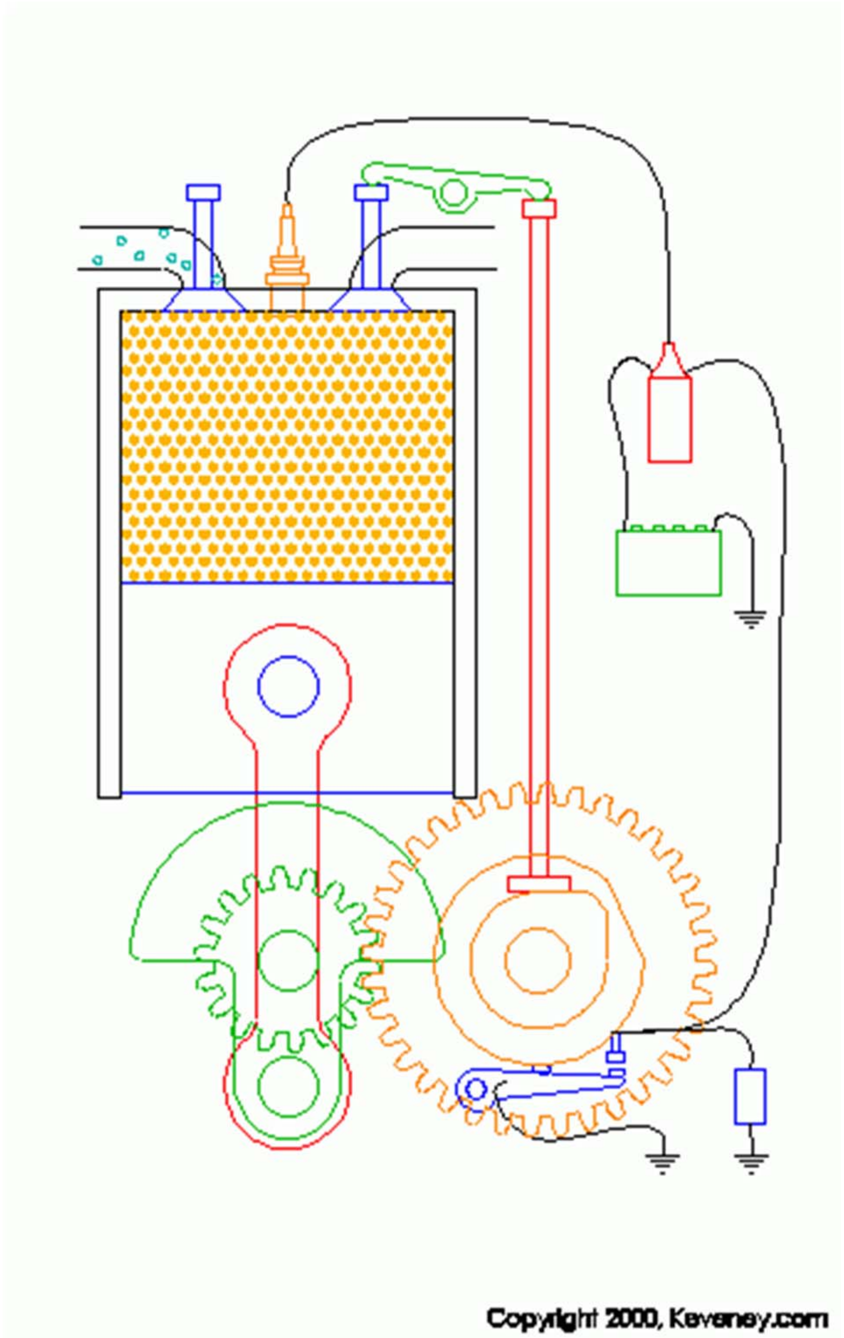
Does each passenger feel the same acceleration?

Applications (continued)

Gears, pulleys and cams, which rotate about fixed axes, are often used in machinery to generate motion and transmit forces. The angular motion of these components must be understood to properly design the system.

How can we relate the angular motions of contacting bodies that rotate about different fixed axes?

Four stroke engine



Otto Cycle

Rigid body motion (section 16.1)

There are cases where an object cannot be treated as a particle. In these cases the size or shape of the body must be considered. Also, rotation of the body about its center of mass requires a different approach.

For example, in the design of gears, cams, and links in machinery or mechanisms, rotation of the body is an important aspect in the analysis of motion.

We will now start to study rigid body motion. The analysis will be limited to planar motion.

A body is said to undergo planar motion when all parts of the body move along paths equidistant from a fixed plane.

Recall

1. Particles:

Definition: A particle is a body of negligible dimensions.

When the dimensions of a body are irrelevant to the description of its motion, the body can be treated as a particle.

Examples:

(a) An airplane: Yes when analyzing the fleight path from LA to NYC. No when the plane rotates.

(b) A space shuttle: Yes when analyzing the orbit of the shuttle. No when the shuttle turns.

(c) Scott Hamilton: Yes when he skates along the rink. No when he does a double toe-loop.

2. Rigid Bodies:

Definition: A rigid body is a body that does not deform and dimensions of the body are not negligible.

When the deformation is much less than the dimensions of the body to be analyzed and the dimensions of a body are relevant to the description of its motion, the body can be treated as a rigid body.

Examples:

(a) An airplane: Yes when analyzing the rotational motion of the airplane. No when analyzing the vibration of the airplane wings.

(b) The Hubble Telescope: Yes when analyzing the unfolding motion of its solar panels. No when analyzing the vibration of the thermal gitters.

(c) Scott Hamilton: Yes when he does a double toe-loop. No when analyzing the contraction of his muscle.

3. Differences Between Particles and Rigid Bodies:

Particles $\mapsto$ No Rotation $\mapsto$ No Moment Equations

Rigid Bodies $\mapsto$ Rotation Exists $\mapsto$ Moment Equations Are Important.

Therefore, we study the motion of particles first and then rigid bodies.

Planar rigid body motion

There are three types of planar rigid body motion.

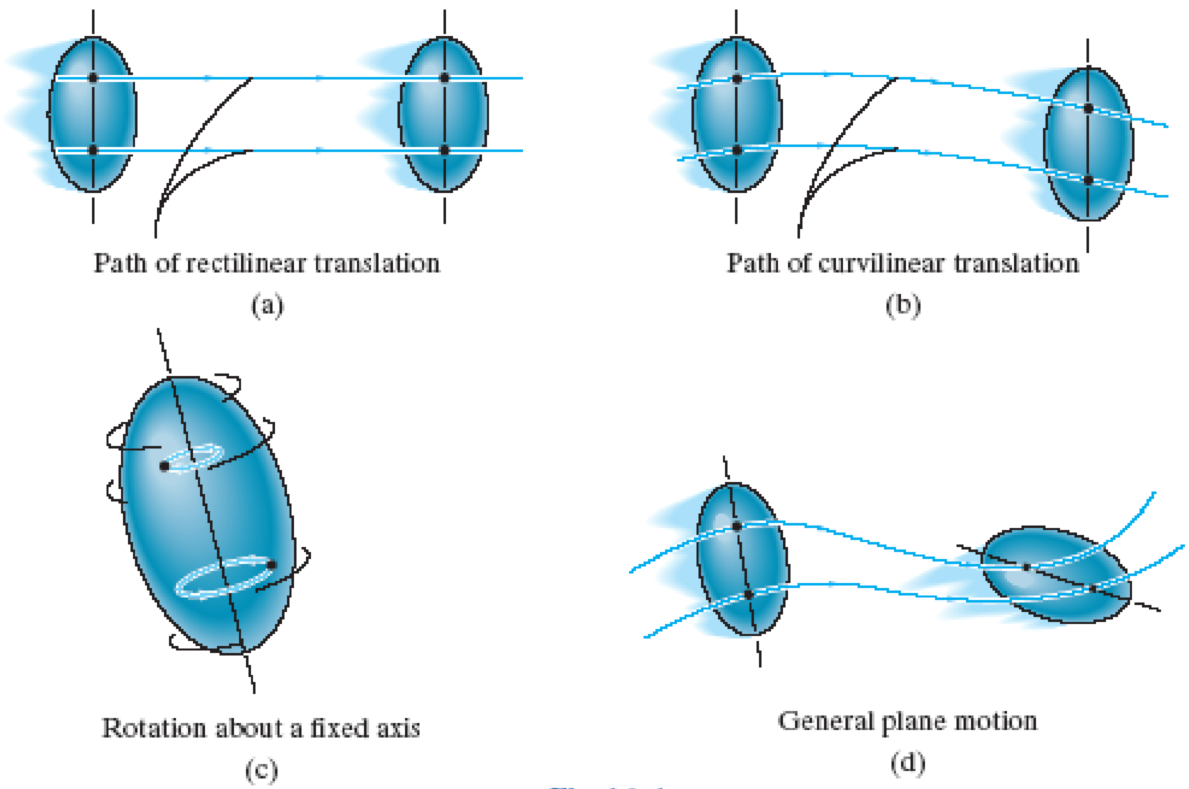
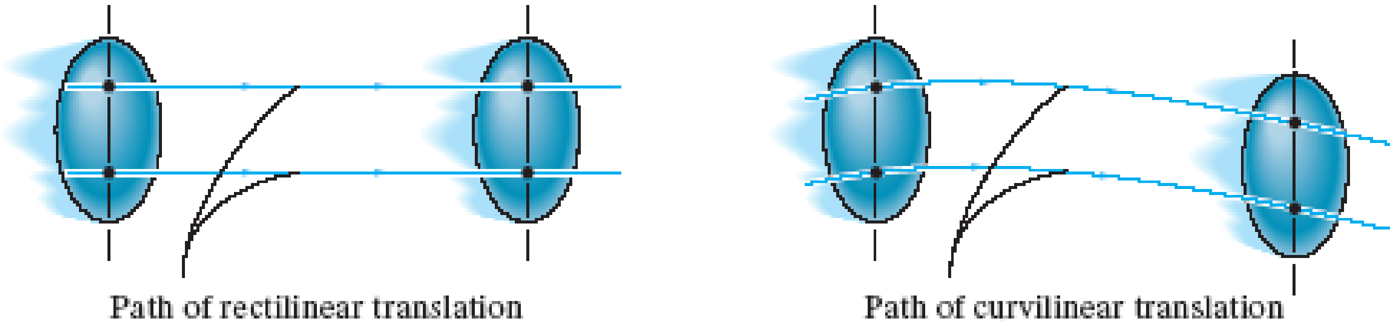


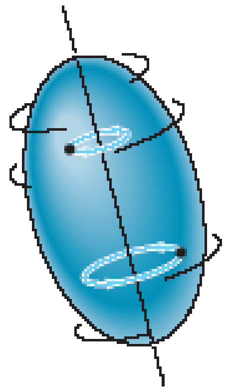
Fig. 16–1

Planar rigid body motion (continues)



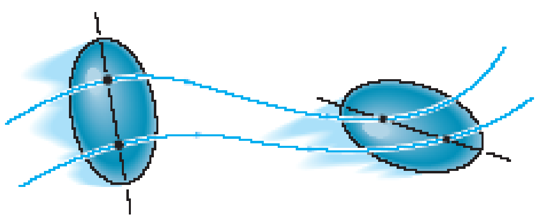
Translation: Translation occurs if every line segment on the body remains parallel to its original direction during the motion. When all points move along straight lines, the motion is called rectilinear translation. When the paths of motion are curved lines, the motion is called curvilinear translation.

Planar rigid body motion (continues)



Rotation about a fixed axis

Rotation about a fixed axis: In this case, all the particles of the body, except those on the axis of rotation, move along circular paths in planes perpendicular to the axis of rotation.

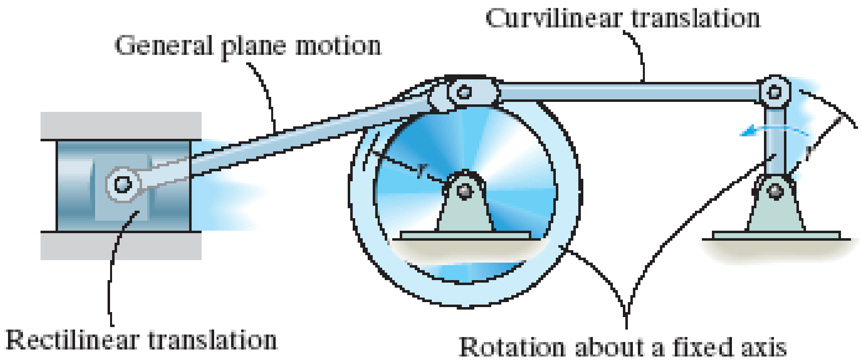


General plane motion

General plane motion: In this case, the body undergoes both translation and rotation. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.

Planar rigid body motion (continues)

An example of bodies undergoing the three types of motion is shown in this mechanism.

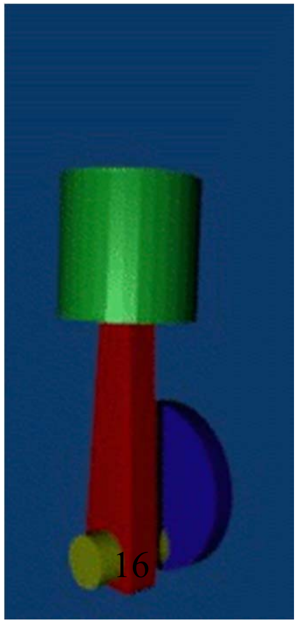


The wheel and crank undergo rotation about a fixed axis. In this case, both axes of rotation are at the location of the pins and perpendicular to the plane of the figure.

The piston undergoes rectilinear translation since it is constrained to slide in a straight line.

The connecting rod undergoes curvilinear translation, since it will remain horizontal as it moves along a circular path.

The connecting rod undergoes general plane motion, as it will both translate and rotate.



Theory: Translation (16.2)

- all points on a rigid body subject to either rectilinear or curvilinear translation (i.e., no rotation) move with the same velocity and acceleration

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$ relative position

$\frac{d}{dt}$

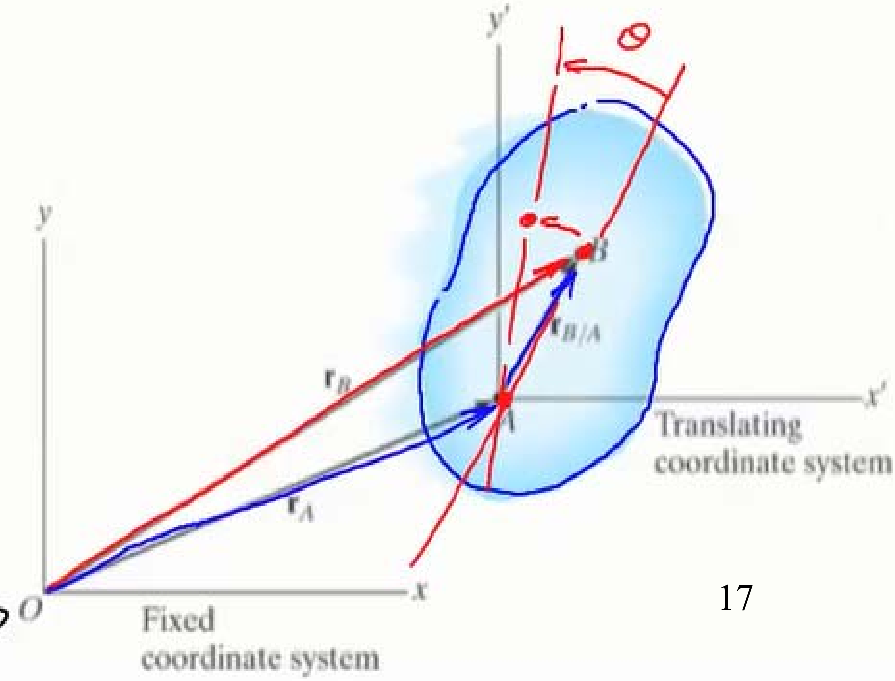
$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$

$\frac{d}{dt}$

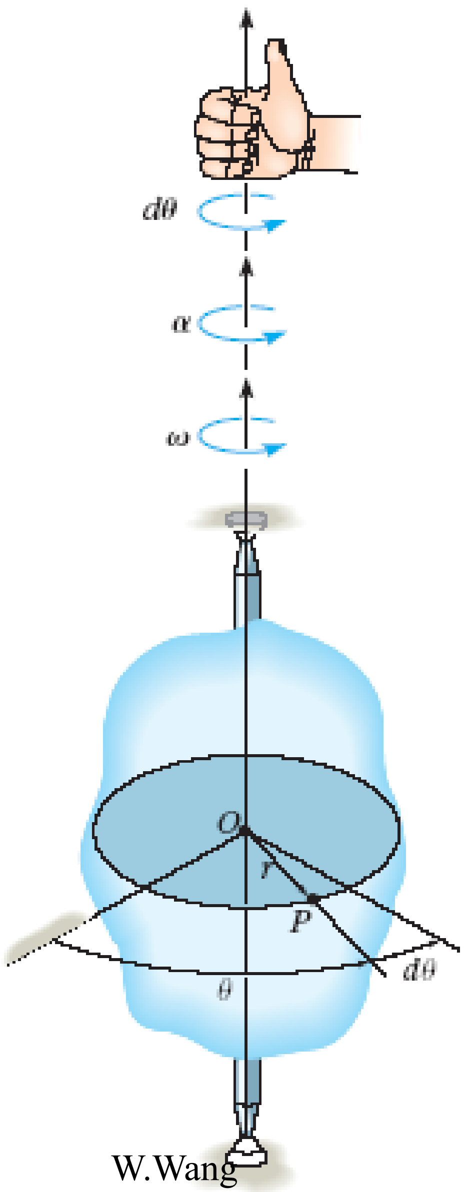
$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$

(i) what is changing?
- distances (A & B)
- orientations

(ii) what coord. system do I use to express these changes?



Rigid body motion – Rotation about a fixed axis (16.3)



When a body rotates about a fixed axis, any point P in the body travels along a circular path. The angular position of P is defined by θ.

The change in angular position, dθ, is called the angular displacement, with units of either radians or revolutions. They are related by

1 revolution = 2π radians

Angular velocity, ω, is obtained by taking the time derivative of angular displacement:

$\omega = d\theta/dt$ (rad/s) +

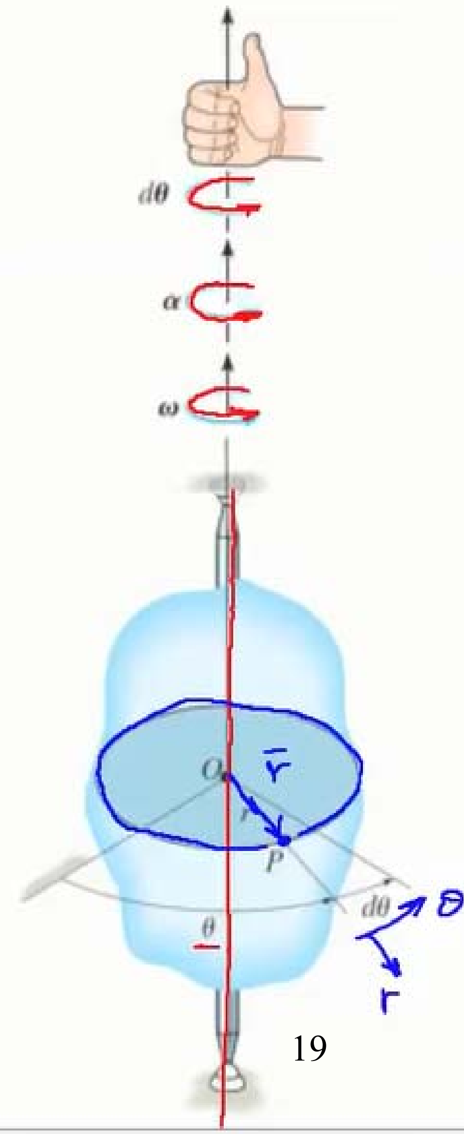
Similarly, angular acceleration is

$\alpha = d^2\theta/dt^2 = d\omega/dt$ or $\alpha = \omega(d\omega/d\theta)$ + rad/s^2

Concept: Rotation about a Fixed Axis (16.3)

- if a body rotates about a fixed axis, then all points on that body follow a circular path
- we define several kinematic quantities:
  - angular position θ
  - angular displacement $d\theta, \Delta\theta$
  - angular velocity ω
  - angular acceleration α

$$\alpha = \dot{\omega} = \ddot{\theta}$$



Theory: (Fixed) Rotational Kinematics

- the kinematic expressions for rotation about a fixed axis are similar to those for particles (and are derived in a similar way)

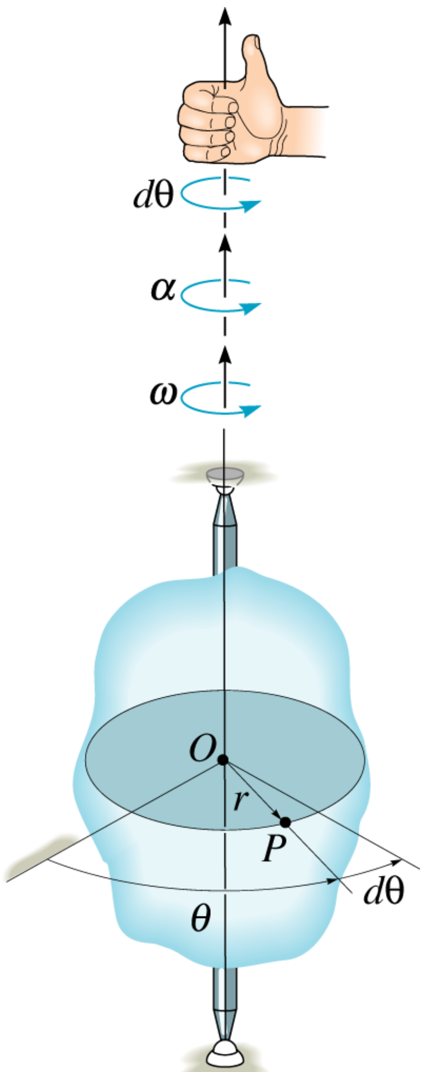
$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt}$$

$$dt = \frac{d\theta}{\omega} \qquad dt = \frac{d\omega}{\alpha}$$

$$\boxed{\alpha \, d\theta = \omega \, d\omega}$$

$[a \, ds = v \, dv]$ particles

Rigid body motion – Rotation about a fixed axis (16.3 continued)



If the angular acceleration of the body is constant, $\alpha = \alpha_C$, the equations for angular velocity and acceleration can be integrated to yield the set of algebraic equations below.

$$\omega = \omega_O + \alpha_C t$$
$$\theta = \theta_O + \omega_O t + 0.5\alpha_C t^2$$
$$\omega^2 = (\omega_O)^2 + 2\alpha_C (\theta - \theta_O)$$

θ_O and ω_O are the initial values of the body's angular position and angular velocity. Note these equations are very similar to the constant acceleration relations developed for the rectilinear motion of a particle.

Theory: Special Case--Constant Acceleration

$\alpha = \alpha_c$

$\omega = \omega_0 + \alpha_c t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

} ~ projectile motion-type equation

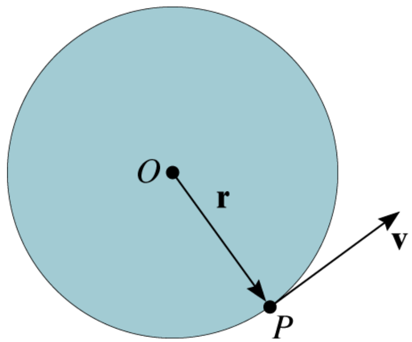
$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$ ⇒ like the particle

$Y = Y_0 + V_{oy}\, t + 1/2\; at^2$

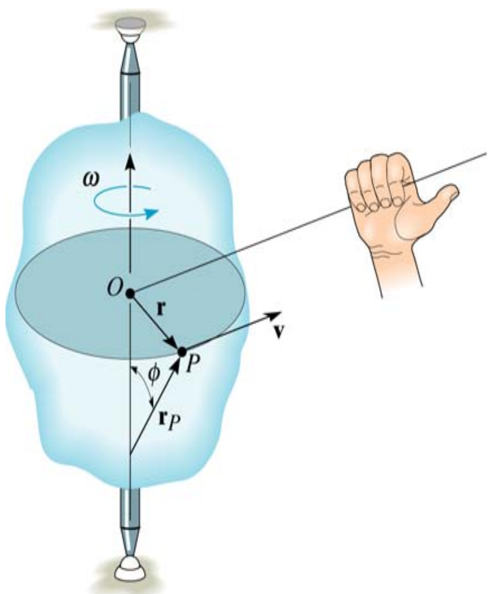
$V_{fy} = V_{oy} + at$

$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$

Rigid body rotation – Velocity of point P



The magnitude of the velocity of P is equal to ωr (the text provides the derivation). The velocity's direction is tangent to the circular path of P.



In the vector formulation, the magnitude and direction of $\boldsymbol{v}$ can be determined from the cross product of $\boldsymbol{\omega}$ and $\boldsymbol{r}_p$. Here $\boldsymbol{r}_p$ is a vector from any point on the axis of rotation to P.

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_p = \boldsymbol{\omega} \times \boldsymbol{r}$$

The direction of $\boldsymbol{v}$ is determined by the right-hand rule.

Theory: Motion of a Point on a Rotating Body

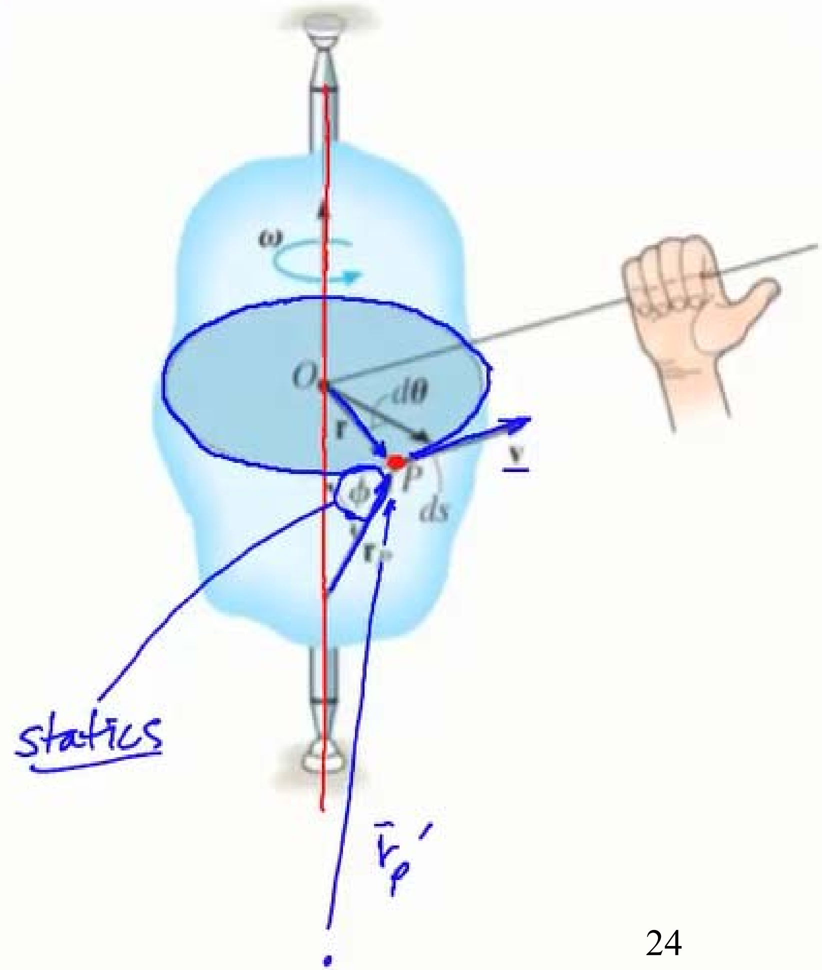
- the motion of a particular point on a body can be expressed using vectors

- velocity:

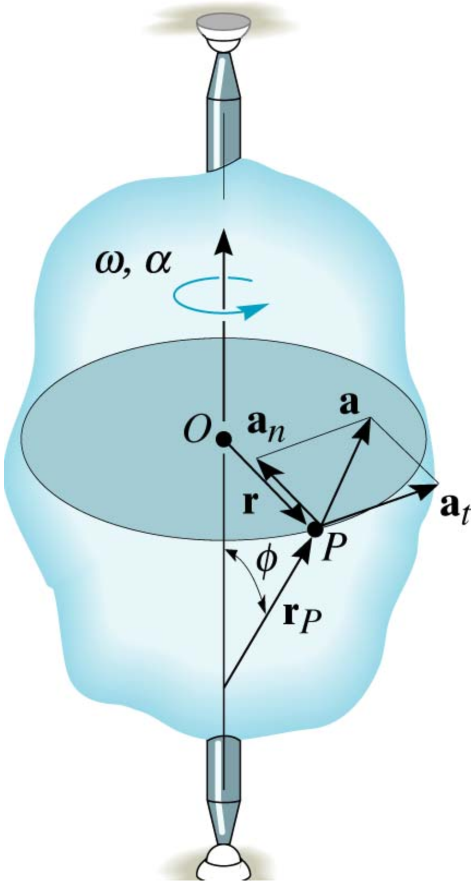
$$\bar{v} = \bar{\omega} \times \bar{r} = \bar{\omega} \times \bar{r}_p = \bar{\omega} \times \bar{r}_p'$$

$$\bar{\omega} \times \bar{r} \neq \bar{r} \times \bar{\omega}$$

order is critical



Rigid body rotation – Acceleration of point P

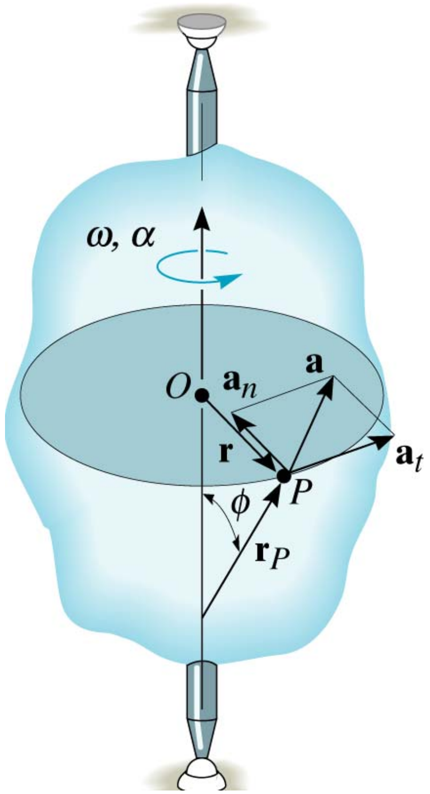


The acceleration of P is expressed in terms of its normal ($\boldsymbol{a}_n$) and tangential ($\boldsymbol{a}_t$) components. In scalar form, these are $a_t = \alpha\ r$ and $a_n = \omega^2\ r$.

The tangential component, $\boldsymbol{a}_t$, represents the time rate of change in the velocity's magnitude. It is directed tangent to the path of motion.

The normal component, $\boldsymbol{a}_n$, represents the time rate of change in the velocity's direction. It is directed toward the center of the circular path.

Rigid body rotation – Acceleration of point P (continued)



Using the vector formulation, the acceleration of P can also be defined by differentiating the velocity. (we derived it earlier in week 2)

$$\boldsymbol{a} = d\boldsymbol{v}/dt = d\boldsymbol{\omega}/dt \times \boldsymbol{r}_P + \boldsymbol{\omega} \times d\boldsymbol{r}_P/dt$$

$$= \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P)$$

It can be shown that this equation reduces to

$$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r} = \boldsymbol{a}_t + \boldsymbol{a}_n$$

The magnitude of the acceleration vector is $a = \sqrt{(a_t)^2 + (a_n)^2}$

Examples: Motion of a Point on a Rotating Body

- acceleration: for rotation about a fixed axis, often expressed using normal and tangential components

P is on curved path $\Rightarrow \bar{a}_n, \bar{a}_t$

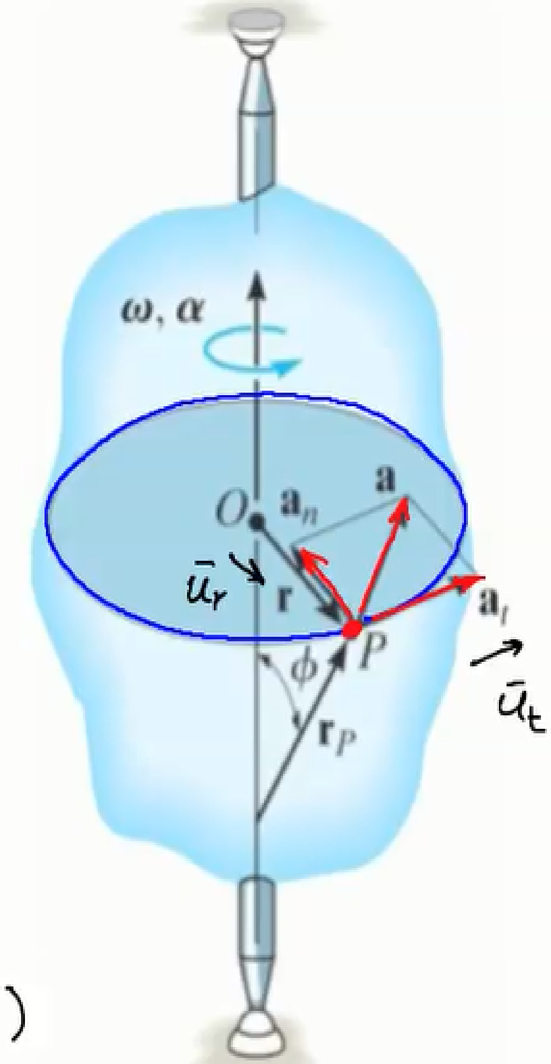
$$\bar{a} = \frac{d}{dt}(\bar{v}) = \frac{d}{dt}(\bar{\omega} \times \bar{r})$$

$$= \underbrace{\bar{\omega} \times \frac{d}{dt}(\bar{r})}_{\bar{\omega} \times \bar{r}} + \underbrace{\frac{d\bar{\omega}}{dt}}_{\Downarrow\ \bar{\alpha}} \times \bar{r}$$

$$\bar{a} = \underbrace{\bar{\alpha} \times \bar{r}}_{\text{tang.}} + \underbrace{\bar{\omega} \times \bar{\omega} \times \bar{r}}_{\text{normal}}$$

tang. $\Downarrow$ mag.: αr $\bar{u}_t$

normal $\Downarrow$ magnitude: $r\omega^2$ (direction $-\bar{r}$) $\Downarrow$ $-\bar{u}_r$



Acceleration in the n-t coordinate system II

The tangential component of acceleration is constant, $a_t = (a_t)_c$.

In this case,

$$s = s_o + v_o t + (1/2)(a_t)_c t^2$$

$$v = v_o + (a_t)_c t$$

$$v^2 = (v_o)^2 + 2(a_t)_c(s - s_o)$$

$$\boldsymbol{a} = \dot{v}\boldsymbol{u}_t + (v^2/\rho)\boldsymbol{u}_n = a_t\boldsymbol{u}_t + a_n\boldsymbol{u}_n$$

Rigid body rotation:

$$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2\boldsymbol{r} = \boldsymbol{a}_t + \boldsymbol{a}_n$$

As before, s_o and v_o are the initial position and velocity of the particle at $t = 0$

Then accleration in polar coordinates:

$$a = \dot{v} = \ddot{r}u_r + \dot{r}\dot{u}_r + \dot{r}\dot{\theta}u_\theta + r\ddot{\theta}u_\theta + r\dot{\theta}\dot{u}_\theta$$

$$= \ddot{r}u_r + \dot{r}\dot{\theta}u_\theta + \dot{r}\dot{\theta}u_\theta + r\ddot{\theta}u_\theta - r\dot{\theta}\dot{\theta}u_r$$

$$= (\ddot{r}u_r - r\dot{\theta}\dot{\theta}u_r) + (2\dot{r}\dot{\theta}u_\theta + r\ddot{\theta}u_\theta)$$

$$= (\ddot{r} - r\dot{\theta}\dot{\theta})\,\boldsymbol{u_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\boldsymbol{u_\theta}$$

Rigid body rotation:

$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2\boldsymbol{r} = \boldsymbol{a}_t + \boldsymbol{a}_n$

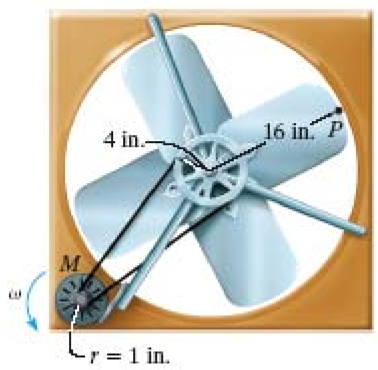
Rotation about a fixed axis - Procedure

- Establish a sign convention along the axis of rotation.
- If a relationship is known between any two of the variables (α, ω, θ, or t), the other variables can be determined from the equations: $\omega = d\theta/dt$ $\alpha = d\omega/dt$ $\alpha\, d\theta = \omega\, d\omega$
- If α is constant, use the equations for constant angular acceleration.
- To determine the motion of a point, the scalar equations $v = \omega r$, $a_t = \alpha r$, $a_n = \omega^2 r$, and $a = \sqrt{(a_t)^2 + (a_n)^2}$ can be used.
- Alternatively, the vector form of the equations can be used (with ***i***, ***j***, ***k*** components).

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_P = \boldsymbol{\omega} \times \boldsymbol{r}$$

$$\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_n = \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P) = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r}$$

Example



Given: The motor M begins rotating at $\omega = 4(1 - e^{-t})$ rad/s, where t is in seconds. The radii of the motor, fan pulleys, and fan blades are 1 in, 4 in, and 16 in, respectively.

Find: The magnitudes of the velocity and acceleration at point P on the fan blade when t = 0.5 s.

Plan:
1) Determine the angular velocity and acceleration of the motor using kinematics of angular motion.
2) Assuming the belt does not slip, the angular velocity and acceleration of the fan are related to the belt's velocity.
3) The magnitudes of the velocity and acceleration of point P can be determined from the scalar equations of motion for a point on a rotating body.

Example (continues)

Solution:

1) Since the angular velocity is given as a function of time, $\omega_m = 4(1 - e^{-t})$, the angular acceleration can be found by differentiation.

$$\alpha_m = d\omega_m/dt = 4e^{-t} \ \text{rad/s}^2$$

When t = 0.5 s,

$\omega_m = 4(1 - e^{-0.5}) = 1.5739$ rad/s, $\alpha_m = 4e^{-0.5} = 2.4261$ rad/s^2

2) Since the belt does not slip (and is assumed inextensible), it must have the same speed and tangential component of acceleration at all points. Thus the pulleys must have the same speed and tangential acceleration at their contact points with the belt. Therefore, the angular velocities of the motor (ω_m) and fan (ω_f) are related as

$$v = \omega_m r_m = \omega_f r_f \Rightarrow (1.5739)(1) = \omega_f(4) \Rightarrow \omega_f = 0.3935 \text{ rad/s}$$

Example (continues)

3) Similarly, the tangential accelerations are related as

$$a_t = \alpha_m r_m = \alpha_f r_f \Rightarrow (2.4261)(1) = \alpha_f(4) \Rightarrow \alpha_f = 0.6065 \text{ rad/s}^2$$

4) The speed of point P on the fan, at a radius of 16 in, is now determined as

$$v_P = \omega_f r_P = (0.3935)(16) = 6.30 \text{ in/s}$$

The normal and tangential components of acceleration of point P are calculated as

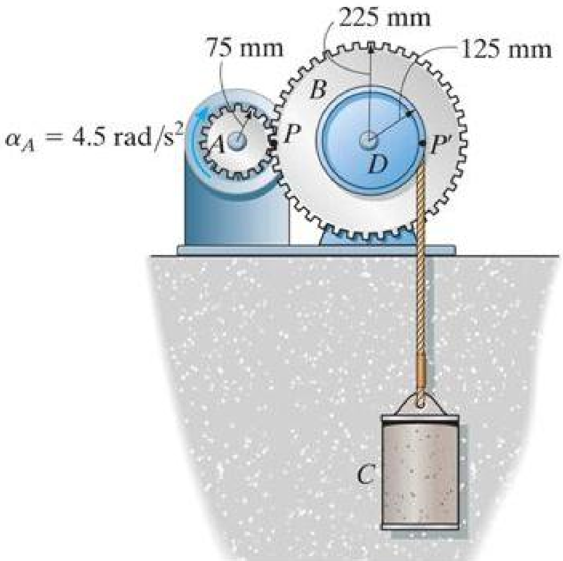
$$a_n = (\omega_f)^2 r_P = (0.3935)^2 (16) = 2.477 \text{ in/s}^2$$

$$a_t = \alpha_f r_P = (0.6065)(16) = 9.704 \text{ in/s}^2$$

The magnitude of the acceleration of P can be determined by

$$a_P = \sqrt{(a_n)^2 + (a_t)^2} = \sqrt{(2.477)^2 + (9.704)^2} = 10.0 \text{ in/s}^2$$

EXAMPLE



Given: The motor turns gear A with a constant angular acceleration, $\alpha_A = 4.5$ rad/s^2, starting from rest. The cord is wrapped around pulley D which is rigidly attached to gear B.

Find: The velocity of cylinder C and the distance it travels in 3 seconds.

Plan:

1) The angular acceleration of gear B (and pulley D) can be related to α_A.
2) The acceleration of cylinder C can be determined by using the equations of motion for a point on a rotating body since $(a_t)_D$ at point P is the same as a_c.
3) The velocity and distance of C can be found by using the constant acceleration equations.

EXAMPLE (continued)

Solution:

1) Gear A and B will have the same speed and tangential component of acceleration at the point where they mesh. Thus,

$$a_t = \alpha_A r_A = \alpha_B r_B \;\Rightarrow\; (4.5)(75) = \alpha_B(225) \;\Rightarrow\; \alpha_B = 1.5 \text{ rad/s}^2$$

Since gear B and pulley D turn together, $\alpha_D = \alpha_B = 1.5 \text{ rad/s}^2$

2) Assuming the cord attached to pulley D is inextensible and does not slip, the velocity and acceleration of cylinder C will be the same as the velocity and tangential component of acceleration along the pulley D.

$$a_C = (a_t)_D = \alpha_D \, r_D = (1.5)(0.125) = 0.1875 \text{ m/s}^2 \uparrow$$

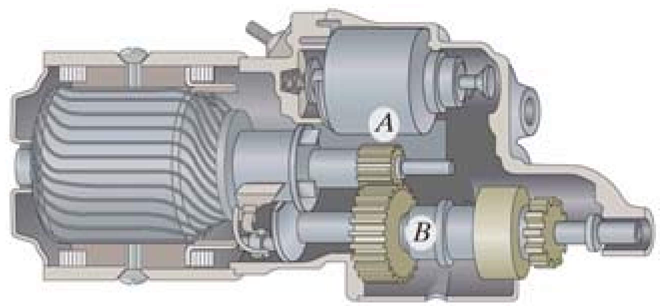
EXAMPLE (continued)

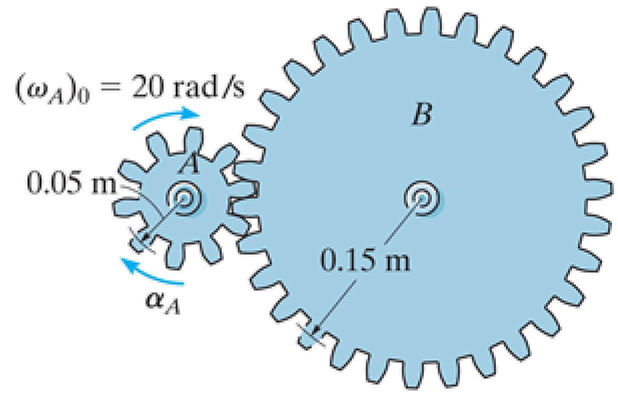
3) Since α_A is constant, α_D and a_C will be constant. The constant acceleration equation for rectilinear motion can be used to determine the velocity and displacement of cylinder C when t = 3 s ($s_0 = v_0 = 0$):

$$v_c = v_0 + a_C\, t = 0 + 0.1875\ \text{m/s}^2 (3\ \text{s}) = 0.563\ \text{m/s} \uparrow$$

$$\begin{aligned} s_c &= s_0 + v_0\, t + (0.5)\, a_C\, t^2 \\ &= 0 + 0 + (0.5)\, 0.1875\ \text{m/s}^2\, (3\ \text{s})^2 = 0.844\ \text{m} \uparrow \end{aligned}$$

Example





Given: Gear A is given an angular acceleration $\alpha_A = 4t^3$ rad/s^2, where t is in seconds, and $(\omega_A)_0 = 20$ rad/s.

Find: The angular velocity and angular displacement of gear B when t = 2 s.

Plan: 1) Apply the kinematic equation of variable angular acceleration to find the angular velocity of gear A.

2) Find the relationship of angular motion between gear A and gear B in terms of time and then use 2 s.

Example (continued)

Solution:

the kinematic equation

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A dt \;\Rightarrow\; \omega_A - 20 = \int_0^t 4t^3 dt = t^4$$

$$\Rightarrow \;\; \omega_A = t^4 + 20$$

$$\int_0^{\theta_A} d\theta_A = \int_0^t \omega_A dt$$

$$\Rightarrow \;\; \theta_A = \int_0^t (t^4 + 20) dt = \frac{1}{5} t^5 + 20\,t$$

When t=2 s, ω_A= 36 rad/s and θ_A= 46.4 rad.

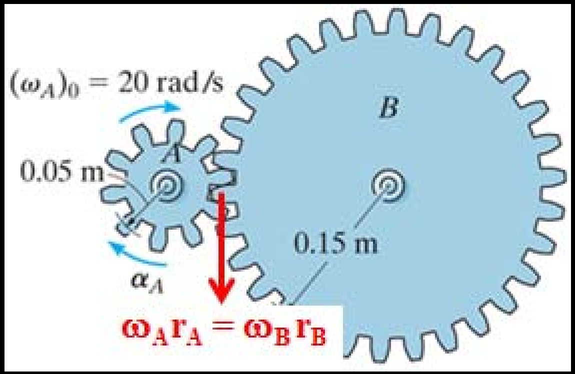
Example (continued)

2) Since gear B meshes with gear A,

$$\omega_A r_A = \omega_B r_B$$

$$\Rightarrow \omega_B = \omega_A (r_A / r_B) = \omega_A (0.05 / 0.15)$$

Similarly, $\theta_B = \theta_A (0.05 / 0.15)$



Since ω_A= 36 rad/s and θ_A= 46.4 rad at t=2 s,

$$\omega_B = 36 (0.05 / 0.15) = 12 \text{ rad/s}$$

$$\theta_B = 46.4 (0.05 / 0.15) = 15.5 \text{ rad}$$

Final Project

1. **Proposal for design project (GROUPS)**

The CS student finally realizes the meaning of the word "deadline".

Deadline: Proposal due This Friday Feb. 14

Homework Assignment

Chapter16- 13, 18, 34
Chapter16- 43,49,50,65,73,91,105,111,114,115

Due next Wednesday !!!

Planar kinematics of a rigid body

Chapter 16

Chapter objectives

- To classify the various types of rigid-body planar motion
- To investigate rigid body translation and analyze it
- Study planar motion
- Relative motion analysis using translating frame of reference
- Find instantaneous center of zero velocity
- Relative motion analysis using rotating frame of reference

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Lecture 14

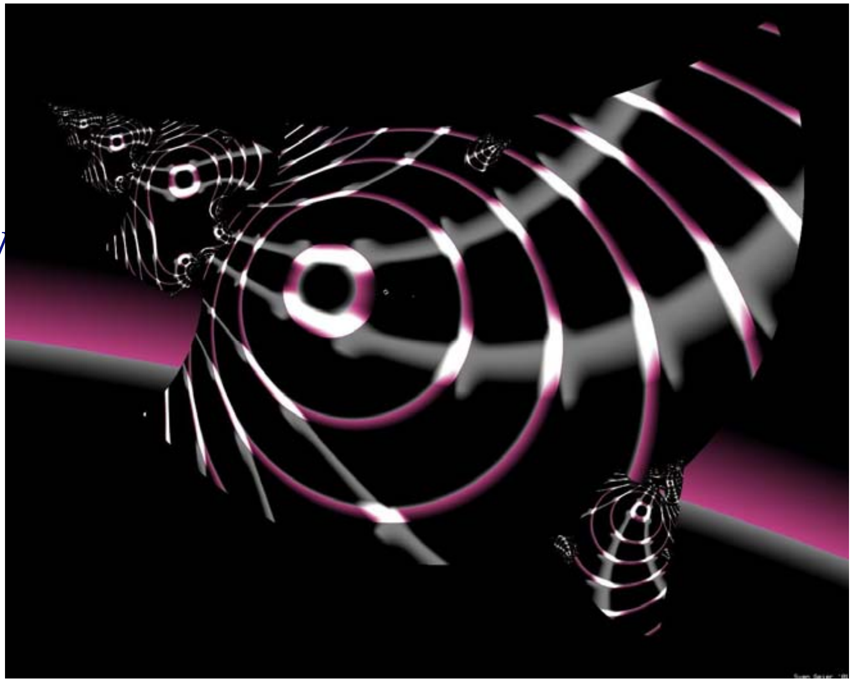
- **Planar kinematics of a rigid body:**

Absolute motion analysis, Relative motion analysis: Velocity, Instantaneous center of zero velocity, Relative motion analysis: Acceleration

- 16.4-16.7

Material covered

- **Planar kinematics of a rigid body :**
- Absolute motion analysis
- Relative motion analysis: Velocity
- Instantaneous center of zero velocity
- Relative motion analysis: Acceleration



…Next lecture…Review 2

Today's Objectives

Students should be able to:

1. **Determine the velocity and acceleration of a rigid body undergoing general plane motion using an absolute motion analysis (16.4)**
2. Describe the velocity of a rigid body in terms of translation and rotation components (16.5)
3. Perform a relative-motion velocity analysis of a point on the body (16.5)
4. Locate the instantaneous center of zero velocity.
5. Use the instantaneous center to determine the velocity of any point on a rigid body in general plane motion (16.6)
6. Resolve the acceleration of a point on a body into components of translation and rotation (16.7)
7. Determine the acceleration of a point on a body by using a relative acceleration analysis (16.7)

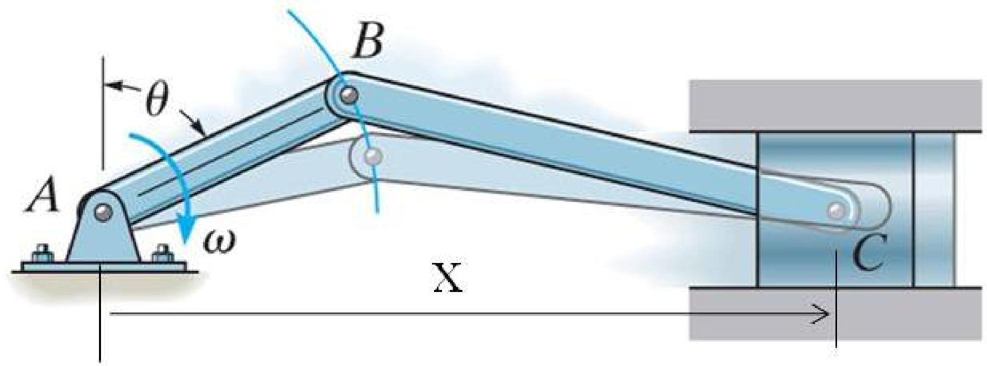
APPLICATIONS

The dumping bin on the truck rotates about a fixed axis passing through the pin at A. It is operated by the extension of the hydraulic cylinder BC.

The angular position of the bin can be specified using the angular position coordinate θ and the position of point C on the bin is specified using the coordinate s.

As a part of the design process for the truck, an engineer had to relate the velocity at which the hydraulic cylinder extends and the resulting angular velocity of the bin.

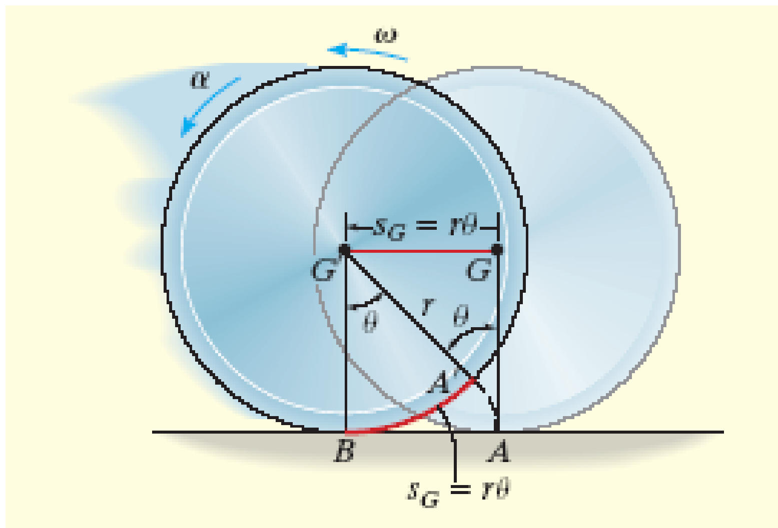
APPLICATIONS (continued)



The position of the piston, x, can be defined as a function of the angular position of the crank, θ. By differentiating x with respect to time, the velocity of the piston can be related to the angular velocity, ω, of the crank. This is necessary when designing an engine.

The stroke of the piston is defined as the total distance moved by the piston as the crank angle varies from 0 to 180°. How does the length of crank AB affect the stroke?

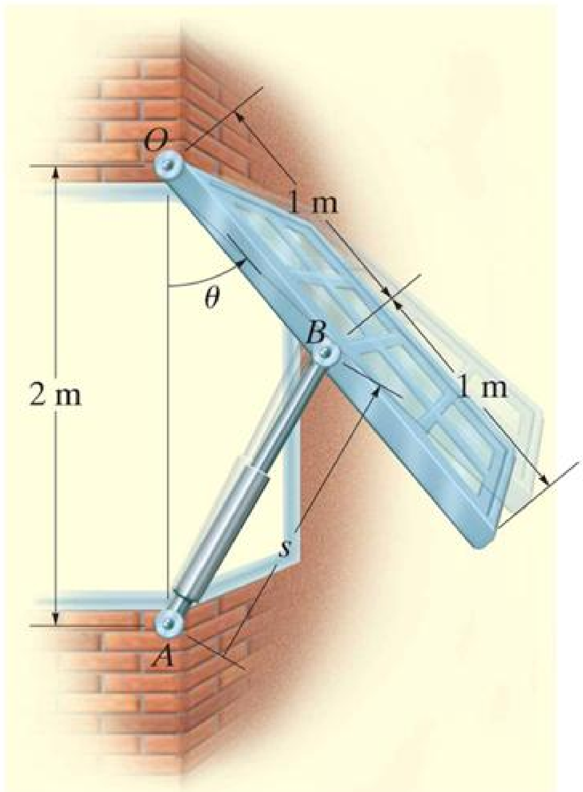
Applications for absolute motion analysis (16.4) (continues)



The rolling of a cylinder is an example of general plane motion.

During this motion, the cylinder rotates clockwise while it translates to the right.

APPLICATIONS (continued)



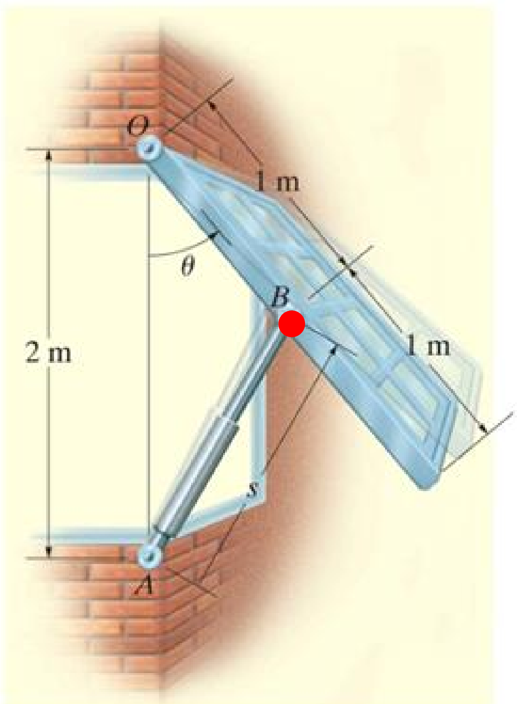
The large window is opened using a hydraulic cylinder AB.

The position B of the hydraulic cylinder rod is related to the angular position, θ, of the window.

A designer has to relate the translational velocity at B of the hydraulic cylinder and the angular velocity and acceleration of the window? How would you go about the task?

ABSOLUTE MOTION ANALYSIS
(Section 16.4)

The figure below shows the window using a hydraulic cylinder AB.



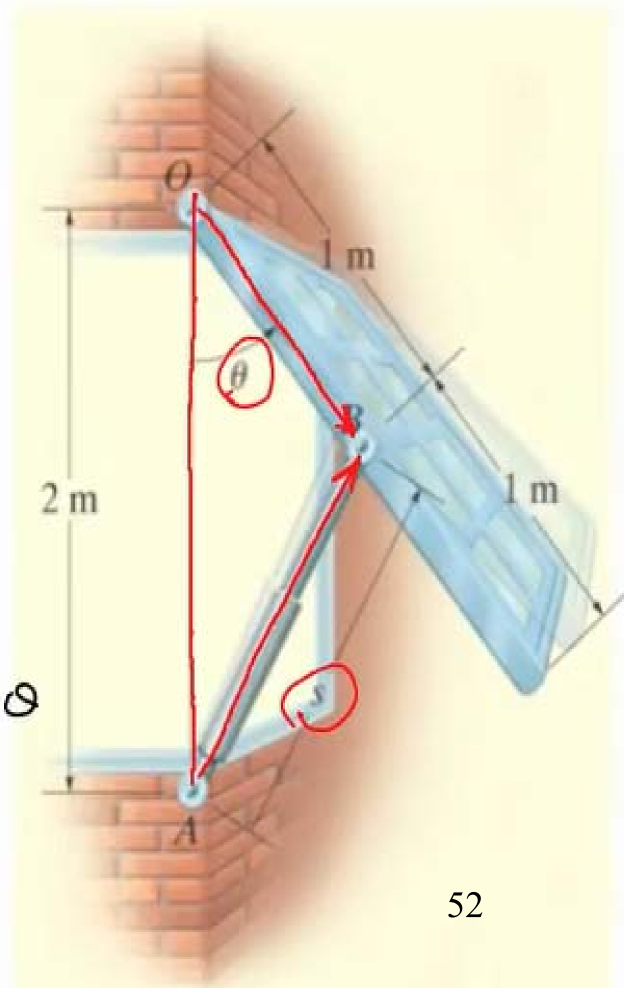
The absolute motion analysis method relates the position of a point, B, on a rigid body undergoing rectilinear motion to the angular position, θ, of a line contained in the body.

Once a relationship in the form of $s_B = f(\theta)$ is established, the velocity and acceleration of point B are obtained in terms of the angular velocity and angular acceleration of the rigid body by taking the first and second time derivatives of the position function.

Usually the chain rule must be used when taking the derivatives of the position coordinate equation.

Theory: Absolute Motion Analysis (16.4)

- a body subjected to general plane motion undergoes simultaneous translation and rotation
- we often use two coordinates: a position coordinate s and an angular coordinate θ
- for instance, example problem 16.5 (law of cosines)



$$s^2 = (2m)^2 + (1m)^2 - 2(2m)(1m)\cos\theta$$

$$s^2 = 5 - 4\cos\theta$$

d/dt

$$\frac{d}{dt}(s^2) = 2s\dot{s} \neq \frac{d}{ds}(s^2) = 2s$$

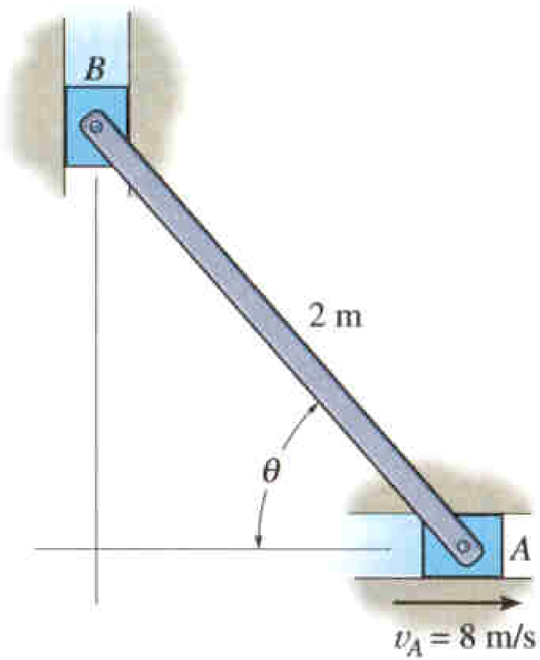
$$\frac{d}{dt}(\cos\theta) = (-\sin\theta)\dot{\theta} \neq \frac{d}{d\theta}\cos\theta = -\sin\theta$$

Absolute motion analysis (16.4)

PROCEDURE FOR ANALYSIS

The absolute motion analysis method (also called the parametric method) relates the position of a point, P, on a rigid body undergoing rectilinear motion to the angular position, θ (parameter), of a line contained in the body. (Often this line is a link in a machine.) Once a relationship in the form of $s_P = f(\theta)$ is established, the velocity and acceleration of point P are obtained in terms of the angular velocity, ω, and angular acceleration, α, of the rigid body by taking the first and second time derivatives of the position function. Usually the chain rule must be used when taking the derivatives of the position coordinate equation.

Example 1 (16.4)

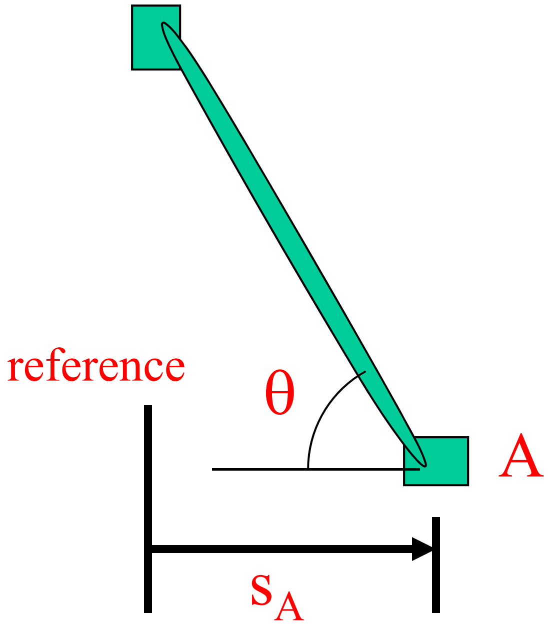


Given: Two slider blocks are connected by a rod of length 2 m. Also, $v_A = 8$ m/s and $a_A = 0$.

Find: Angular velocity, ω, and angular acceleration, α, of the rod when θ = 60°.

Plan: Choose a fixed reference point and define the position of the slider A in terms of the parameter θ. Notice from the position vector of A, positive angular position θ is measured clockwise.

Example 1 (16.4) continues



Solution:

By geometry, $s_A = 2 \cos \theta$

By differentiating with respect to time,

$$v_A = -2 \omega \sin \theta$$

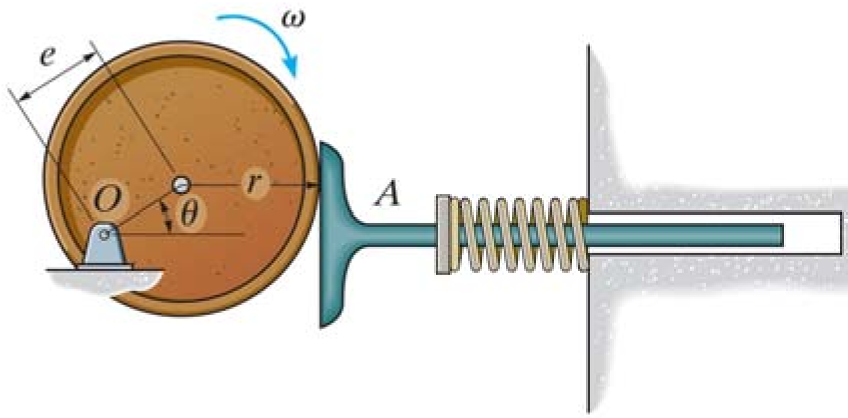
Using $\theta = 60°$ and $v_A = 8$ m/s and solving for ω:

$$\omega = 8/(-2 \sin 60°) = - 4.62 \text{ rad/s}$$

(The negative sign means the rod rotates counterclockwise as point A goes to the right.) Differentiating v_A and solving for α,

$$a_A = -2\alpha \sin \theta - 2\omega^2 \cos \theta = 0$$
$$\alpha = - \omega^2/\tan \theta = -12.32 \text{ rad/s}^2$$

EXAMPLE II



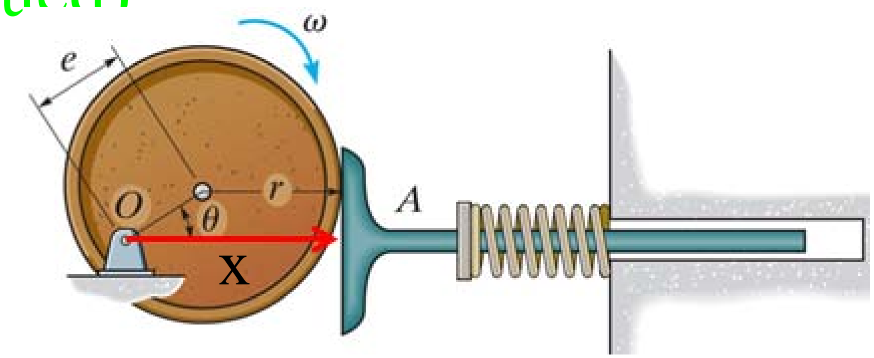
Given: A circular cam is rotating clockwise about O with a constant ω.

Find: The velocity and acceleration of the follower rod A as a function of θ.

Plan: Set the coordinate x to be the distance between O and the rod A. Relate x to the angular position, θ. Then take time derivatives of the position equation to find the velocity and acceleration relationships.

EXAMPLE II

Solution: (continued)



Relate x, the distance between O and the rod, to θ.

$$x = e\cos\theta + r$$

Take time derivatives of the position to find the velocity and acceleration.

$$\dot{x} = e(-\sin\theta)\dot{\theta} + \dot{r}$$

Since r = constant $\Rightarrow$ $\dot{x} = -e(\sin\theta)\dot{\theta}$

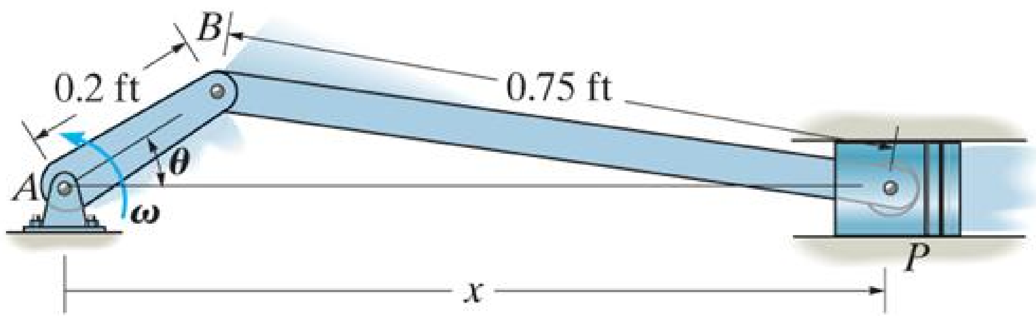
$$\ddot{x} = -e(\cos\theta)\,\dot{\theta}^2 - e(\sin\theta)\ddot{\theta}$$

Since $\dot{\theta}$ = constant $\Rightarrow$ $\ddot{x} = -e(\cos\theta)\dot{\theta}^2$

Notice that the cam is rotating clockwise. $\Rightarrow \dot{\theta} = -\omega$

Therefore, $\dot{x} = e\,\omega(\sin\theta)$ and $\ddot{x} = e\,\omega^2(\cos\theta)$

EXAMPLE III

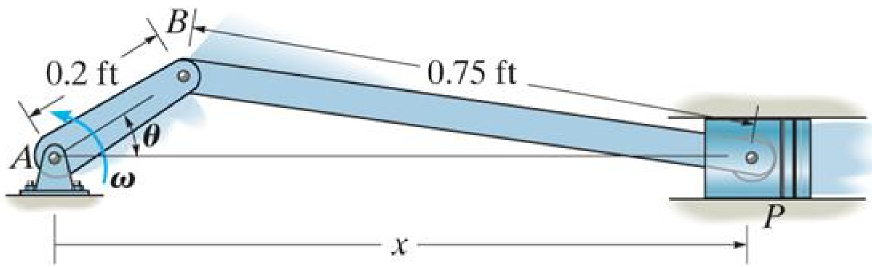


Given: Crank AB rotates at a constant velocity of ω = 150 rad/s .

Find: The velocity of point P when $\theta = 30°$.

Plan: Define x as a function of θ and differentiate with respect to time.

EXAMPLE III (continued)



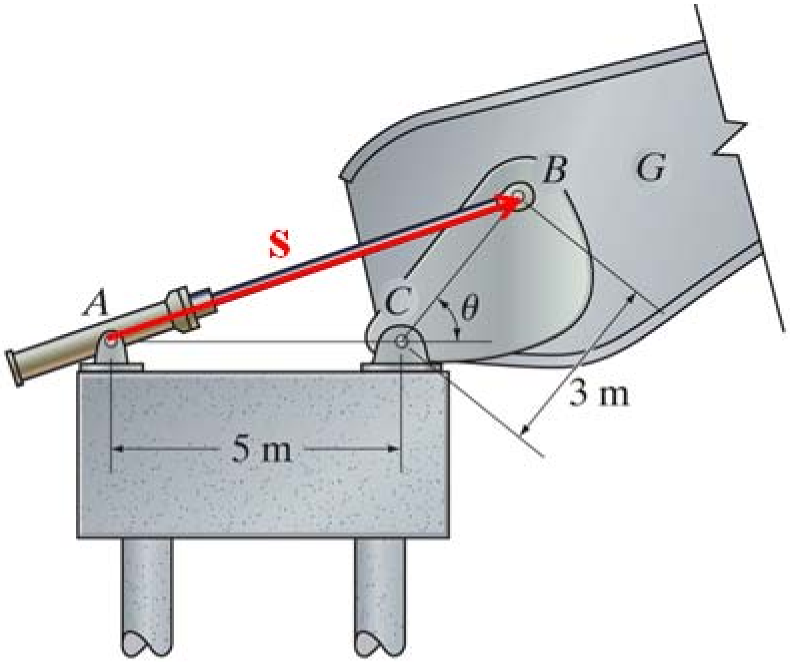
Solution: $x_P = 0.2 \cos \theta + \sqrt{(0.75)^2 - (0.2 \sin \theta)^2}$

$$v_P = -0.2\omega \sin \theta + (0.5)[(0.75)^2 - (0.2\sin \theta)^2]^{-0.5}(-2)(0.2\sin \theta)(0.2\cos \theta)\,\omega$$

$$v_P = -0.2\omega \sin \theta - [0.5(0.2)^2 \sin 2\theta\, \omega] / \sqrt{(0.75)^2 - (0.2 \sin \theta)^2}$$

At $\theta = 30°$, $\omega = 150$ rad/s and $v_P = -18.5$ ft/s $= 18.5$ ft/s $\leftarrow$

EXAMPLE IV



Given: The hydraulic cylinder AB shortens at a constant rate of 0.15 m/s so that girder G of a bascule bridge is raised.

Find: The angular velocity of the bridge girder when $\theta = 60°$.

Plan: Set the coordinate s to be distance AB. Then relate s to the angular position, θ. Take time derivative of this position relationship to find the angular velocity.

EXAMPLE IV

Solution: (continued)

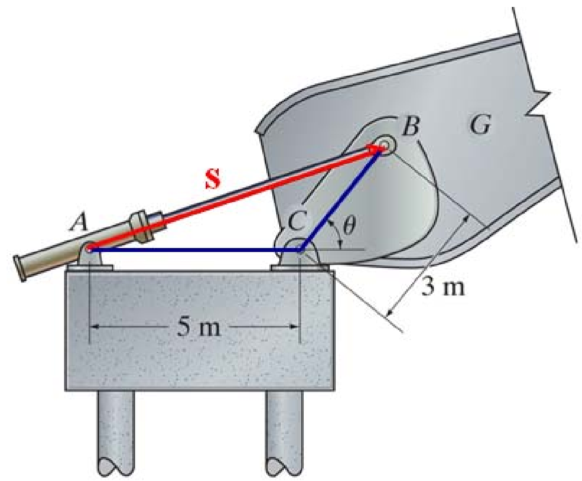
Relate s, the distance AB, to θ applying the law of cosines to the geometry.

$$AB^2 = BC^2 + AC^2 - 2(BC)(AC)\cos(180-\theta)$$

$$s^2 = 3^2 + 5^2 - 2\,(3)\,(5)\cos(180-\theta)$$

$$= 34 - 30\cos(180-\theta)$$

$$s^2 = 34 + 30\cos(\theta)$$



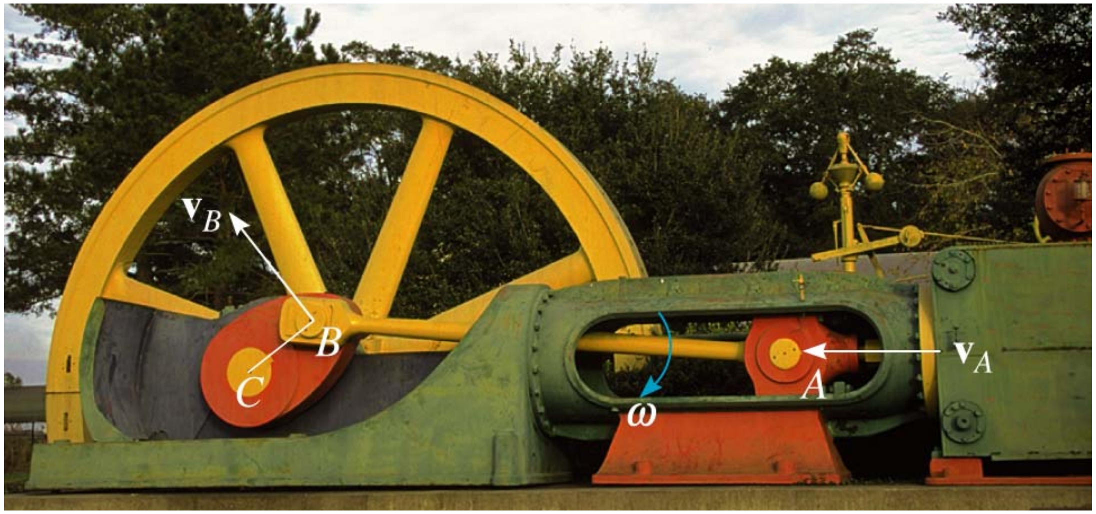
Take time derivatives of s^2 to find the velocity, $\dot{s}$.

$$2\,s\,\dot{s} = -30\sin(\theta)\dot{\theta} \quad \Rightarrow \quad \dot{\theta} = -s\,\dot{s}/(15\sin\theta)$$

When $\theta = 60° \Rightarrow \dot{s} = -0.15$ m/s, $s = \sqrt{34 + 30\cos 60} = 7$ m.

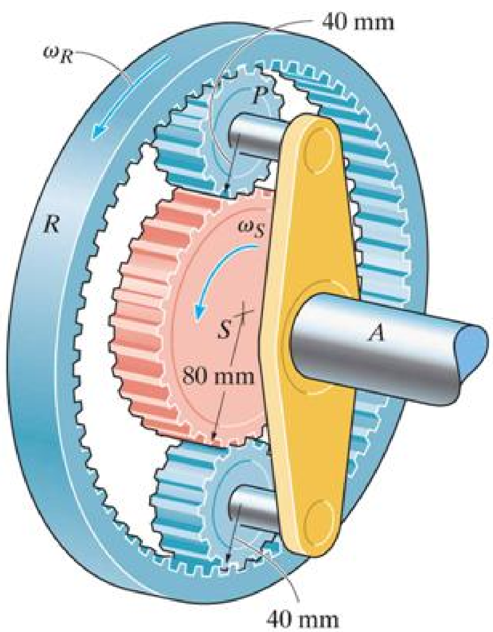
Angular velocity $\dot{\theta} = -(7)(-0.15)/(15\sin 60) = 0.0808$ rad/s

Applications for Relative motion analysis: Velocity (16.5)



As the slider block A moves horizontally to the left with $\boldsymbol{v}_A$, it causes the link CB to rotate counterclockwise. Thus $\boldsymbol{v}_B$ is directed tangent to its circular path.

APPLICATIONS (continued)



Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.

How can we relate the angular velocities of the various gears in the system?

Relative Motion

16.4 Absolute Motion: Coordinates are fixed in space.

16.5 Relative Motion: Coordinates are moving.

(1) Translating Coordinates

- F-14 takes off from carrier
- Jump shots of Michael Jordan
- Missile fired from a jet fighter

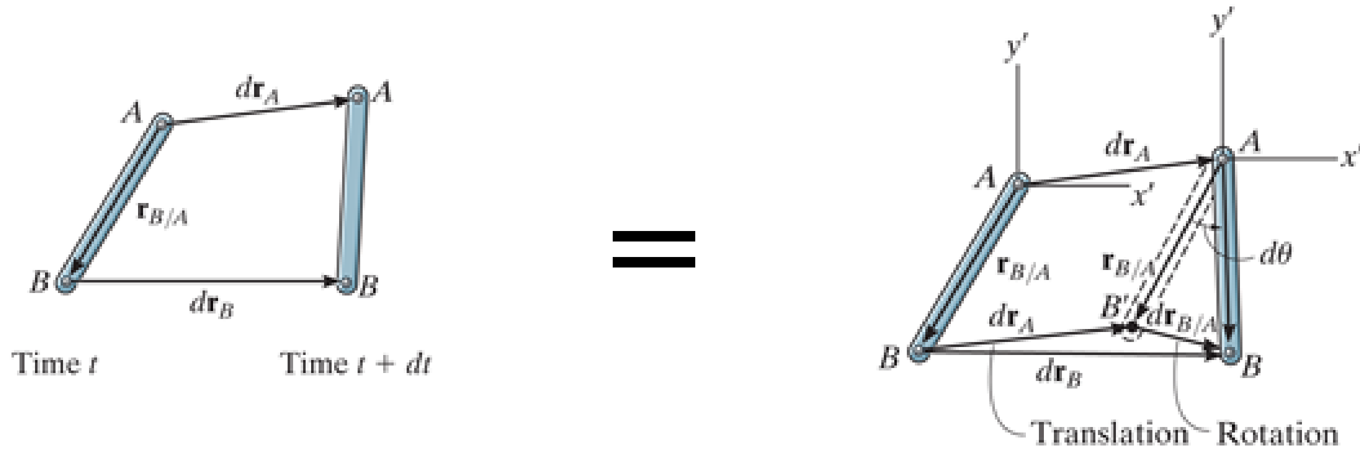
(2) Rotating Coordinates

- Dining in Space Needle Restaurant
- Foucault Pendulum
- Sunrise & Sunset

Why do we need to use relative motion? Easier way to solve complicated kinematic problem!!

Relative motion analysis (16.5)

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.



Point A is called the base point in this analysis. It generally has a known motion. The x'-y' frame translates with the body, but does not rotate. The displacement of point B can be written:

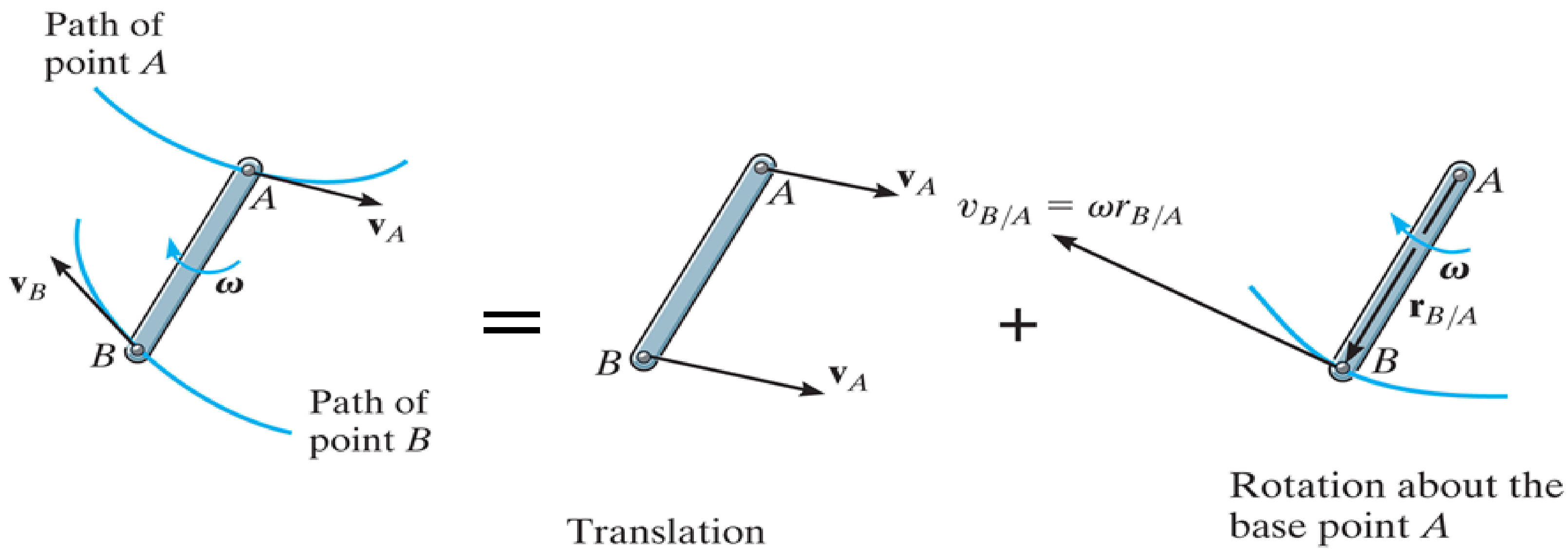
Disp. due to translation

$$d\boldsymbol{r}_B = d\boldsymbol{r}_A + d\boldsymbol{r}_{B/A}$$

Disp. due to translation and rotation

Disp. due to rotation

Relative motion analysis: Velocity (16.5)



The velocity at B is given as : $(d\boldsymbol{r}_B/dt) = (d\boldsymbol{r}_A/dt) + (d\boldsymbol{r}_{B/A}/dt)$ or

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$$

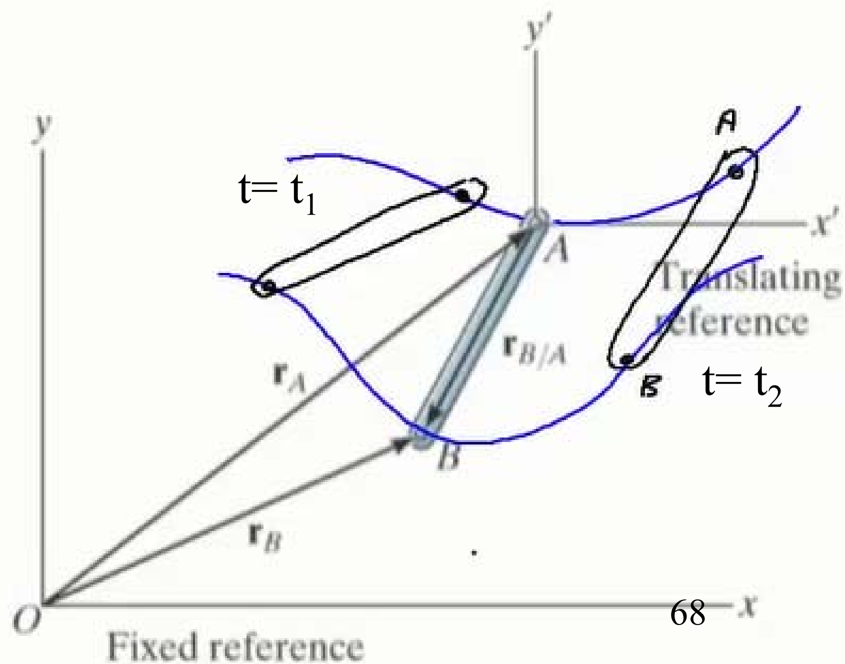
Since the body is taken as rotating about A,

$$\boldsymbol{v}_{B/A} = d\boldsymbol{r}_{B/A}/dt = \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

Here $\boldsymbol{\omega}$ will only have a $\boldsymbol{k}$ component since the axis of rotation is perpendicular to the plane of translation.

Theory: Relative Motion Analysis (16.5)

- another way to characterize this general = translation + rotation motion is to use a moving coordinate frame
- consider a rigid body (bar) AB, whose motion is "general"
- we attach a moving (translating!) reference frame to point A, and look at the rotation of B around the origin of this moving frame
- the moving reference frame does not rotate



Theory: Relative Motion Analysis (16.5)

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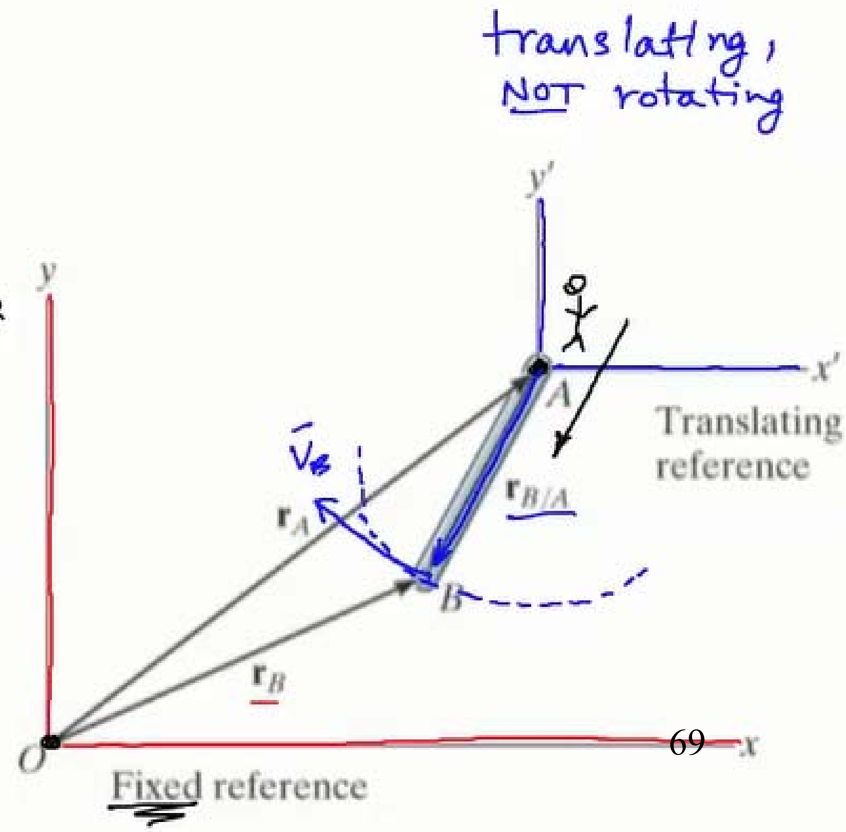
- the moving reference frame does not rotate

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A} \Rightarrow \text{translating frame}$$

$\bar{V}_{B/A}$ ⇓ pure rotation $\bar{\omega}_{AB} \times \bar{r}_{B/A}$



Theory: Position at Two Times

① pure translation, $d\bar{r}_A \Rightarrow$

② rotating B with respect to A

"pure" rotation, $d\theta$

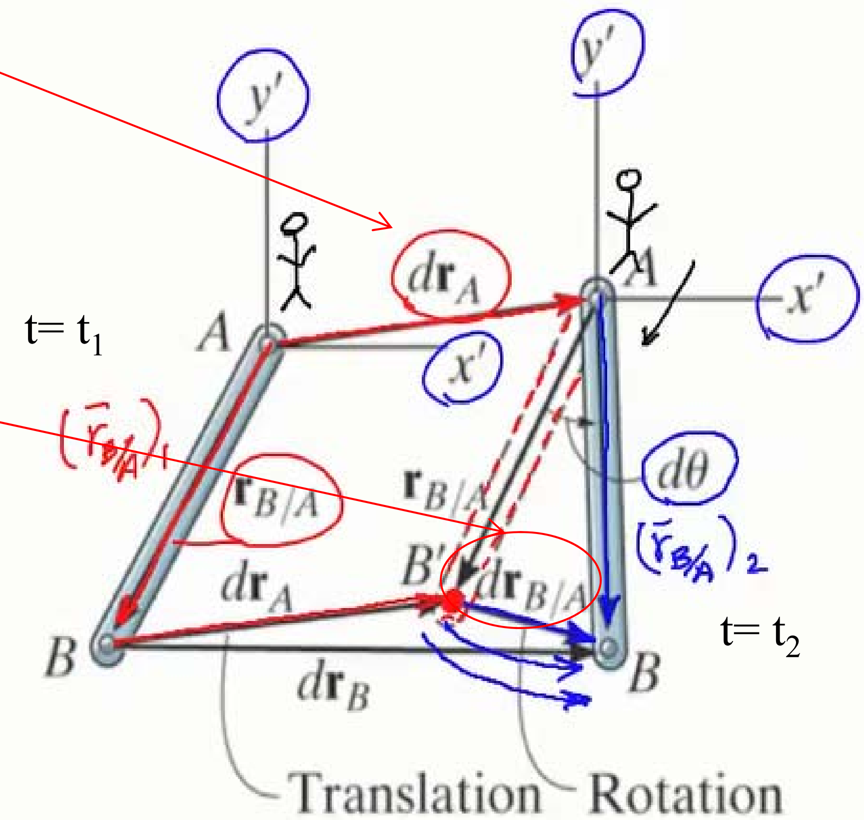
$d\bar{r}_{B/A} \Rightarrow \bar{\omega} \times \bar{r}$

general motion: ① + ②

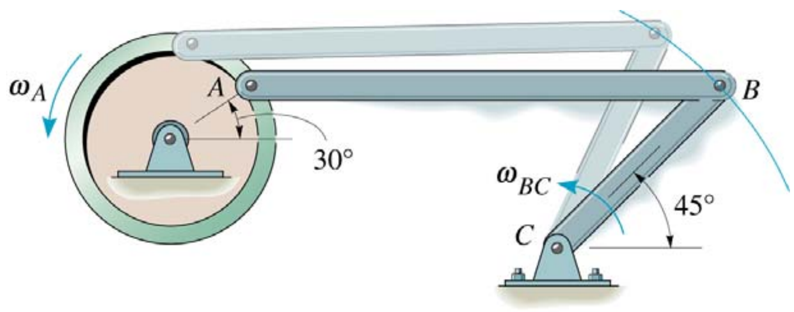
$$\bar{V}_{B/A} = \bar{\omega}_{AB} \times \bar{r}_{B/A}$$

"circular motion"

↳ rigid body



Relative motion analysis: Velocity (16.5) continues

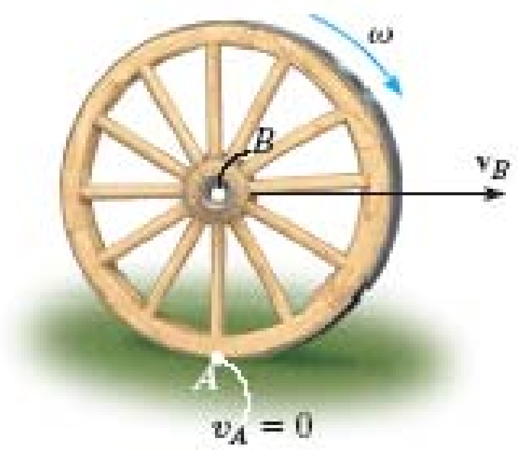


$$v_B = v_A + \omega \times r_{B/A}$$

When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.

Here both points A and B have circular motion since the disk and link BC move in circular paths. The directions of v_A and v_B are known since they are always tangent to the circular path of motion.

Relative motion analysis: Velocity (16.5) continues



$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground. Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $\boldsymbol{v}_B$ has a known direction, e.g., parallel to the surface.

Relative motion analysis: Analysis procedure (16.5)

The relative velocity equation can be applied using either a Cartesian vector analysis or by writing scalar x and y component equations directly.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector $\boldsymbol{v}_{B/A}$.
2. Write the equation $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ and by using the kinematic diagram, underneath each term represent the vectors graphically by showing their magnitudes and directions.
3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.

Relative motion analysis: Analysis procedure (16.5) continues

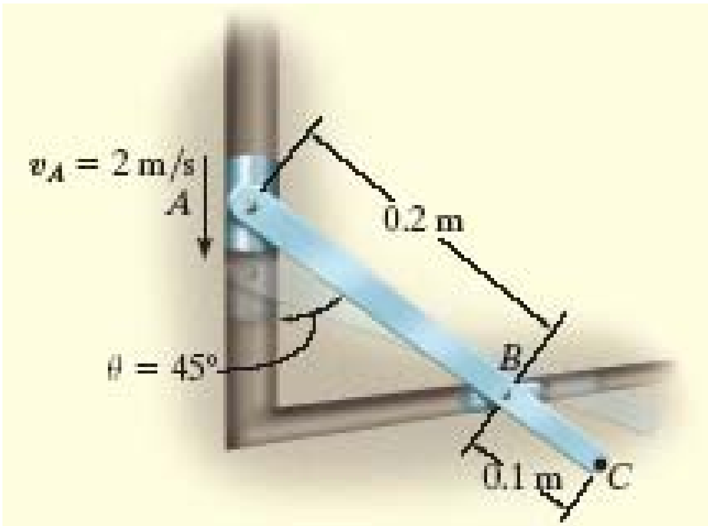
Vector Analysis:

1. Establish the fixed x-y coordinate directions and draw the kinematic diagram of the body, showing the vectors $\boldsymbol{v}_A$, $\boldsymbol{v}_B$, $\boldsymbol{r}_{B/A}$ and $\boldsymbol{\omega}$. If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in Cartesian vector form and substitute into $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$. Evaluate the cross product and equate respective $\boldsymbol{i}$ and $\boldsymbol{j}$ components to obtain two scalar equations.

3. If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.

Relative motion analysis: Velocity (16.5) Problem 2



Given: Block A is moving down at 2 m/s.

Find: The velocity of B at the instant θ = 45°.

Plan:
1. Establish the fixed x-y directions and draw a kinematic diagram.
2. Express each of the velocity vectors in terms of their ***i***, ***j***, ***k*** components and solve $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$.

Relative motion analysis: Velocity (16.5) Problem 2 continues

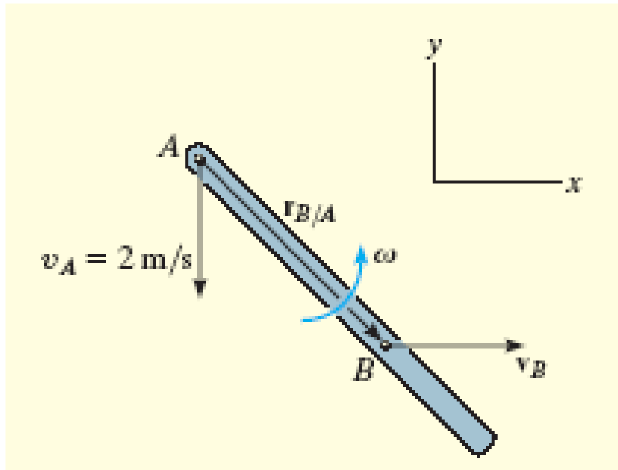
Solution:

$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{B/A}$

$v_B\,\boldsymbol{i} = -2\,\boldsymbol{j} + (\omega\,\boldsymbol{k} \times (0.2 \sin 45\,\boldsymbol{i} - 0.2 \cos 45\,\boldsymbol{j}))$

$v_B\,\boldsymbol{i} = -2\,\boldsymbol{j} + 0.2\,\omega \sin 45\,\boldsymbol{j} + 0.2\,\omega \cos 45\,\boldsymbol{i}$

Equating the ***i*** and ***j*** components gives:



$v_B = 0.2\,\omega \cos 45$

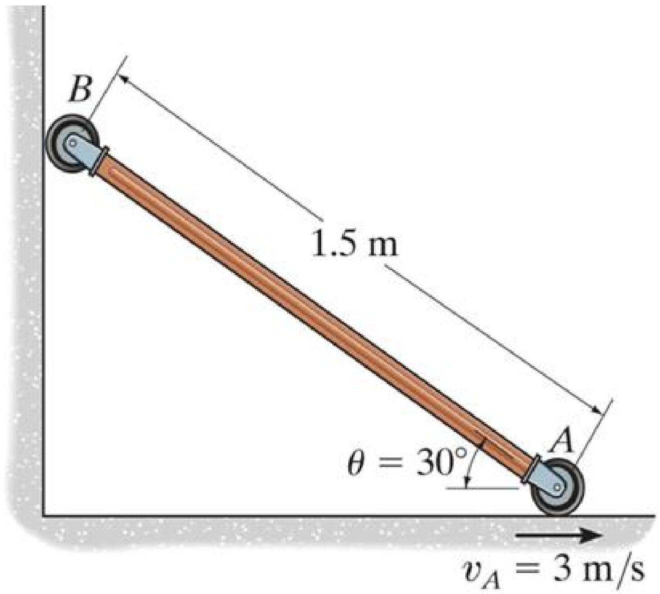
$0 = -2 + 0.2\,\omega \sin 45$

Solving:

$\omega = 14.1$ rad/s or $\boldsymbol{\omega}_{AB} = 14.1$ rad/s $\boldsymbol{k}$

$v_B = 2$ m/s or $\boldsymbol{v}_B = 2$ m/s $\boldsymbol{i}$

EXAMPLE I



Given: Roller A is moving to the right at 3 m/s.

Find: The velocity of B at the instant $\theta = 30°$.

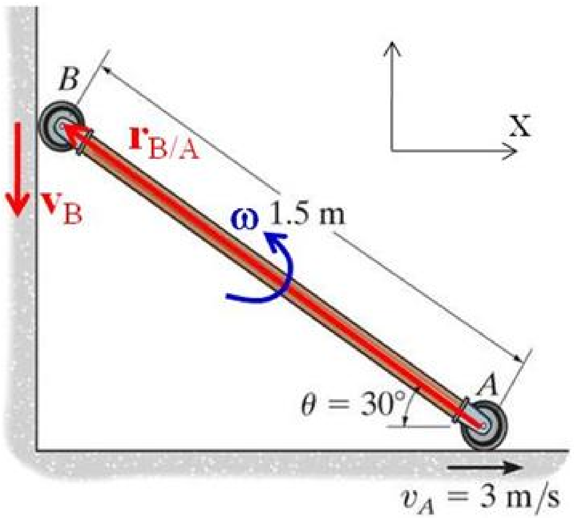
Plan:

1. Establish the fixed x - y directions and draw a kinematic diagram of the bar and rollers.
2. Express each of the velocity vectors for A and B in terms of their *i*, *j*, *k* components and solve $v_B = v_A + \omega \times r_{B/A}$.

EXAMPLE I (continued)

Solution:

Kinematic diagram:



Express the velocity vectors in CVN

$$v_B = v_A + \omega \times r_{B/A}$$

$$-v_B\, j = 3\, i + [\, \omega k \times (-1.5\cos 30 i + 1.5\sin 30 j\,)]$$

$$-v_B\, j = 3\, i - 1.299\, \omega\, j - 0.75\, \omega\, i$$

Equating the *i* and *j* components gives:

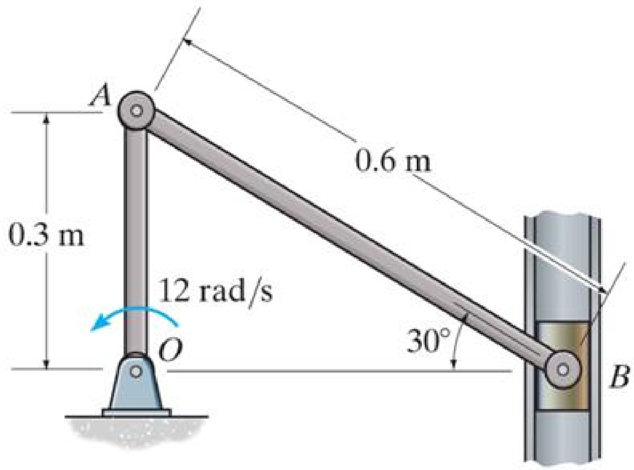
$$0 = 3 - 0.75\, \omega$$

$$-v_B = -1.299\, \omega$$

Solving: $\omega = 4$ rad/s or $\omega = 4$ rad/s k

$v_B = 5.2$ m/s or $v_B = -5.2$ m/s j

EXAMPLE II



Given: Crank rotates OA with an angular velocity of 12 rad/s.

Find: The velocity of piston B and the angular velocity of rod AB.

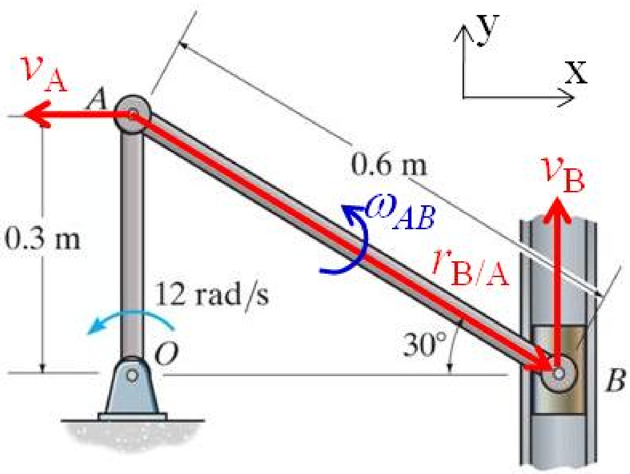
Plan:

Notice that point A moves on a circular path. The directions of v_A is tangent to its path of motion. Draw a kinematic diagram of rod AB and use $v_B = v_A + \omega_{AB} \times r_{B/A}$.

EXAMPLE II (continued)

Solution:

Kinematic diagram of AB:



Since crack OA rotates with an angular velocity of 12 rad/s, the velocity at A will be: $v_A = -0.3(12)\ i = -3.6\ i$ m/s

Rod AB. Write the relative-velocity equation:

$$v_B = v_A + \omega_{AB} \times r_{B/A}$$

$$v_B j = -3.6\ i + \omega_{AB} k \times (0.6\cos 30°\ i - 0.6\sin 30°\ j)$$

$$v_B j = -3.6\ i + 0.5196\ \omega_{AB}\ j + 0.3\ \omega_{AB}\ i$$

By comparing the i, j components:

i: $0 = -3.6 + 0.3\ \omega_{AB}$ $\Rightarrow$ $\omega_{AB} = 12$ rad/s

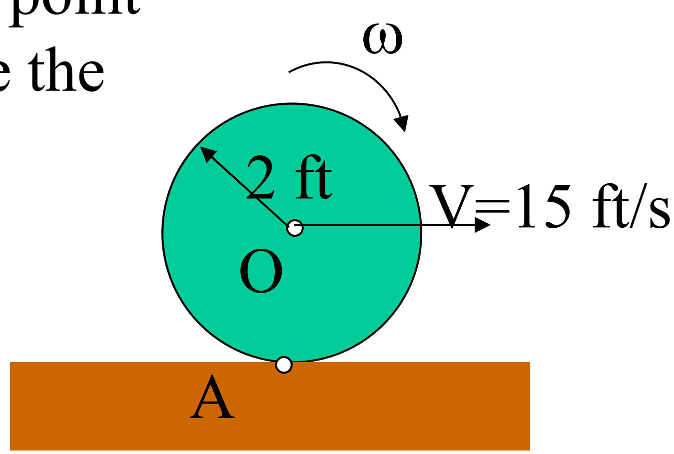
j: $v_B = 0.5196\ \omega_{AB}$ $\Rightarrow$ $v_B = 6.24$ m/s

COMCEPT QUIZ

1. If the disk is moving with a velocity at point O of 15 ft/s and ω = 2 rad/s, determine the velocity at A.

   A) 0 ft/s   B) 4 ft/s

   C) 15 ft/s   D) 11 ft/s

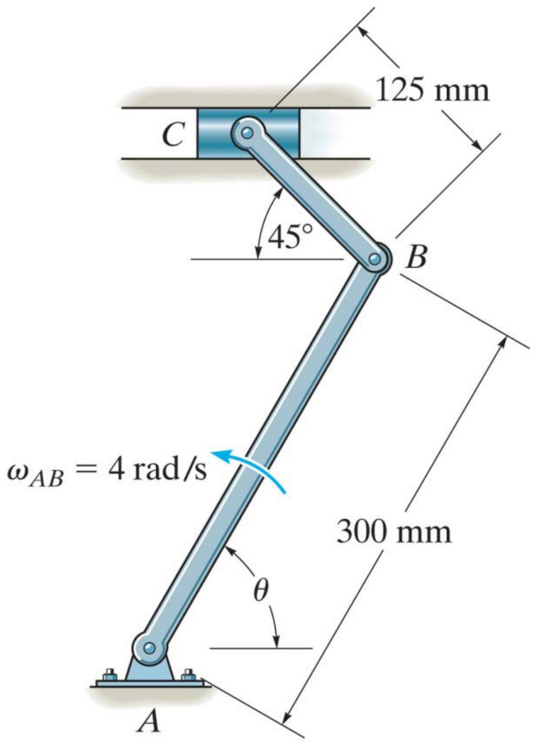


2. If the velocity at A is zero, then determine the angular velocity, ω.

   A) 30 rad/s   B) 0 rad/s

   C) 7.5 rad/s   D) 15 rad/s

Example



Given: The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. The link AB is rotating at $\omega_{AB} = 4$ rad/s.

Find: The velocity of the slider block C when $\theta = 60°$.

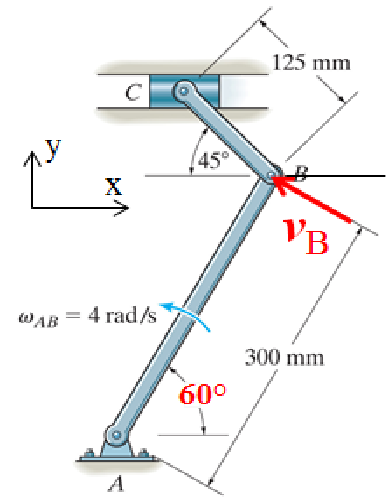
Plan: Notice that link AB rotates about a fixed point A. The directions of v_B is tangent to its path of motion. Draw a kinematic diagram of rod BC. Then, apply the relative velocity equations to the rod and solve for unknowns.

Example (continued)

Solution:

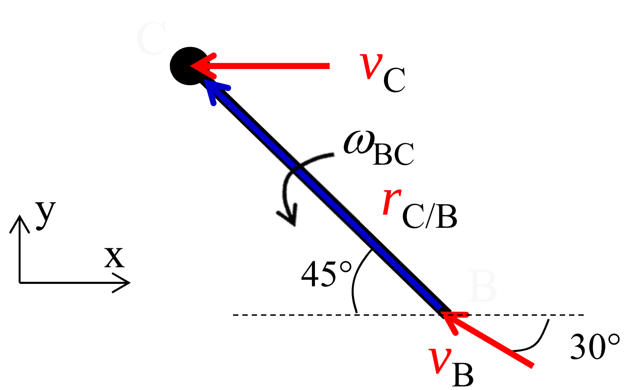
Since link AB is rotating at $\omega_{AB} = 4$ rad/s, the velocity at point B will be:

$$v_B = 4\ (300) = 1200\ \text{mm/s}$$



At $\theta = 60°$, $v_B = -1200 \cos 30°\ i + 1200 \sin 30°\ j$
$= (-1039\ i + 600\ j)$ mm/s

Draw a kinematic diagram of rod BC

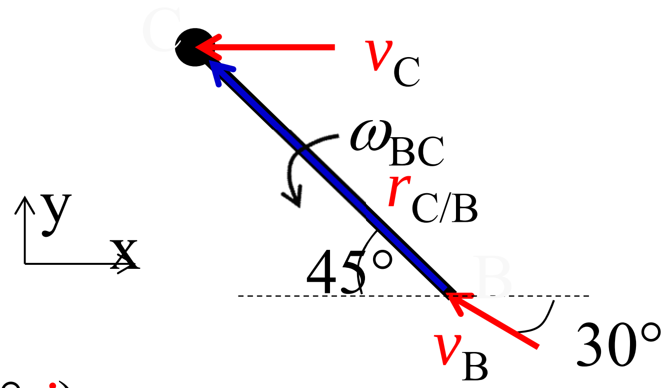


Notice that the slider block C has a horizontal motion.

Example(continued)

Solution continued:

Apply the relative velocity equation in order to find the velocity at C.



$$v_C = v_B + \omega_{BC} \times r_{C/B}$$

$$v_C\, i = (-\ 1039\ i + 600\ j) + \omega_{BC}\, k \times (-125 \cos 45°\ i + 125 \sin 45°\ j)$$

$$v_C\, i = (-1039 - 125\ \omega_{BC} \sin 45°)i + (600 - 125\ \omega_{BC} \cos 45°)j$$

Equating the *i* and *j* components yields:

$$v_C = -1039 - 125\ \omega_{BC} \sin 45°$$

$$0 = 600 - 125\ \omega_{BC} \cos 45°$$

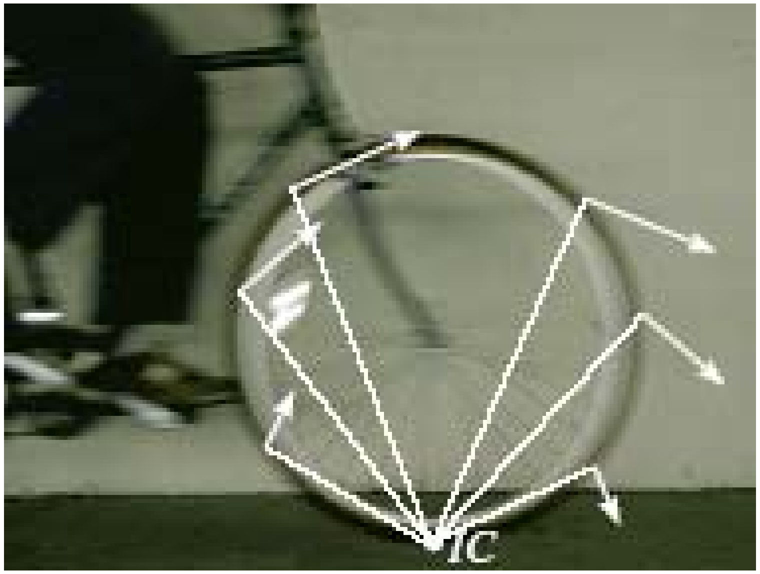
$\omega_{BC} = 4.8$ rad/s

$v_C = -\ 1639$ mm/s $= 1639$ mm/s $\leftarrow$

I might skip 16.6

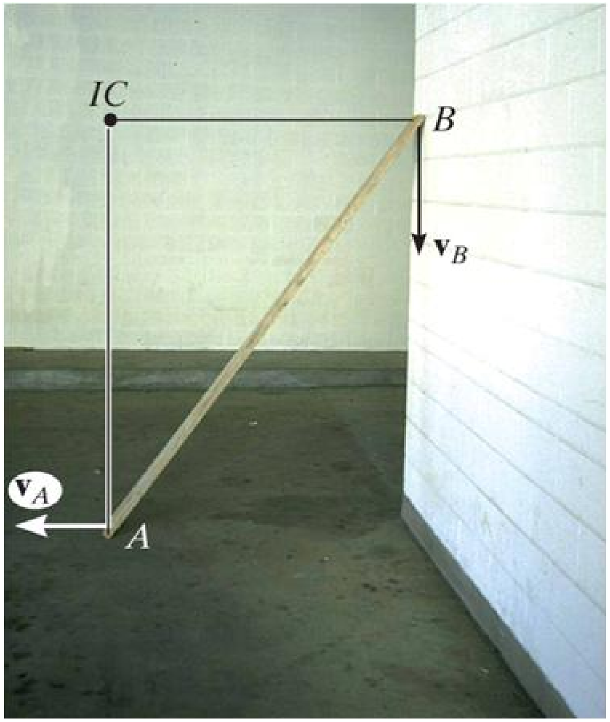
Instantaneous center of zero velocity (16.6)

Applications



The instantaneous center (IC) of zero velocity for this bicycle wheel is at the point in contact with ground. The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.

APPLICATIONS (continued)



As the board slides down the wall (to the left), it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

How can this result help you analyze other situations?

What is the direction of the velocity of the center of gravity of the board?

Instantaneous center of zero velocity (16.6)

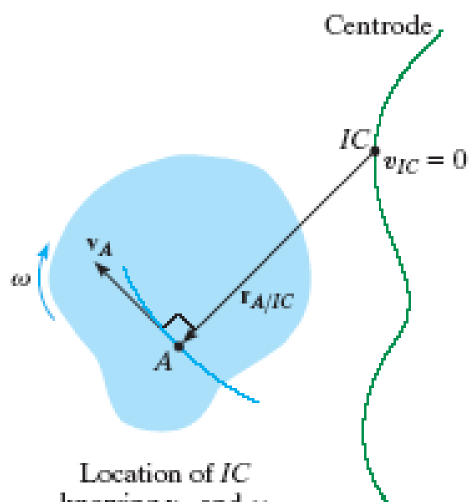
For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it were rigidly connected to the body).

This point is called the instantaneous center of zero velocity, or IC. It may or may not lie on the body!

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

Location of center of zero velocity (16.6)

To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist.

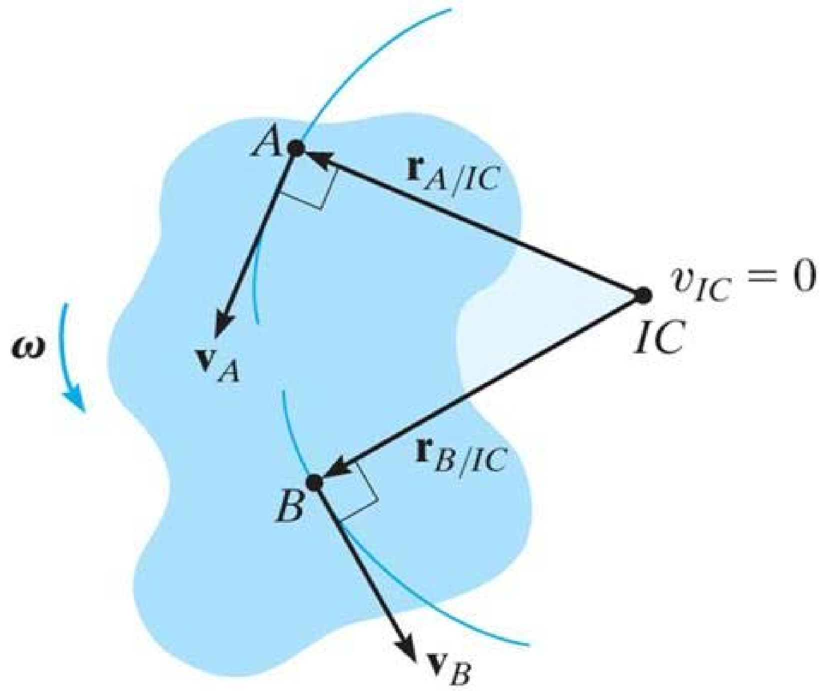


Location of IC knowing $\mathbf{v}_A$ and ω

First, consider the case when velocity $\boldsymbol{v}_A$ of a point A on the body and the angular velocity $\boldsymbol{\omega}$ of the body are known.

In this case, the IC is located along the line drawn perpendicular to $\boldsymbol{v}_A$ at A, a distance $r_{A/IC} = v_A/\omega$ from A. Note that the IC lies up and to the right of A since $\boldsymbol{v}_A$ must cause a clockwise angular velocity $\boldsymbol{\omega}$ about the IC.

Location of center of zero velocity (16.6) continues

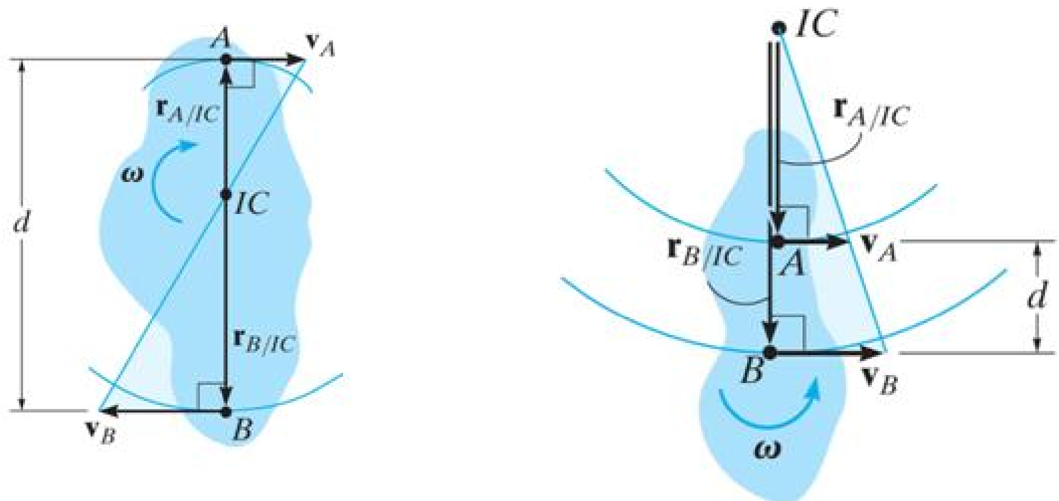


Location of *IC* knowing the directions of $\mathbf{v}_A$ and $\mathbf{v}_B$

A second case is when the lines of action of two non-parallel velocities, $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$, are known.

First, construct line segments from A and B perpendicular to $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$. The point of intersection of these two line segments locates the IC of the body.

Location of center of zero velocity (16.6) continues



Location of *IC*
knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

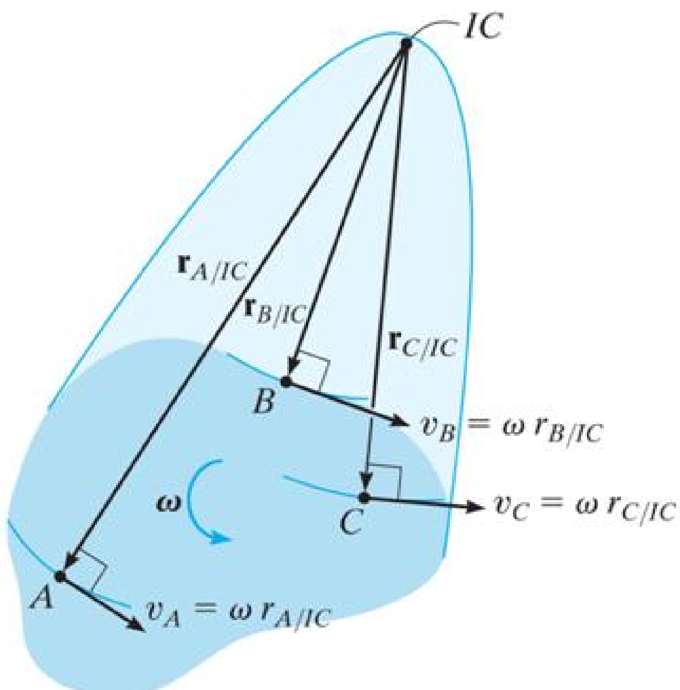
A third case is when the magnitude and direction of two parallel velocities at A and B are known.

Here the location of the IC is determined by proportional triangles. As a special case, note that if the body is translating only ($\boldsymbol{v}_A = \boldsymbol{v}_B$), then the IC would be located at infinity. Then ω equals zero, as expected.

W.Wang

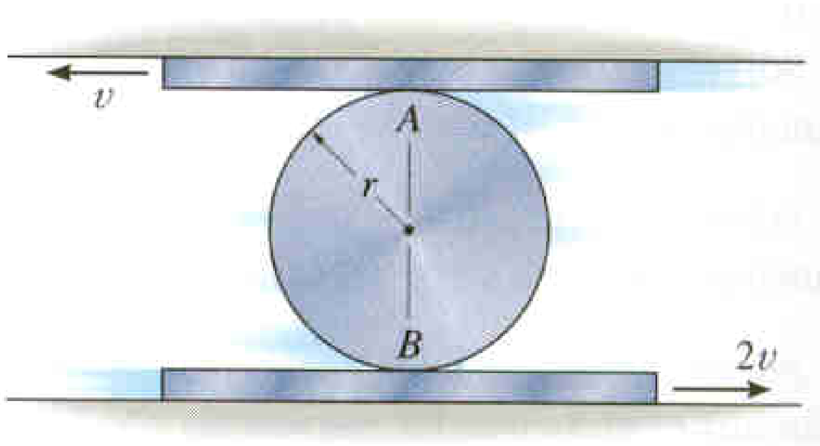
Velocity analysis (16.6)

The velocity of any point on a body undergoing general plane motion can be determined easily once the instantaneous center of zero velocity of the body is located.



Since the body seems to rotate about the IC at any instant, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is $v = \omega\, r$, where r is the radial distance from the IC to the point. The velocity's line of action is perpendicular to its associated radial line. Note the velocity has a sense of direction which tends to move the point in a manner consistent with the angular rotation direction.

Example (16.6)



Given: The disk rolls without slipping between two moving plates.

$v_B = 2v$ →

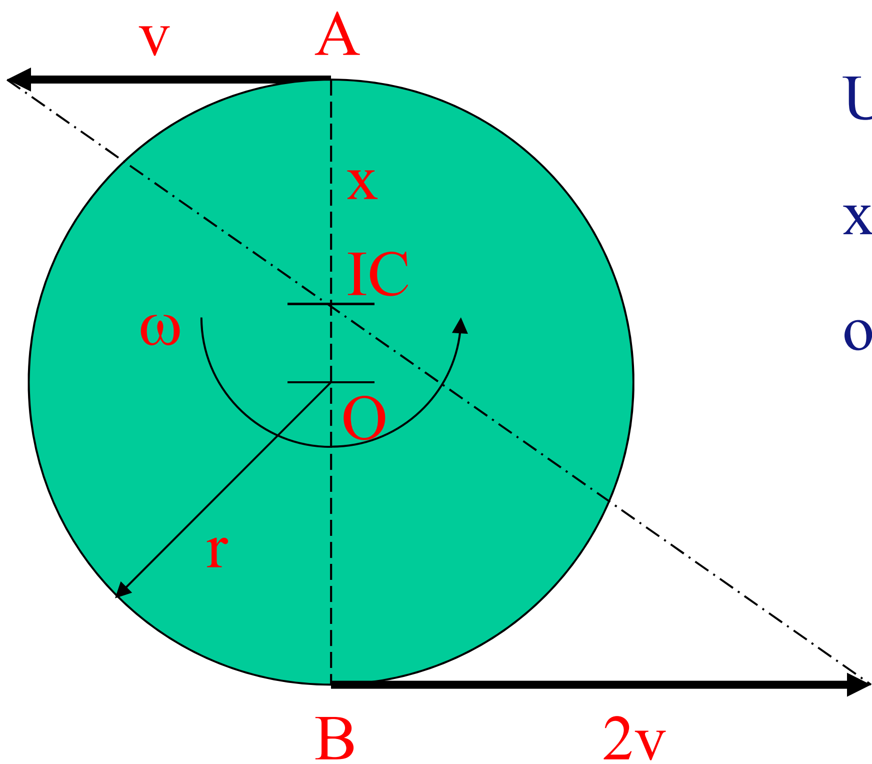
$v_A = v$ ←

Find: The angular velocity of the disk.

Plan: This is an example of the third case discussed in the lecture notes. Locate the IC of the disk using geometry and trigonometry. Then calculate the angular velocity.

Example Continues (16.6)

Solution:



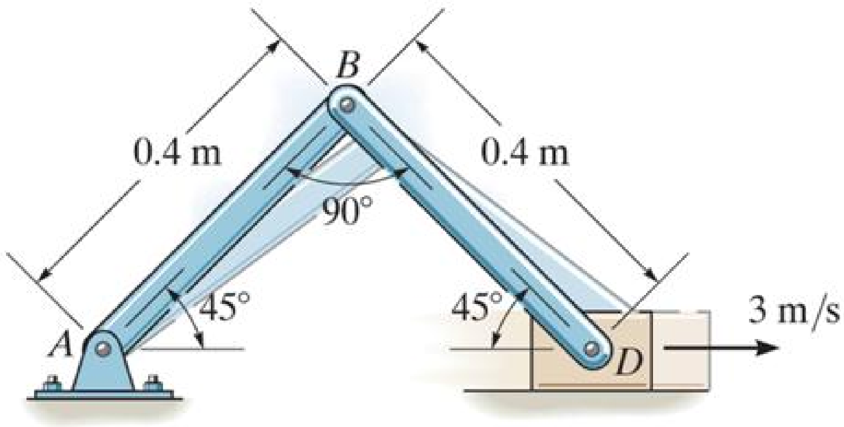
Using similar triangles:

$x/v = (2r-x)/(2v)$

or $x = (2/3)r$

Therefore $\omega = v/x = 1.5(v/r)$

EXAMPLE I

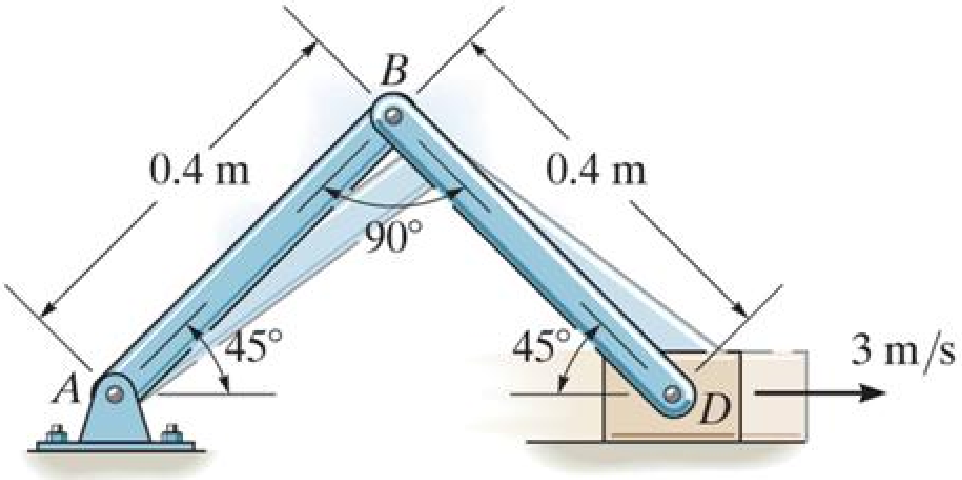


Given: A linkage undergoing motion as shown. The velocity of the block, v_D, is 3 m/s.

Find: The angular velocities of links AB and BD.

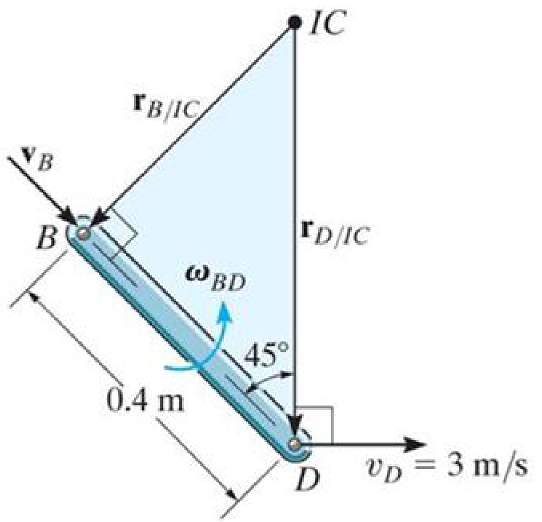
Plan: Locate the instantaneous center of zero velocity of link BD and then solve for the angular velocities.

EXAMPLE I



Solution: Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to $\boldsymbol{v}_B$ and $\boldsymbol{v}_D$. Note that $\boldsymbol{v}_B$ is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.

EXAMPLE I (continued)



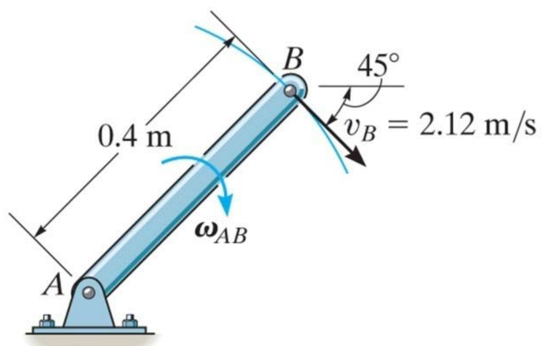
Using these facts,

$r_{B/IC} = 0.4 \tan 45° = 0.4$ m

$r_{D/IC} = 0.4/\cos 45° = 0.566$ m

Since the magnitude of v_D is known, the angular velocity of link BD can be found from $v_D = \omega_{BD}\, r_{D/IC}$.

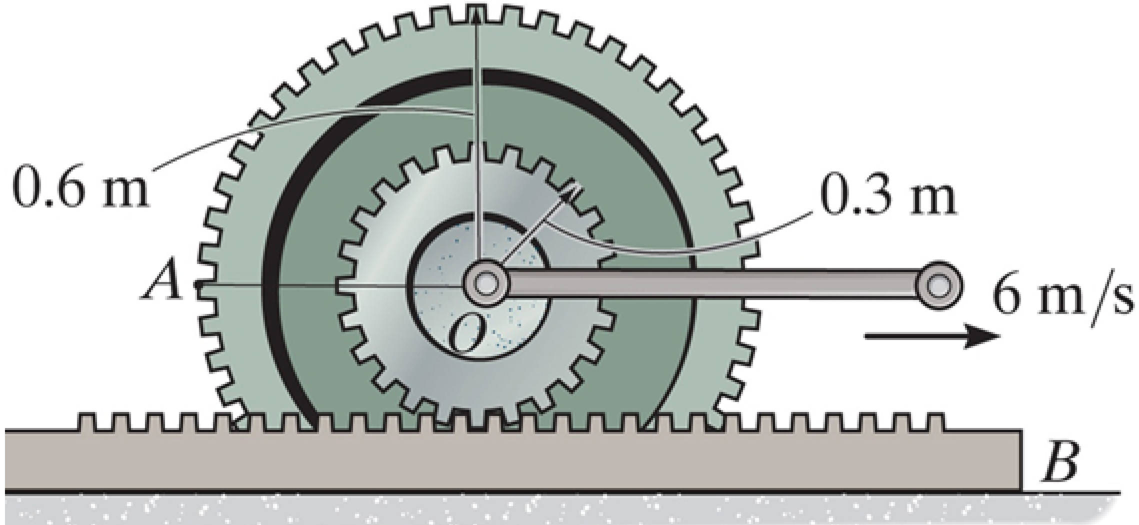
$\omega_{BD} = v_D/r_{D/IC} = 3/0.566 = 5.3$ rad/s



Link AB is subjected to rotation about A.

$\omega_{AB} = v_B/r_{B/A} = (r_{B/IC})\omega_{BD}/r_{B/A} = 0.4(5.3)/0.4 = 5.3$ rad/s

EXAMPLE II



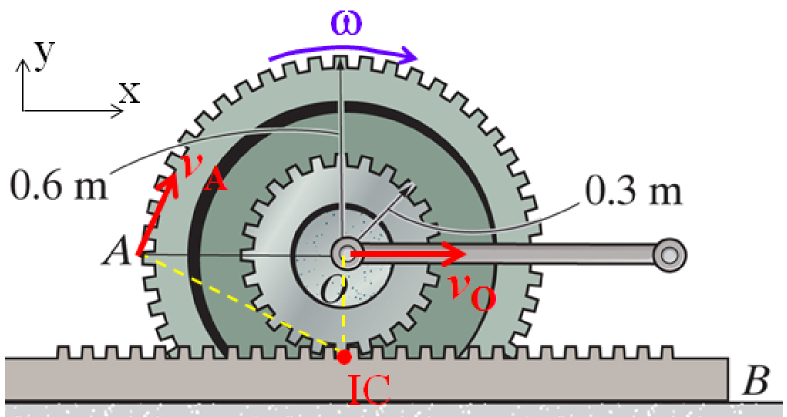
Given: The center O of the gear set rolls with v_O = 6 m/s. The gear rack B is fixed.

Find: The velocity of point A on the outer gear.

Plan: Locate the IC of the smaller gear. Then calculate the velocities at A.

EXAMPLE II
(continued)

Solution:



Note that the gear rolls without slipping. Thus, the IC is at the contact point with the gear rack B.

The angular velocity of the wheel can be found from

$\omega = v_O/r_{O/IC} = 6/0.3 = 20$ rad/s (↻ or CW)

The velocity at A will be

$\boldsymbol{v}_A = \boldsymbol{\omega} \times \boldsymbol{r}_{A/IC} = (-20)\,\boldsymbol{k} \times (-0.6\,\boldsymbol{i} + 0.3\boldsymbol{j}) = (6\,\boldsymbol{i} + 12\,\boldsymbol{j})$ m/s

$v_A = \sqrt{6^2 + 12^2} = 13.4$ m/s

$\theta = \tan^{-1}(12/6) = 63.4°$

CONCEPT QUIZ

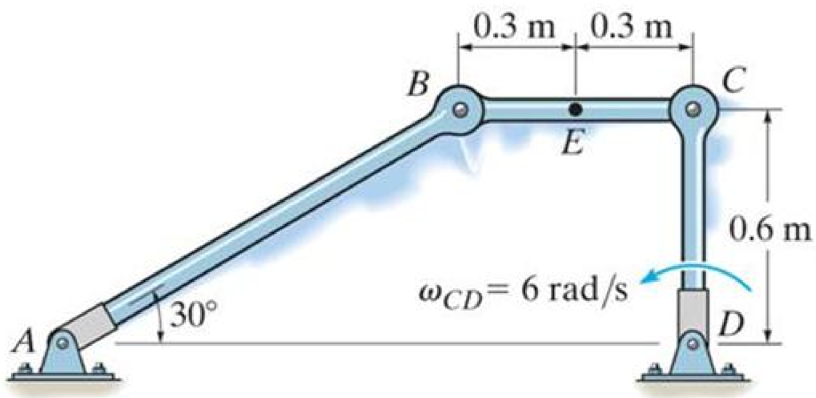
1. When the velocities of two points on a body are equal in magnitude and parallel but in opposite directions, the IC is located at

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

2. When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

Example III



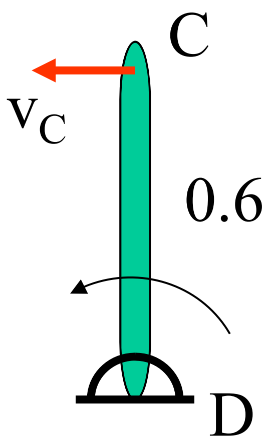
Given: The four bar linkage is moving with ω_{CD} equal to 6 rad/s CCW.

Find: The velocity of point E on link BC and angular velocity of link AB.

Plan: This is an example of the second case in the lecture notes. Since the direction of Point B's velocity must be perpendicular to AB, and Point C's velocity must be perpendicular to CD, the location of the instantaneous center, I, for link BC can be found.

Example III
(continued)

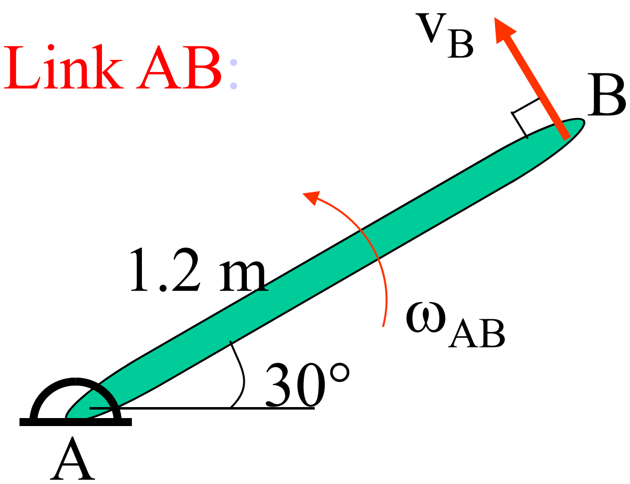
Link CD:



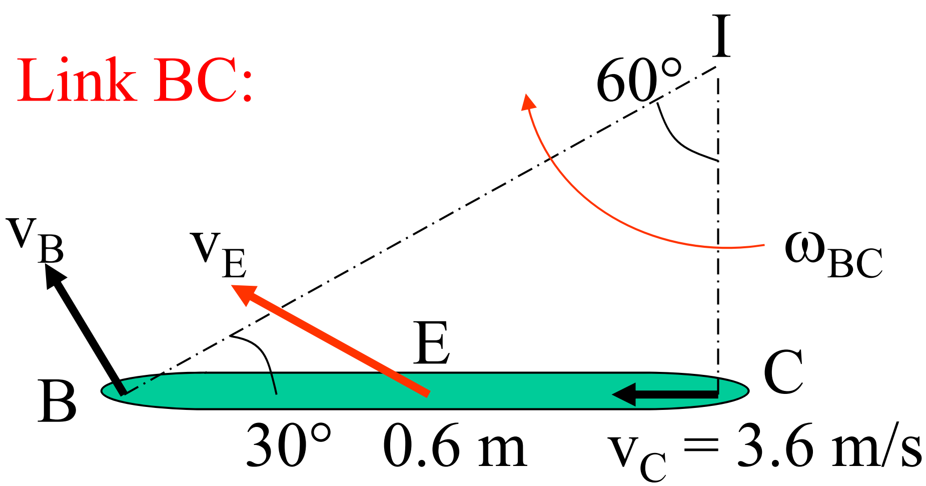
$\omega_{CD} = 6$ rad/s

$v_C = 0.6(6) = 3.6$ m/s

Link AB:



Link BC:



From triangle CBI

IC = 0.346 m

IB = 0.6/sin 60° = 0.693 m

$v_C = (IC)\omega_{BC}$

$\omega_{BC} = v_C/IC = 3.6/0.346$

$\omega_{BC} = 10.39$ rad/s

Example III
(continued)

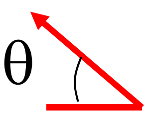
$v_B = (IB)\omega_{BC} = 0.693(10.39) = 7.2$ m/s

From link AB, v_B is also equal to $1.2\ \omega_{AB}$.

Therefore $7.2 = 1.2\ \omega_{AB}$ => $\omega_{AB} = 6$ rad/s

$v_E = (IE)\omega_{BC}$ where distance IE $= \sqrt{0.3^2 + 0.346^2} = 0.458$ m

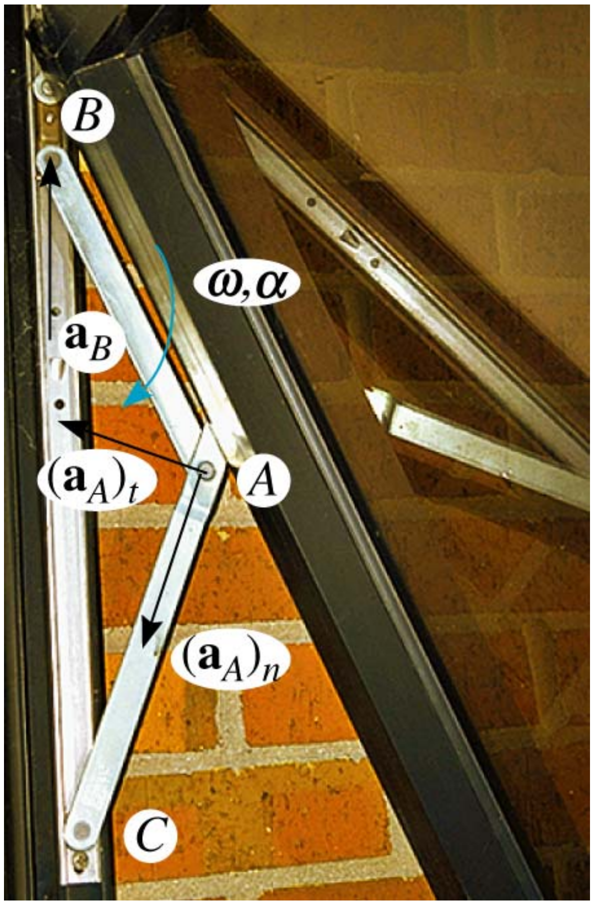
$v_E = 0.458(10.39) = 4.76$ m/s θ



where $\theta = \tan^{-1}(0.3/0.346) = 40.9°$

I might skip 16.6

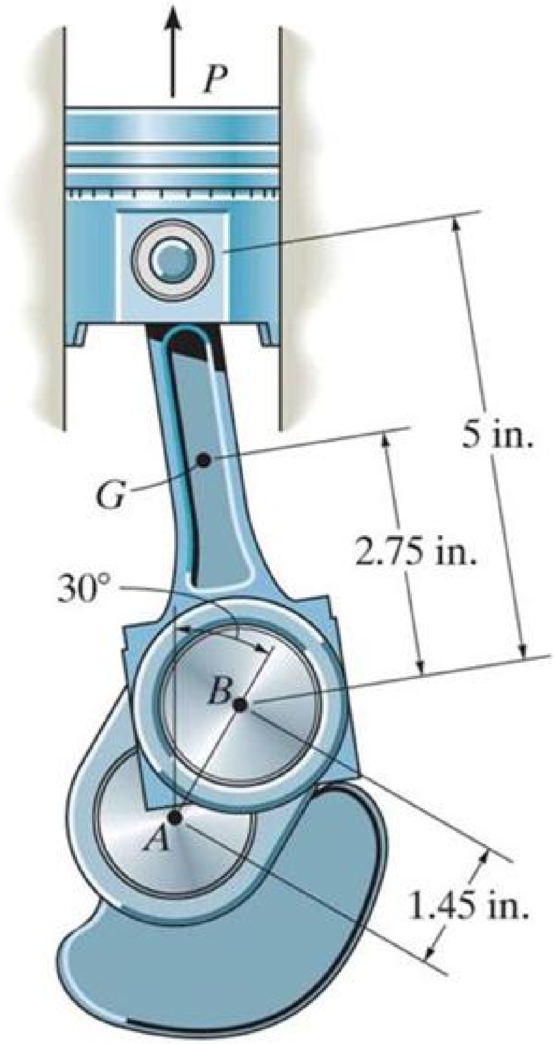
Relative motion analysis: Acceleration (16.7) Applications



In the mechanism for a window, link AC rotates about a fixed axis through C, while point B slides in a straight track. The components of acceleration of these points can be inferred since their motions are known.

To prevent damage to the window, the accelerations of the links must be limited.

APPLICATIONS (continued)



In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?

Relative motion analysis: Acceleration (16.7)

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

$$\frac{dv_B}{dt} = \frac{dv_A}{dt} + \frac{dv_{B/A}}{dt}$$

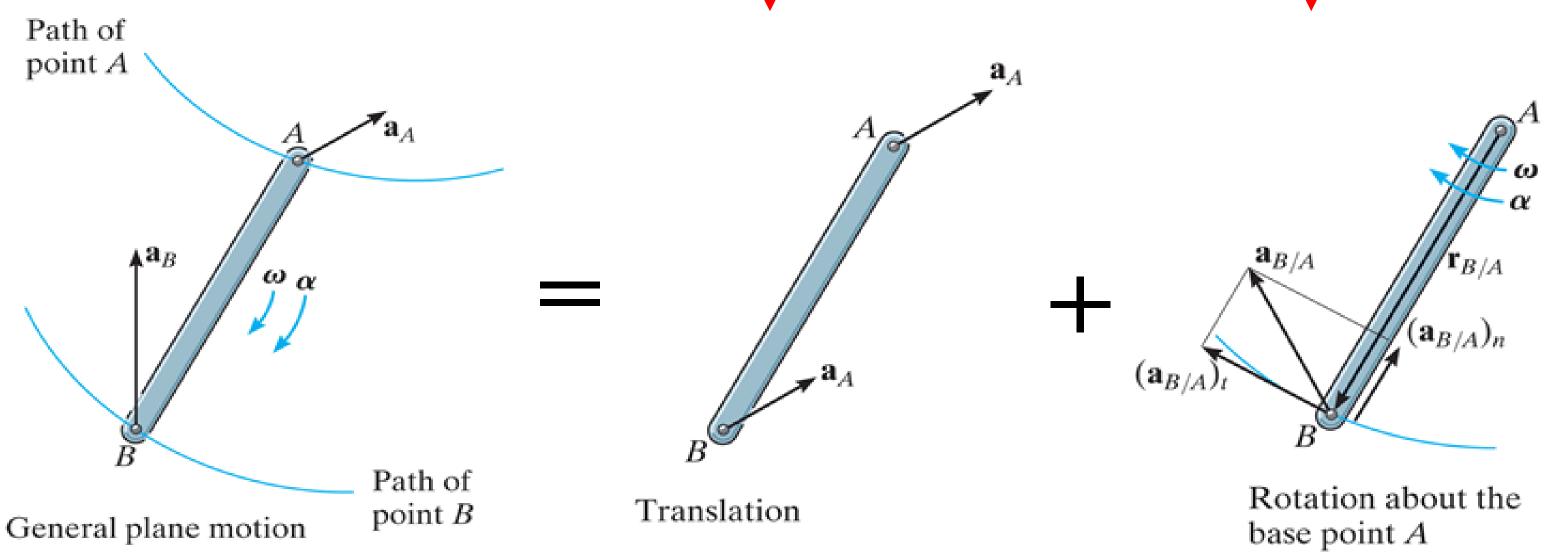
These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A. It will develop tangential and normal components.

The result is $\boldsymbol{a}_B = \boldsymbol{a}_A + (\boldsymbol{a}_{B/A})_t + (\boldsymbol{a}_{B/A})_n$

Relative motion analysis: Acceleration (16.7) continues

Graphically: $\boldsymbol{a}_B = \boldsymbol{a}_A + (\boldsymbol{a}_{B/A})_t + (\boldsymbol{a}_{B/A})_n$



The relative tangential acceleration component $(\boldsymbol{a}_{B/A})_t$ is $(\boldsymbol{\alpha} \times \boldsymbol{r}_{B/A})$ and perpendicular to $\boldsymbol{r}_{B/A}$.

The relative normal acceleration component $(\boldsymbol{a}_{B/A})_n$ is $(-\omega^2 \boldsymbol{r}_{B/A})$ and the direction is always from B towards A.

Relative motion analysis: Acceleration (16.7) continues

Since the relative acceleration components can be expressed as $(\boldsymbol{a}_{B/A})_t = \alpha \times \boldsymbol{r}_{B/A}$ and $(\boldsymbol{a}_{B/A})_n = -\omega^2 \boldsymbol{r}_{B/A}$ the relative acceleration equation becomes

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \alpha \times \boldsymbol{r}_{B/A} - \omega^2 \boldsymbol{r}_{B/A}$$

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, ω^2) and the relative position vector, $\boldsymbol{r}_{B/A}$.

Theory: Relative Motion, Acceleration (16.7)

- acceleration analysis for general motion also follows the "relative" form, by differentiation of the relative velocity equation:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\underbrace{\bar{a}_B = \bar{a}_A}_{\text{absolute!}} + \underbrace{\bar{a}_{B/A}}_{\text{relative}}$$

$$\bar{a}_{B/A} = \underbrace{\bar{\alpha}_{AB} \times \bar{r}_{B/A}}_{\text{tang.}} + \underbrace{\bar{\omega}_{AB} \times (\vec{\omega}_{AB} \times \bar{r}_{B/A})}_{\text{normal}}$$

Theory: Relative Motion Analysis (16.5)

- another way to characterize this general = translation + rotation motion is to use a moving coordinate frame
- consider a rigid body (bar) AB, whose motion is "general"
- we attach a moving (translating!) reference frame to point A, and look at the rotation of B around the origin of this moving frame
- the moving reference frame does not rotate

translating, NOT rotating

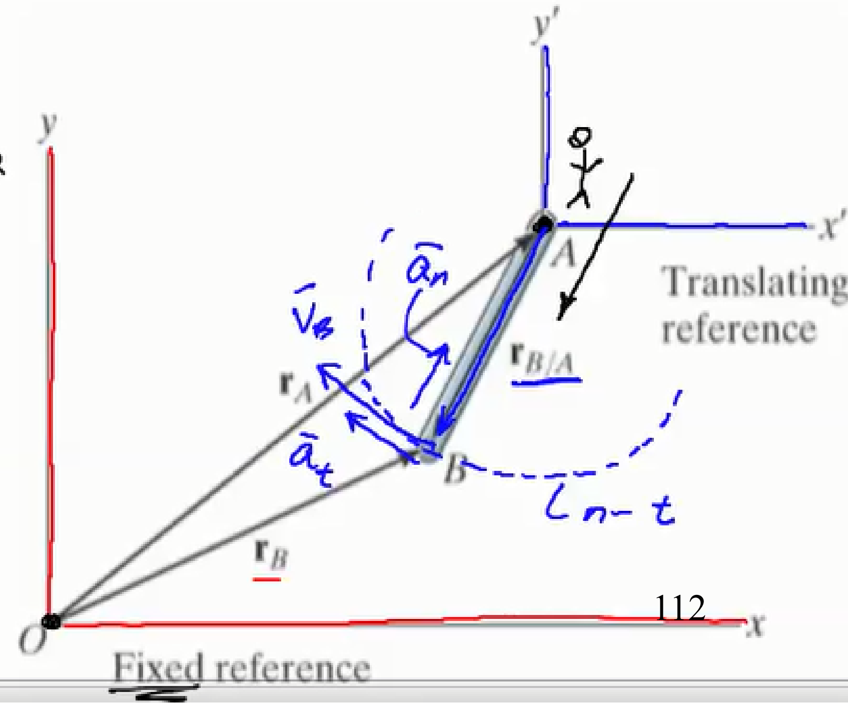
$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$$

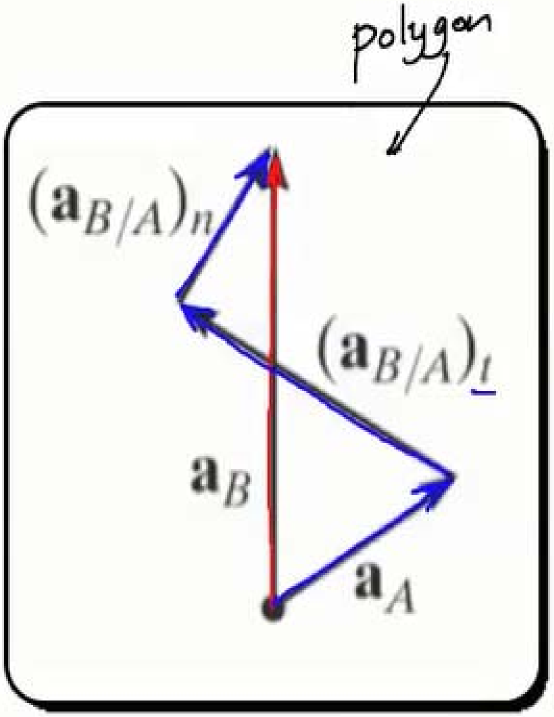
$\bar{V}_{B/A}$ ⇒ translating frame

⇓ pure rotation

$\bar{\omega}_{AB} \times \bar{r}_{B/A}$



Theory: Acceleration Diagram



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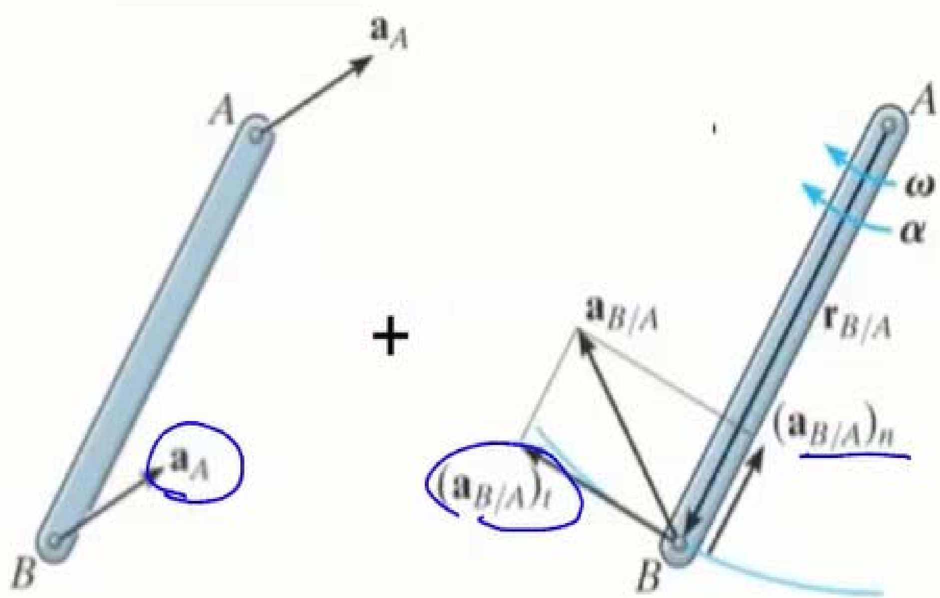


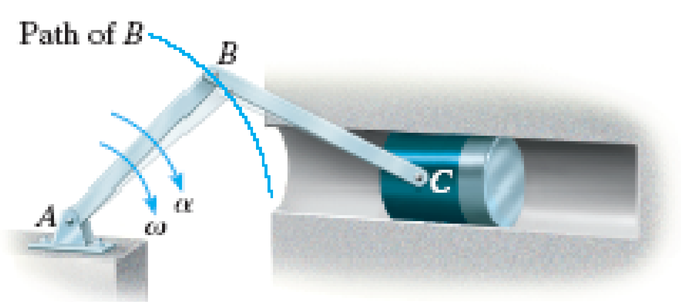
Translation

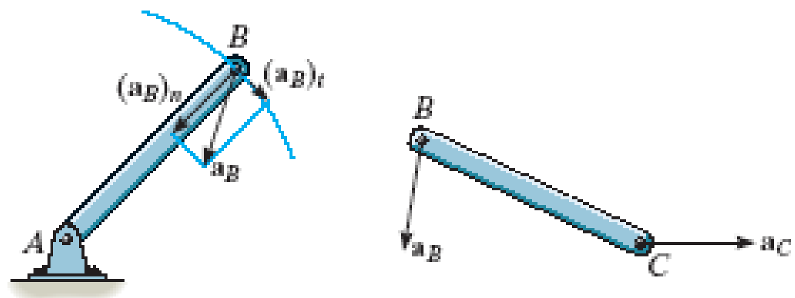
Rotation about the base point A

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$$

Application of relative acceleration equation

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.





In this mechanism, point B is known to travel along a circular path, so $\boldsymbol{a}_B$ can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, $\boldsymbol{a}_C$ is directed horizontally.

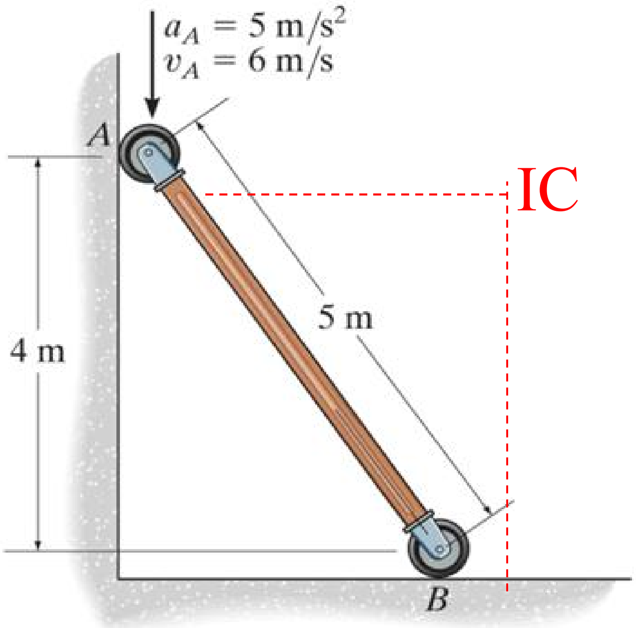
Procedure of analysis (16.7)

1. Establish a fixed coordinate system.
2. Draw the kinematic diagram of the body.
3. Indicate on it $\boldsymbol{a}_A$, $\boldsymbol{a}_B$, $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, and $\boldsymbol{r}_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $\boldsymbol{a}_A = (\boldsymbol{a}_A)_t + (\boldsymbol{a}_A)_n$ and $\boldsymbol{a}_B = (\boldsymbol{a}_B)_t + (\boldsymbol{a}_B)_n$.
4. Apply the relative acceleration equation:

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \alpha \times \boldsymbol{r}_{B/A} - \omega^2 \, \boldsymbol{r}_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, it indicates the sense of direction of the vector is opposite to that shown on the diagram.

EXAMPLE I



Given: Point A on rod AB has an acceleration of 5 m/s^2 and a velocity of 6 m/s at the instant shown.

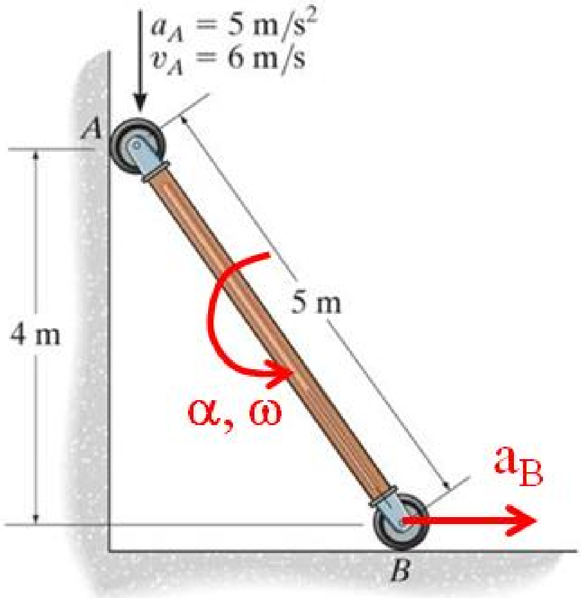
Find: The angular acceleration of the rod and the acceleration at B at this instant.

Plan: Follow the problem solving procedure!

Solution: First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB, we can determine ω:

$\omega = v_A/r_{A/IC} = v_A/\,(3) = 2$ rad/s

EXAMPLE I (continued)



Since points A and B both move along straight-line paths,

$a_A = -5\, j$ m/s^2

$a_B = a_B\, i$ m/s^2

Applying the relative acceleration equation

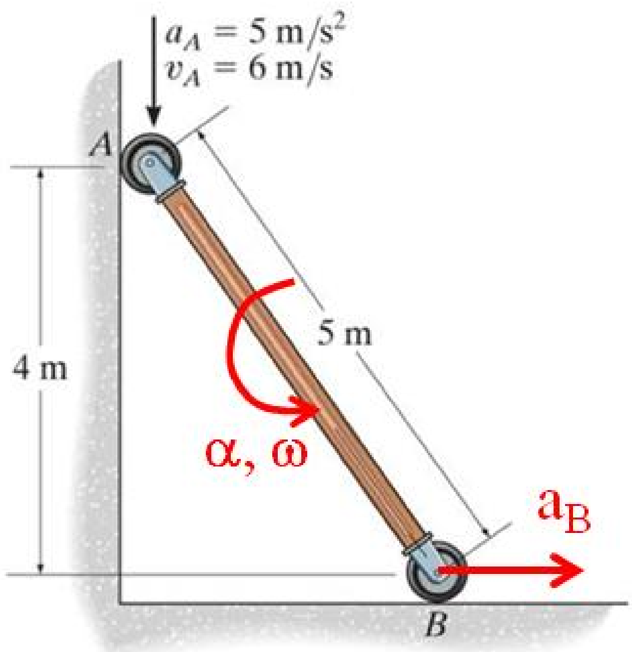
$$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

$$a_B\, i = -\,5\, j + \alpha\, k \times (3\, i - 4\, j) - 2^2\,(3\, i - 4\, j)$$

$$a_B\, i = -\,5\, j + 4\,\alpha\, i + 3\,\alpha\, j - (12\, i - 16\, j)$$

EXAMPLE I (continued)

So with $a_B\, i = -5\, j + 4\,\alpha\, i + 3\,\alpha\, j - (12\; i - 16\, j)$, we can solve for a_B and α.



By comparing the *i, j* components;

$$a_B = 4\,\alpha - 12$$

$$0 = 11 + 3\alpha$$

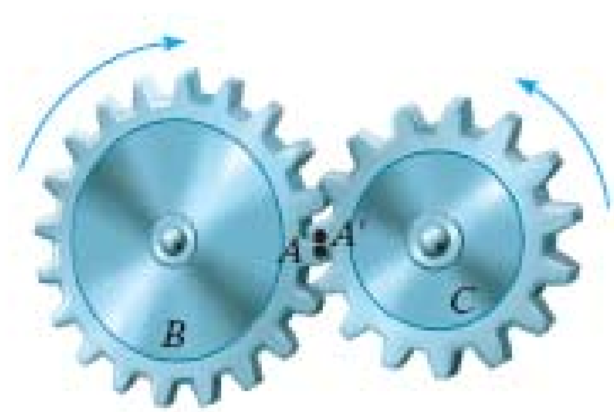
Solving:

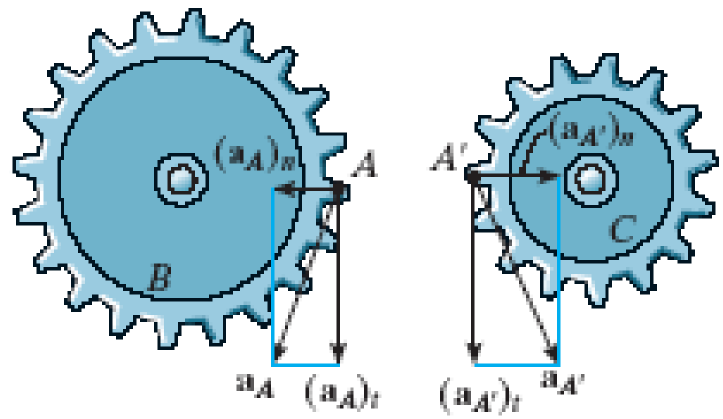
$$a_B = -26.7\ \text{m/s}^2$$

$$\alpha = -3.67\ \text{rad/s}^2$$

Bodies in contact (16.7)

Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.





In this case, the tangential components of acceleration will be the same, i. e.,

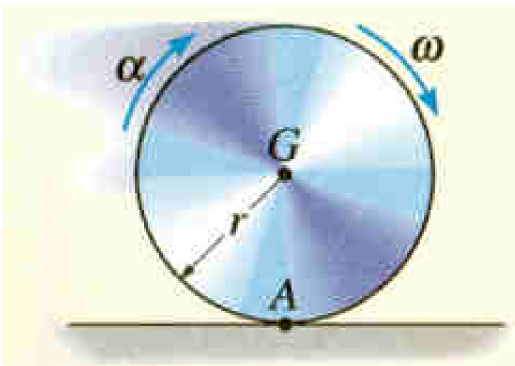
$$(\boldsymbol{a}_A)_t = (\boldsymbol{a}_{A'})_t \text{ (which implies } \alpha_B r_B = \alpha_C r_C \text{).}$$

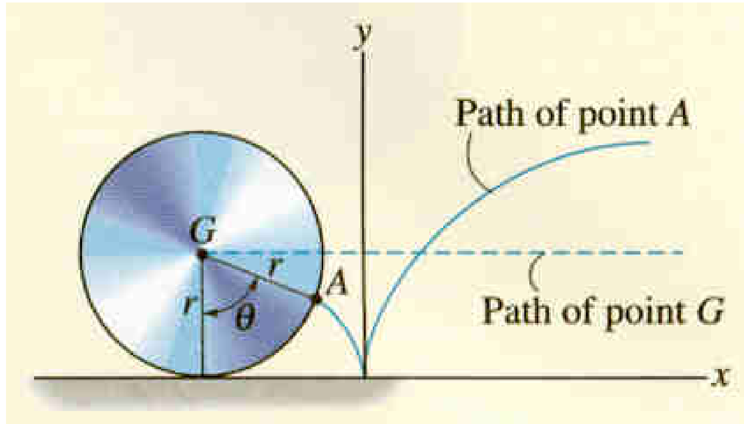
The normal components of acceleration will not be the same.

$$(\boldsymbol{a}_A)_n \neq (\boldsymbol{a}_{A'})_n \text{ so } \boldsymbol{a}_A \neq \boldsymbol{a}_{A'}$$

Rolling motion (16.7)

Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball or disk rolling along a flat surface without slipping. This problem can be analyzed using relative velocity and acceleration equations.

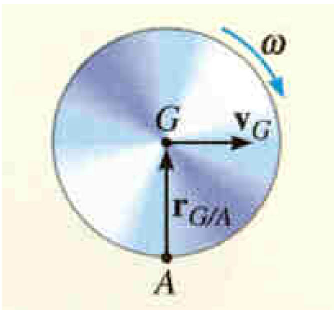




As the cylinder rolls, point G (center) moves along a straight line, while point A, on the rim of the cylinder, moves along a curved path called a cycloid. If ω and α are known, the relative velocity and acceleration equations can be applied to these points, at the instant A is in contact with the ground.

Rolling motion (16.7) continues

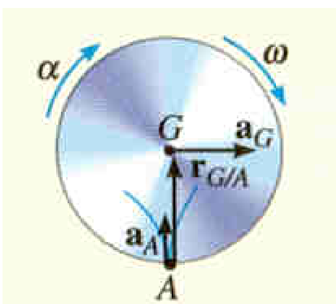
- Velocity:



Since no slip occurs, $\boldsymbol{v}_A = \boldsymbol{0}$ when A is in contact with ground. From the kinematic diagram:

$$\boldsymbol{v}_G = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{G/A}$$
$$v_G \boldsymbol{i} = \boldsymbol{0} + (-\omega \boldsymbol{k}) \times (r \boldsymbol{j})$$
$$v_G = \omega r \quad \text{or} \quad \boldsymbol{v}_G = \omega r \boldsymbol{i}$$

- Acceleration:



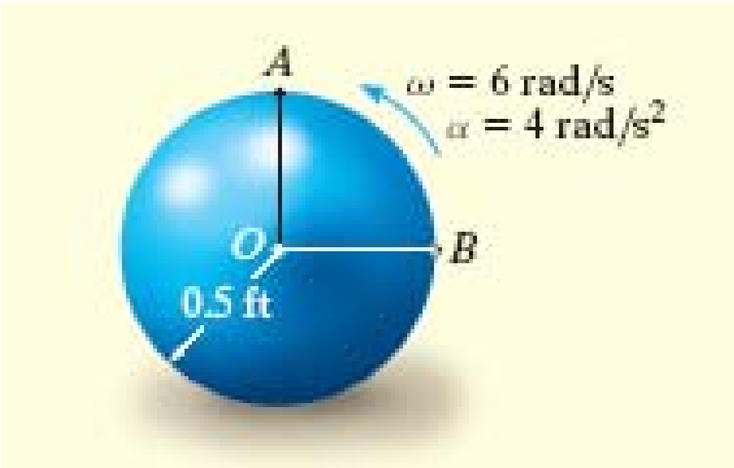
Since G moves along a straight-line path, $\boldsymbol{a}_G$ is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.

$$\boldsymbol{a}_G = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2 \boldsymbol{r}_{G/A} \Rightarrow a_G \boldsymbol{i} = a_A \boldsymbol{j} + (-\alpha \boldsymbol{k}) \times (r \boldsymbol{j}) - \omega^2 (r \boldsymbol{j})$$

Evaluating and equating $\boldsymbol{i}$ and $\boldsymbol{j}$ components:

$$a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad \boldsymbol{a}_G = \alpha r \boldsymbol{i} \quad \text{and} \quad \boldsymbol{a}_A = \omega^2 r \boldsymbol{j}$$

Example (16.7)



Given: The ball rolls without slipping.

Find: The accelerations of points A and B at this instant.

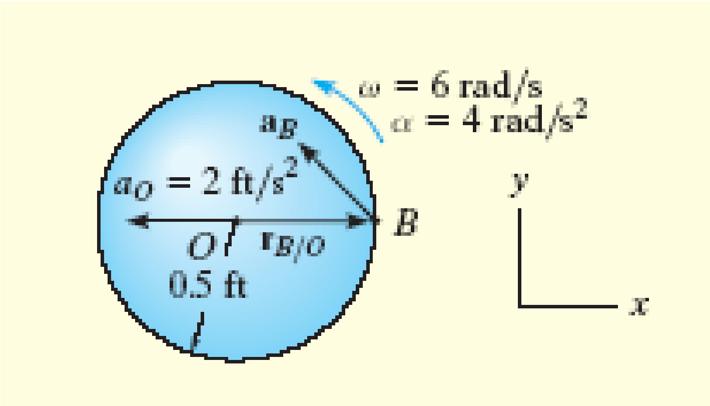
Plan: Follow the solution procedure.

Solution: Since the ball is rolling without slip, $\boldsymbol{a}_O$ is directed to the left with a magnitude of:

$$a_O = \alpha r = (4 \text{ rad/s}^2)(0.5 \text{ ft}) = 2 \text{ ft/s}^2$$

Example continues

Now, apply the relative acceleration equation between points O and B.

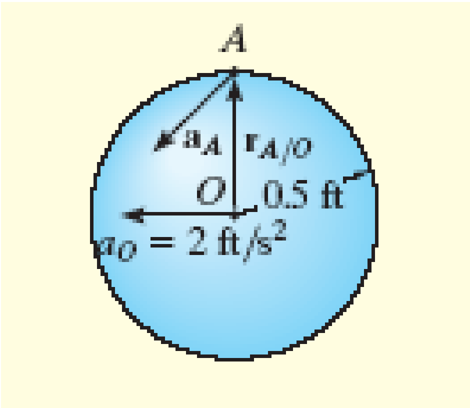


$$\boldsymbol{a}_B = \boldsymbol{a}_O + \boldsymbol{\alpha} \times \boldsymbol{r}_{B/O} - \omega^2 \boldsymbol{r}_{B/O}$$

$$\boldsymbol{a}_B = -2\boldsymbol{i} + (4\boldsymbol{k}) \times (0.5\boldsymbol{i}) - (6)^2(0.5\boldsymbol{i})$$

$$= (-20\boldsymbol{i} + 2\boldsymbol{j}) \text{ ft/s}^2$$

Now do the same for point A.

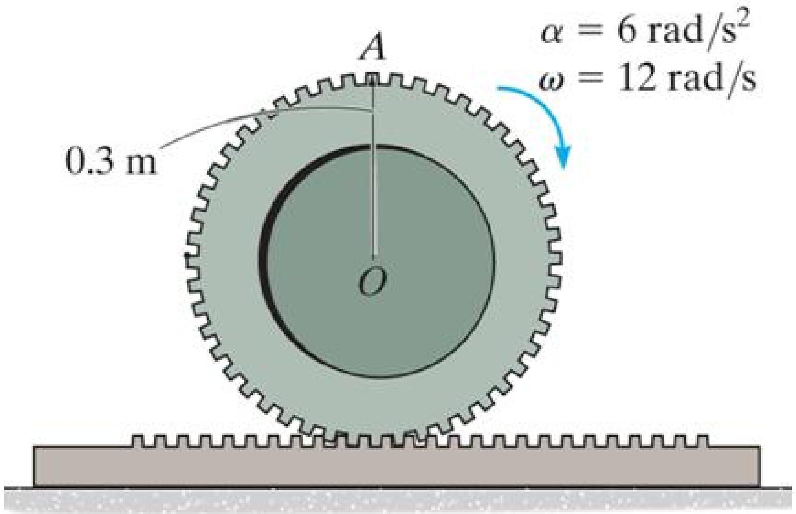


$$\boldsymbol{a}_A = \boldsymbol{a}_O + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/O} - \omega^2 \boldsymbol{r}_{A/O}$$

$$\boldsymbol{a}_A = -2\boldsymbol{i} + (4\boldsymbol{k}) \times (0.5\boldsymbol{i}) - (6)^2(0.5\boldsymbol{j})$$

$$= (-4\boldsymbol{i} - 18\boldsymbol{j}) \text{ ft/s}^2$$

EXAMPLE II



Given: The gear with a center at O rolls on the fixed rack.

Find: The acceleration of point A at this instant.

Plan:

Follow the solution procedure!

Solution: Since the gear rolls on the fixed rack without slip, a_O is directed to the right with a magnitude of

$$a_O = \alpha r = (6 \text{ rad/s}^2)(0.3 \text{ m}) = 1.8 \text{ m/s}^2.$$

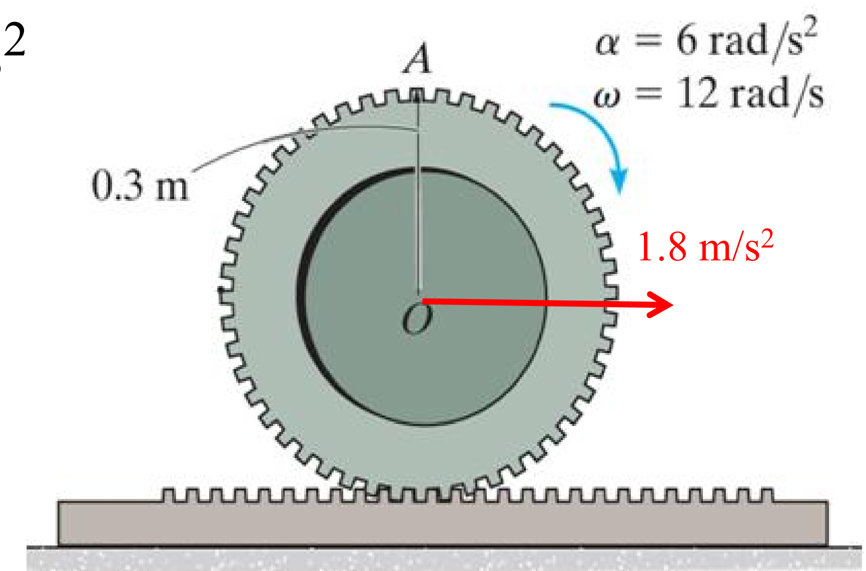
EXAMPLE II (continued)

So now with a_O = 1.8 m/s^2, we can apply the relative acceleration equation between points O and A.

$$a_A = a_O + \alpha \times r_{A/O} - \omega^2 r_{A/O}$$

$$a_A = 1.8i + (-6k)\times(0.3j) - 12^2 (0.3j)$$

$$= (3.6\,i - 43.2j) \text{ m/s}^2$$



Final Project

1. Proposal for design project (GROUPS)

The CS student finally realizes the meaning of the word "deadline".

Deadline: Proposal due Today

Homework Assignment

Chapter16- 13, 18, 34
Chapter16- 43,49,50,65,73,91,105,111,114,115

Due next Wednesday !!!


What a
Weekend!