ME 230 Kinematics and Dynamics

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Planar kinetics of a rigid body: Work and Energy
Chapter 18

Chapter objectives

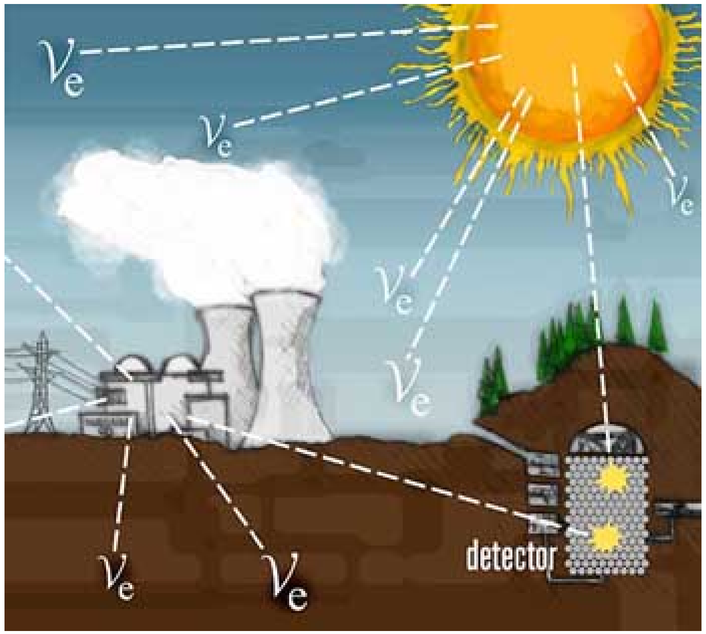
- Develop formulations for the kinetic energy of a body, and define the various ways a force and couple do work.
- Apply the principle of work and energy to solve rigid-body planar kinetic problems that involve force, velocity and displacement
- Show how the conservation of energy can be used to solve rigid-body planar kinetic problems

Lecture 20

- **Planar kinetics of a rigid body: Work and Energy**

Conservation of Energy

- 18.5



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Material covered

- **Planar kinetics of a rigid body :Work and Energy**

  18.5: Conservation of energy

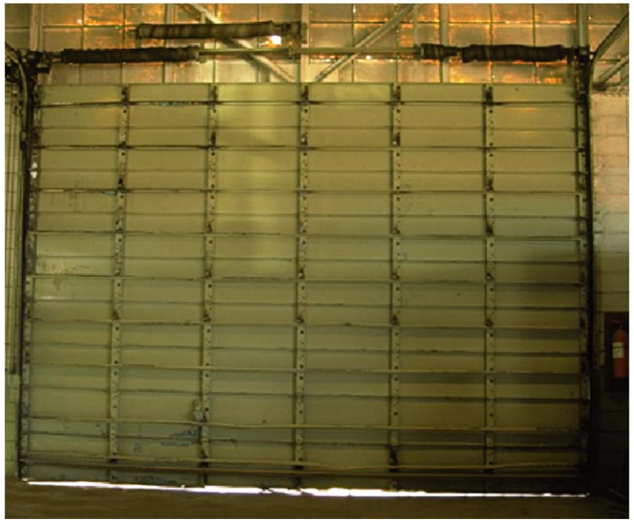
…Next lecture…Ch.19

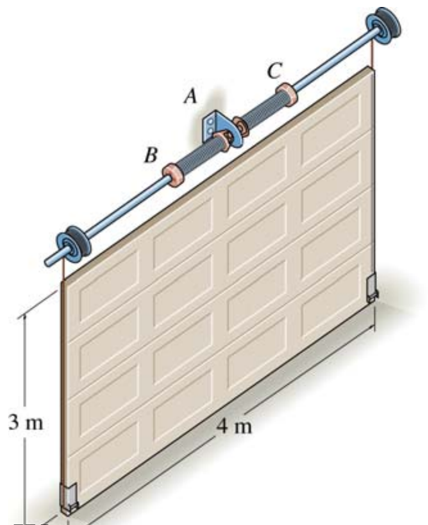
Today's Objectives

Students should be able to:

1) Determine the potential energy of conservative forces.
2) Apply the principle of conservation of energy.

Applications 1





The torsional spring located at the top of the garage door winds up as the door is lowered.

When the door is raised, the potential energy stored in the spring is transferred into the gravitational potential energy of the door’s weight, thereby making it easy to open.

Are parameters such as the torsional spring stiffness and initial rotation angle of the spring important when you install a new door?

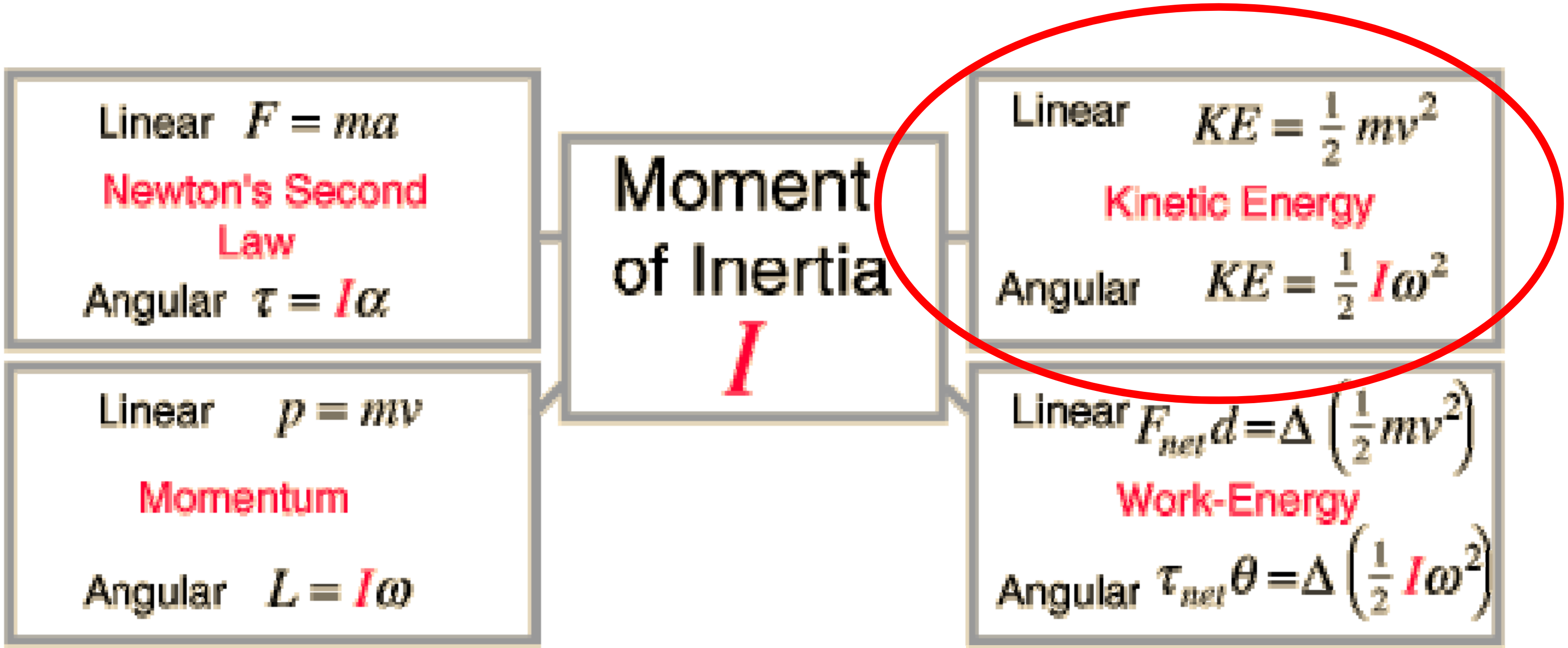
APPLICATIONS (continued)

Two torsional springs are used to assist in opening and closing the hood of this truck.

Assuming the springs are uncoiled when the hood is opened, can we determine the stiffness of each spring so that the hood can easily be lifted, i.e., there is practically no external force applied to the hood, when a person is opening it?

Are the gravitational potential energy of the hood and the torsional spring stiffness related to each other? If so, how?

Moment of Inertia



Moment of inertia is the name given to rotational inertia, the rotational analog of mass for linear motion. For a point mass the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis, $I = mr^2$.

Conservation of energy (18.5)

The conservation of energy theorem is a "simpler" energy method (recall that the principle of work and energy is also an energy method) for solving problems.

Once again, the problem parameter of distance is a key indicator of when conservation of energy is a good method for solving the problem.

If it is appropriate, conservation of energy is easier to use than the principle of work and energy.

This is because the calculation of the work of a conservative force is simpler. But, what makes a force conservative?

Conservative forces

A force *F* is conservative if the work done by the force is independent of the path.

In this case, the work depends only on the initial and final positions of the object with the path between positions of no consequence.

Typical conservative forces encountered in dynamics are gravitational forces (i.e., weight) and elastic forces (i.e., springs).

What is a common force that is not conservative?

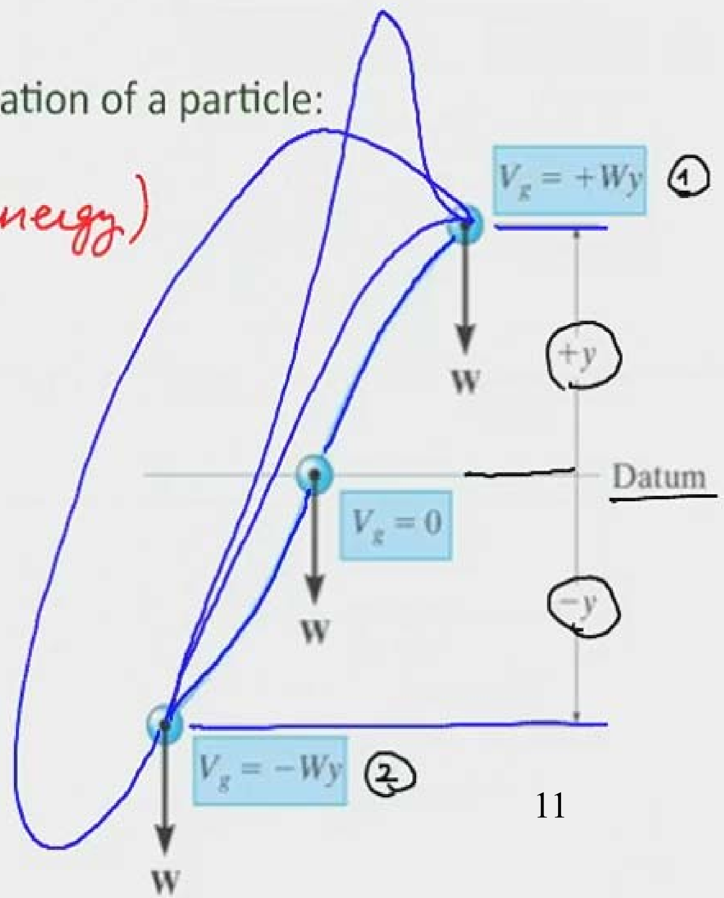
Theory: Conservative Forces and Potential En. (14.5)

- a force is called "conservative" if the work of that force is independent of path and instead depends only upon the starting and ending points on the path
- weight of a particle and spring force are both conservative forces
- potential energy is the amount of work a conservative force can do when it moves from a given position in the datum (the capacity to do work...)
- gravitational potential energy is related to the vertical location of a particle:

$$V_g = Wy \quad (V: \text{potential energy})$$

- elastic potential energy is related to spring deformation:

$$V_e = \frac{1}{2} k s^2$$



Frictional force is a non-conservative force because it depends on the path.

Conservation of energy

When a rigid body is acted upon by a system of conservative forces, the work done by these forces is conserved. Thus, the sum of kinetic energy and potential energy remains constant. This principle is called conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 = \text{Constant}$$

$T = 1/2\ m\ (v_G)^2 + 1/2\ I_G\ \omega^2$ $V = V_g + V_e$, where $V_g = W\ y_G$ and $V_e = 1/2\ k\ s^2$.

In other words, as a rigid body moves from one position to another when acted upon by only conservative forces, kinetic energy is converted to potential energy and vice versa.

Theory: Projectile Motion (12.6)

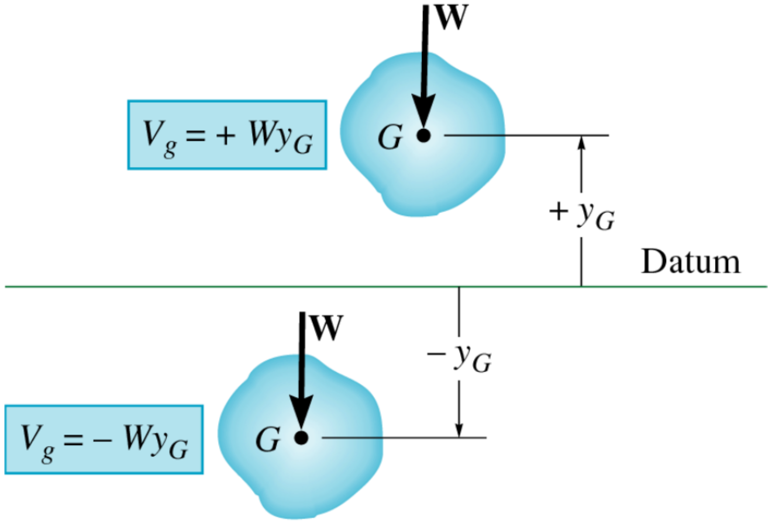
- projectile motion is a special case of erratic motion usually modeled using Cartesian vectors
- it is a special case, because projectiles move in the presence of a constant gravitational acceleration in one direction (up/down), and (usually) negligible acceleration in another (horizontally); projectiles are modeled as particles
- we solve the problem using Cartesian coordinates, in two parts

Vertical motion	Horizontal motion
Y direction	X direction
$V_{oy} = V_o \sin\theta$	$V_{ox} = V_o \cos\theta$
$V_{fy}{}^2 = V_{oy}{}^2 + 2aY$	$V_{fx}{}^2 = V_{ox}{}^2$
$V_{fy} = V_{oy} + at$	$V_{fx} = V_{ox}$
$Y = V_{oy}\, t + 1/2\, at^2$	$X = V_{ox}\, t$
$Y = 1/2(V_{fy} + V_{oy})t$	$X = 1/2(V_{fx} + V_{ox})t$

Since we know V_c we can find V_{cy} fron V_{cx} and since V_{Ax} is known then t can be solved

Gravitational potential energy

The gravitational potential energy of an object is a function of the height of the body's center of gravity above or below a datum.



The gravitational potential energy of a body is found by the equation

$$V_g = W\,y_G$$

Gravitational potential energy is positive when y_G is positive, since the weight has the ability to do positive work when the body is moved back to the datum.

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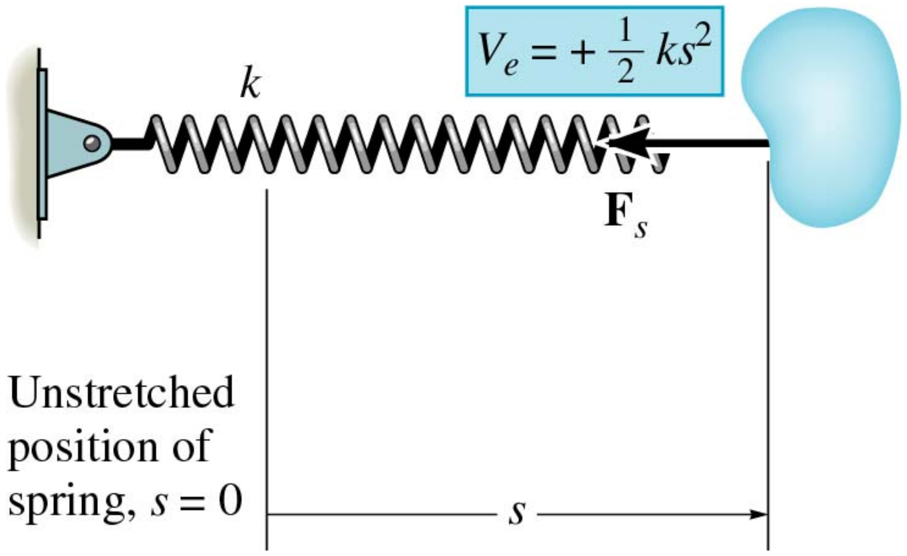
Elastic potential energy

Spring forces are also conservative forces.



Elastic potential energy

The potential energy of a spring force (F = ks) is found by the equation

$$V_e = \frac{1}{2} ks^2$$

Notice that the elastic potential energy is always positive.

Procedure of analysis

Problems involving velocity, displacement and conservative force systems can be solved using the conservation of energy equation.

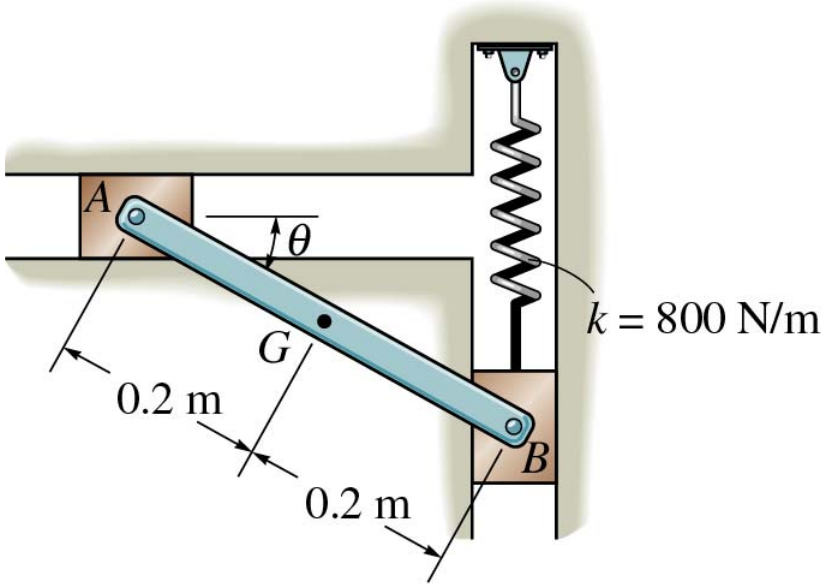
- Potential energy: Draw two diagrams: one with the body located at its initial position and one at the final position. Compute the potential energy at each position using

$$V = V_g + V_e, \text{ where } V_g = W\, y_G \text{ and } V_e = 1/2\, k\, s^2.$$

- Kinetic energy: Compute the kinetic energy of the rigid body at each location. Kinetic energy has two components: translational kinetic energy ($\frac{1}{2}\, m(v_G)^2$) and rotational kinetic energy ($\frac{1}{2}\, I_G\, \omega^2$).

- Apply the conservation of energy equation.

Example



Given: The rod AB has a mass of 10 kg. Piston B is attached to a spring of constant k = 800 N/m. The spring is un-stretched when θ = 0°. Neglect the mass of the pistons.

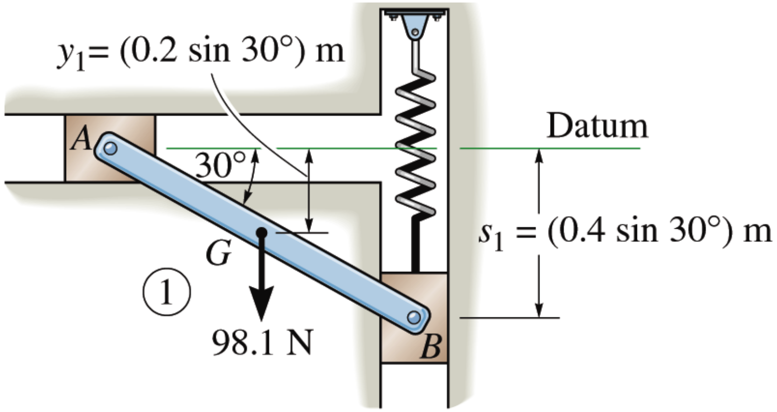
Find: The angular velocity of rod AB at θ = 0° if the rod is released from rest when θ = 30°.

Plan: Use the energy conservation equation since all forces are conservative and distance is a parameter (represented here by θ). The potential energy and kinetic energy of the rod at states 1 and 2 will have to be determined.

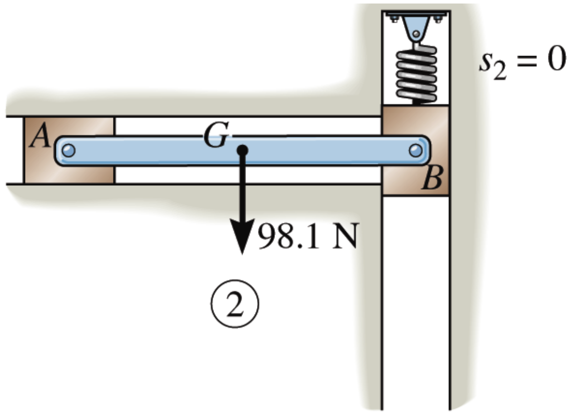
Example continued

Solution:

Initial Position



Final Position



Potential Energy:

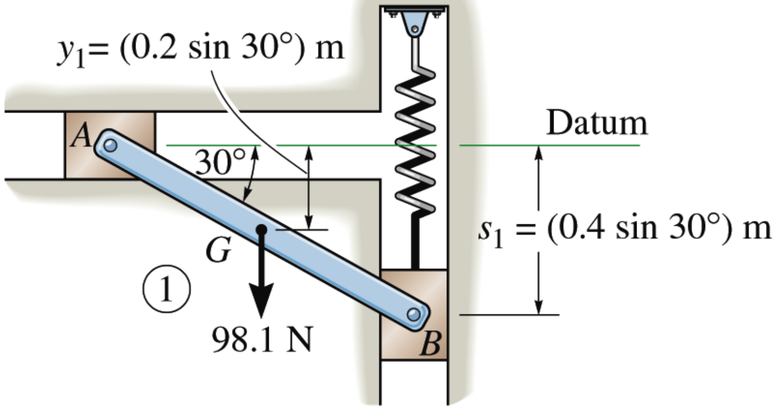
Let’s put the datum in line with the rod when θ = 0°.
Then, the gravitational potential energy and the elastic potential energy will be zero at position 2. => V_2 = 0 (potential energy@2)

Gravitational potential energy at 1: - (10)(9.81) ½ (0.4 sin 30)
Elastic potential energy at 1: ½ (800) (0.4 sin 30)^2

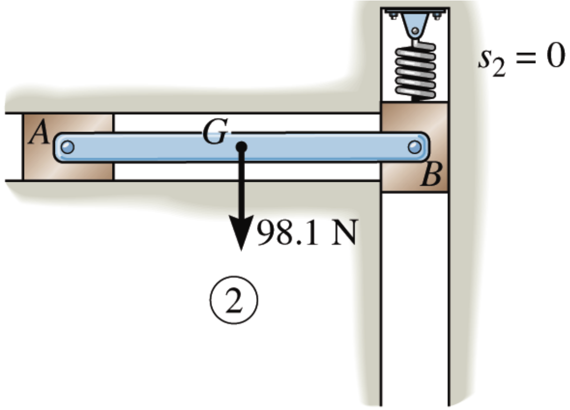
So V_1 = - 9.81 J + 16.0 J = 6.19 J

Example continued

Initial Position

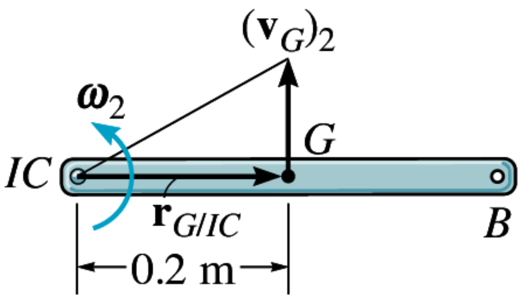


Final Position



Kinetic Energy:

The rod is released from rest from position 1 (so $v_{G1} = 0$, $\omega_1 = 0$). Therefore, $T_1 = 0$.



At position 2, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

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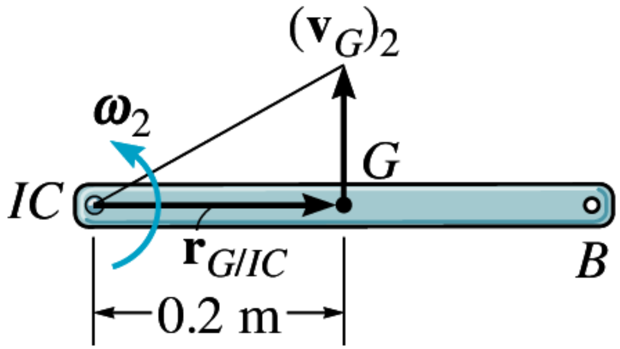
Example continued

Therefore,

$T_2 = ½\,(10)(v_{G2})^2 + ½\,(1/12)(10)(0.4^2)(\omega_2)^2$



At position 2, point A is the instantaneous center of rotation. Hence, $v_{G2} = r\,\omega = 0.2\,\omega_2$.

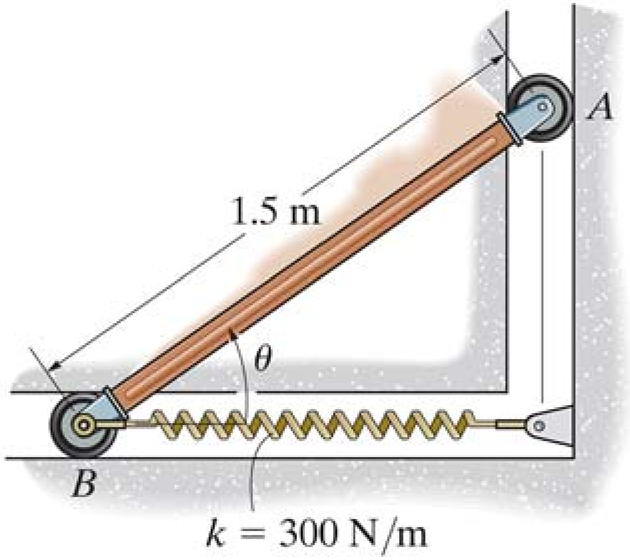
Then, $T_2 = 0.2\,\omega_2^2 + 0.067\,\omega_2^2 = 0.267\,\omega_2^2$

Now apply the conservation of energy equation and solve for the unknown angular velocity, ω_2.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6.19 = 0.267\omega_2^2 + 0 \quad \Rightarrow \quad \omega_2 = 4.82 \text{ rad/s}$$

EXAMPLE I



Given: The rod AB has a mass of 30 kg. The spring is un-stretched when $\theta = 45°$. The spring constant k is 300 N/m.

Find: The angular velocity of rod AB at $\theta = 0°$, if the rod is released from rest when $\theta = 45°$.

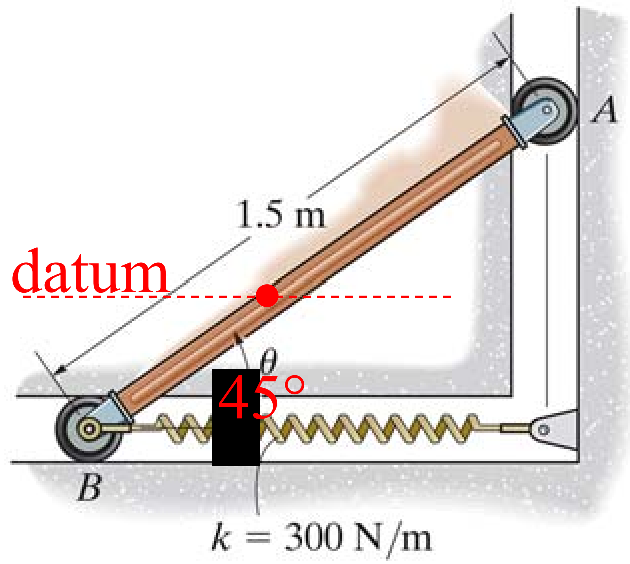
Plan: Use the energy conservation equation since all forces are conservative and distance is a parameter (represented here by θ). The potential energy and kinetic energy of the rod at states 1 and 2 will have to be determined.

EXAMPLE I (continued)

Solution:

Potential Energy

Let's put the datum in line with the rod when $\theta = 45°$. Then, the gravitational potential energy and the elastic potential energy will be zero at position 1. So, $V_1 = 0$.



Gravitational potential energy at 2 (when θ=0°):

$-(30)\ (9.81)\ ½\ (1.5 \sin 45°)$ from $V_g = W\ y_G = m\ g\ y_G$

Elastic potential energy at 2:

$½\ (300)\ (1.5 - 1.5 \cos 45°)^2$ from $V_e = 1/2\ k\ s^2$

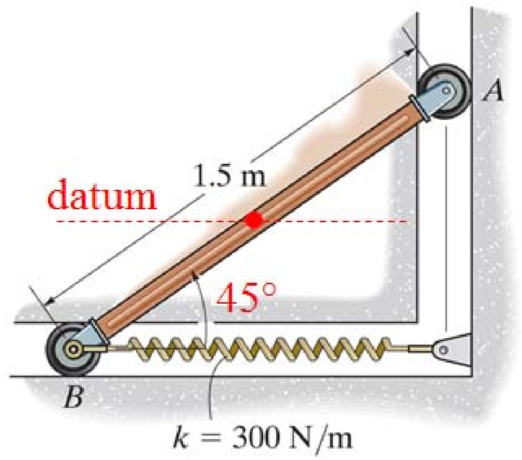
So $V_2 = -156.1 + 28.95 = -127.2$ N·m

EXAMPLE I (continued)

Kinetic Energy:

The rod is released from rest at position 1 (when θ=45°).

Therefore, $T_1 = 0$.



At position 2 (when θ=0°), the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

Therefore, $T_2 = ½ (30)(v_{G2})^2 + ½ (1/12)(30)(1.5^2)(\omega_2)^2$

$T_2 = 15 (v_{G2})^2 + 2.813 (\omega_2)^2$

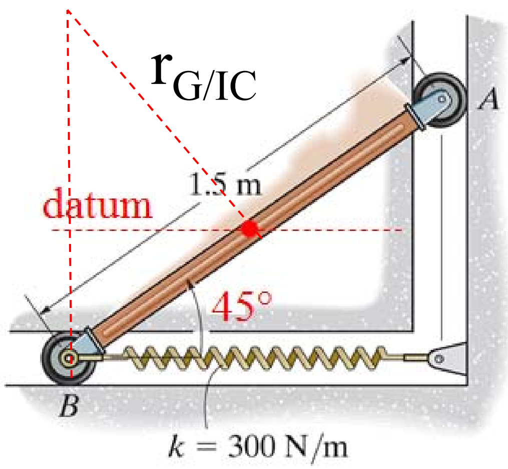
EXAMPLE I (continued)

At position 2(when θ=0°), point A is the instantaneous center of rotation.

Hence, $v_{G2} = r_{G/IC}\ \omega = (0.75/\tan 45^o)\omega_2$. Then,

$$T_2 = 15\ (0.75\ \omega_2)^2 + 2.813\ (\omega_2)^2$$

$$T_2 = 11.25\ (\omega_2)^2$$

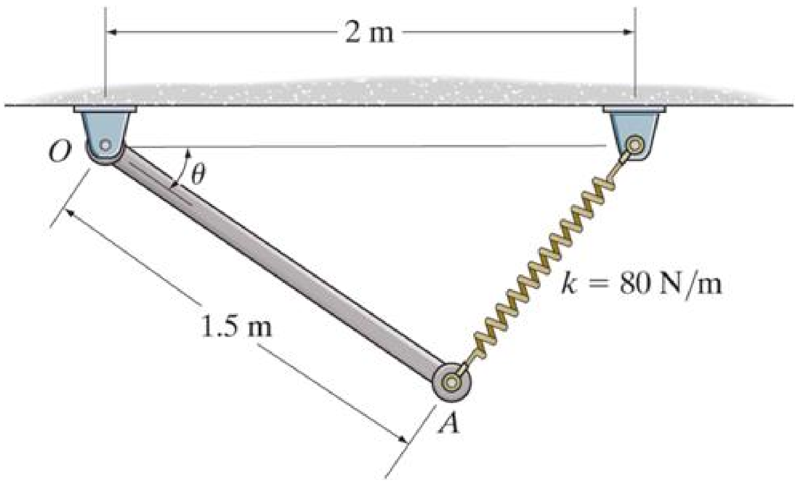


Now apply the conservation of energy equation and solve for the unknown angular velocity, ω_2.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 11.25(\omega_2)^2 - 127.2 \quad \Rightarrow \quad \omega_2 = 3.36 \text{ rad/s}$$

EXAMPLE II



Given: The 30 kg rod is released from rest when $\theta = 0°$. The spring is unstretched when $\theta = 0°$.

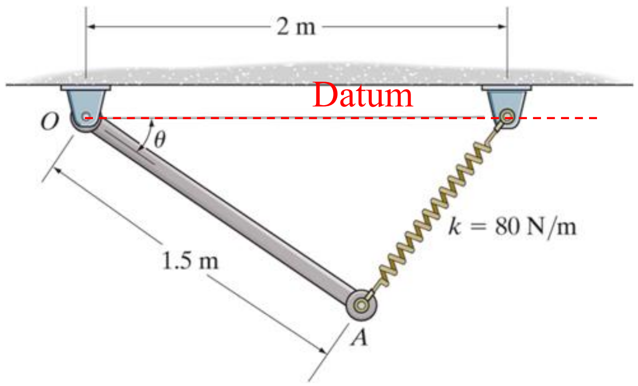
Find: The angular velocity of the rod when $\theta = 30°$.

Plan:

Since distance is a parameter and all forces doing work are conservative, use conservation of energy. Determine the potential energy and kinetic energy of the system at both positions and apply the conservation of energy equation.

EXAMPLE II (continued)

Solution:



Potential Energy:

Let's put the datum in line with the rod when $\theta = 0°$.

Then, the gravitational potential energy when $\theta = 30°$ is

$$V_{G2} = -30\,(9.81)\,(\tfrac{1}{2}\,1.5 \sin 30°) = -110.4 \text{ N·m}$$

The elastic potential energy at $\theta = 0°$ is zero since the spring is un-stretched. The un-stretched length of the spring is 0.5 m.

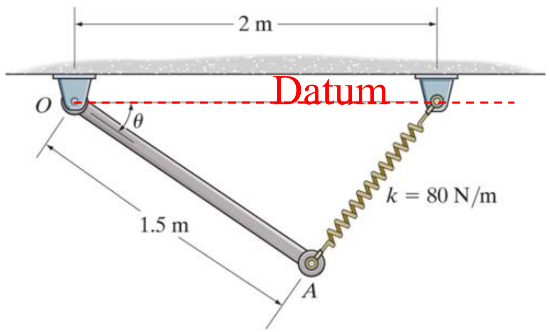
The elastic potential energy at $\theta = 30°$ is

$$V_{E2} = \tfrac{1}{2}\,80\,\left(\sqrt{0.5^2 + (1.5 \sin 30°)^2} - 0.5\right)^2 = 6.444 \text{ N·m}$$

EXAMPLE II (continued)

Kinetic Energy:

The rod is released from rest at $\theta = 0°$, so $v_{G1} = 0$ and $\omega_1 = 0$. Thus, the kinetic energy at position 1 is $T_1 = 0$.



At $\theta = 30°$, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

$$T_2 = ½\, m\, (v_{G2})^2 + ½\, I_G\, (\omega_2)^2$$

$$= ½\, (30)\, (v_{G2})^2 + ½\, \{(1/12)\, 30\, (1.5)^2\}\, (\omega_2)^2$$

Since $v_{G2} = (0.75\, \omega_2)$,

$$T_2 = ½(30)\, (0.75\, \omega_2)^2 + ½\{(1/12)30(1.5)^2\}\, (\omega_2)^{2}$$

$$T_2 = 11.25\, (\omega_2)^2$$

EXAMPLE II
(continued)

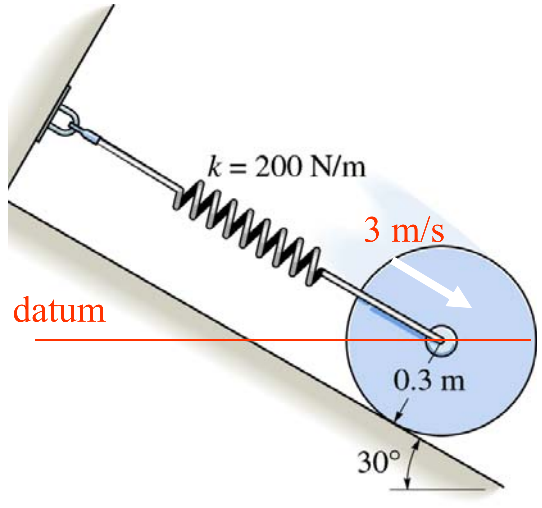
Now all terms in the conservation of energy equation have been formulated. Writing the general equation and then substituting into it yields:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 11.25\,(\omega_2)^2 + (-110.4 + 6.444)$$

Solving , $\omega_2 = 3.04$ rad/s

UNDERSTANDING QUIZ



1. At the instant shown, the spring is undeformed. Determine the change in potential energy if the 20 kg disk (k_G = 0.5 m) rolls 2 revolutions without slipping.

   A) $\frac{1}{2}(200)(1.2\pi)^2 + (20)9.81(1.2\pi \sin 30°)$

   B) $- \frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30°)$

   C) $\frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30°)$

   D) $\frac{1}{2}(200)(1.2\pi)^2$

2. Determine the kinetic energy of the disk at this instant.

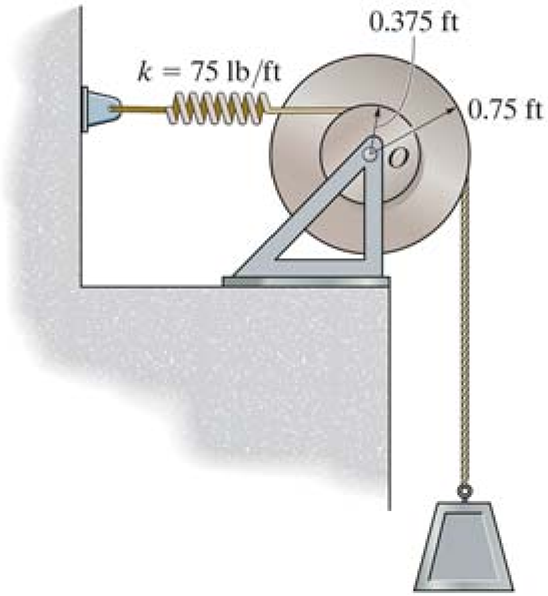
   A) $(\frac{1}{2})(20)(3)^2$

   B) $\frac{1}{2}(20)(0.5^2)(10)^2$

   C) Answer A + Answer B

   D) None of the above

Example III



Given: A 250-lb block is released from rest when the spring is unstretched. The drum has a weight of 50 lb and a radius of gyration of $k_O = 0.5$ ft about its center of mass O.

Find: The velocity of the block after it has descended 5 feet.

Plan: Conservative forces and distance leads to the use of conservation of energy. First, determine the potential energy and kinetic energy for both the start and end positions. Then apply the conservation of energy equation.

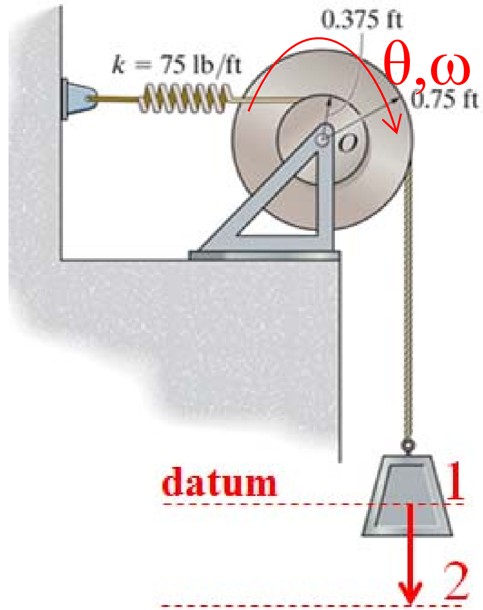
Example III (continued)

Solution:

Potential Energy:

Let's put the datum when the spring is unstretched. Thus, the gravitational potential energy is zero and the elastic potential energy will be zero. So,

$$V_1 = 0$$



Use geometry to find the spring stretch at 2:

$$0.75(\theta) = y \Rightarrow \theta = (5 / 0.75) = 6.667 \text{ rad.}$$

Thus, the spring elongation is $\Delta_s = 0.375\,(6.667) = 2.5$ ft

Distance travel by the rope is same as the distance of the disk rotate due to the rope if there is no slipping.
θ is same but distance travels by the two concentric pulley is different because of different radii.

Gravitational potential energy at 2:

$$V_{G2} = W\, y_{G2} - 250\,(5) = -1250 \text{ ft·lb}$$

Elastic potential energy at 2 is :

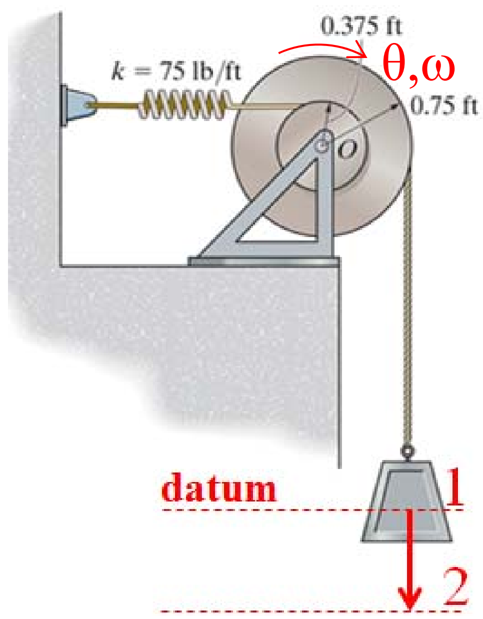
$$V_{E2} = 1/2\, k\, s^2 = \tfrac{1}{2}\,(75)\, 2.5^2 = 234.4 \text{ ft·lb}$$

Example III (continued)

Kinetic Energy:

The block is at rest at position 1.

Thus, $T_1 = 0$.



Kinetic energy of the block at 2.

$T_{block\ 2} = (1/2)\ m\ (v_2)^2$

Kinetic energy of the disk at 2

$T_{disk\ 2} = (1/2)\ I_O\ (\omega_2)^2$

where $I_O = (50\ /\ 32.2)\ 0.5^2 = 0.3882$ slug·m^2

and $v_2 = (0.75)\ \omega_2$

$T_2 = (1/2)\ (250\ /\ 32.2)\ (v_2)^2 + (1/2)\ 0.3882\ (v_2\ /\ 0.75)^2$

$T_2 = 4.227(v_2)^2$

Example III (continued)

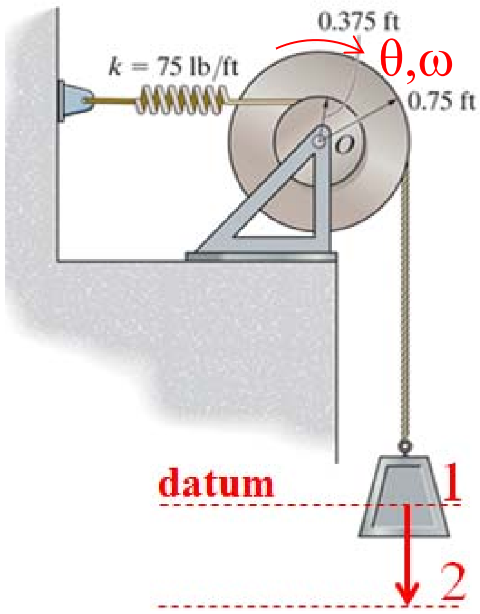
Now, substitute into the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 4.227(v_2)^2 + (-1250 + 234.4)$$

Solving for v_2 yields

$v_2 = 15.5$ ft/s $\downarrow$



Homework Assignment

Chapter18- 17, 37, 43, 47
Due Wednesday !!!

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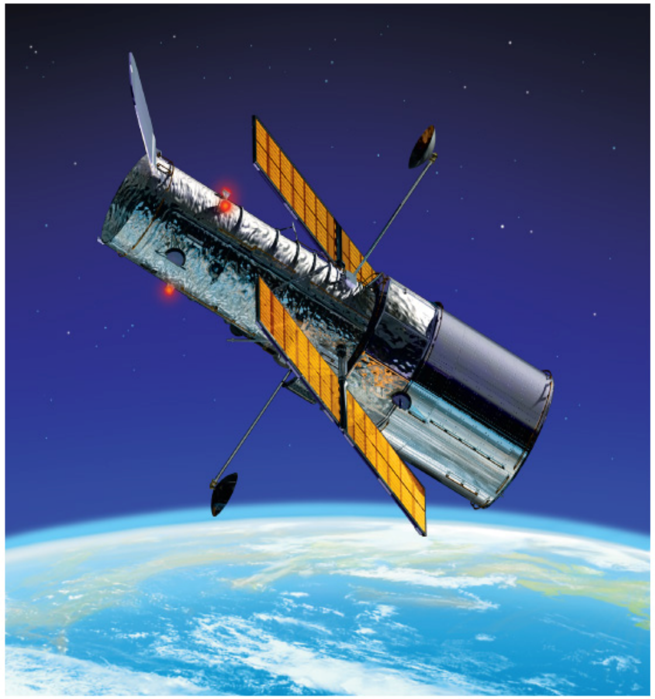
Planar kinetics of a rigid body: Impulse and Momentum

Chapter 19

Chapter objectives

- Develop formulations for the linear and angular momentum of a body
- Apply the principles of linear and angular impulse and momentum to solve rigid body planar kinetic problems that involve force, velocity and time
- To discuss the application of the conservation of momentum



Lecture 21

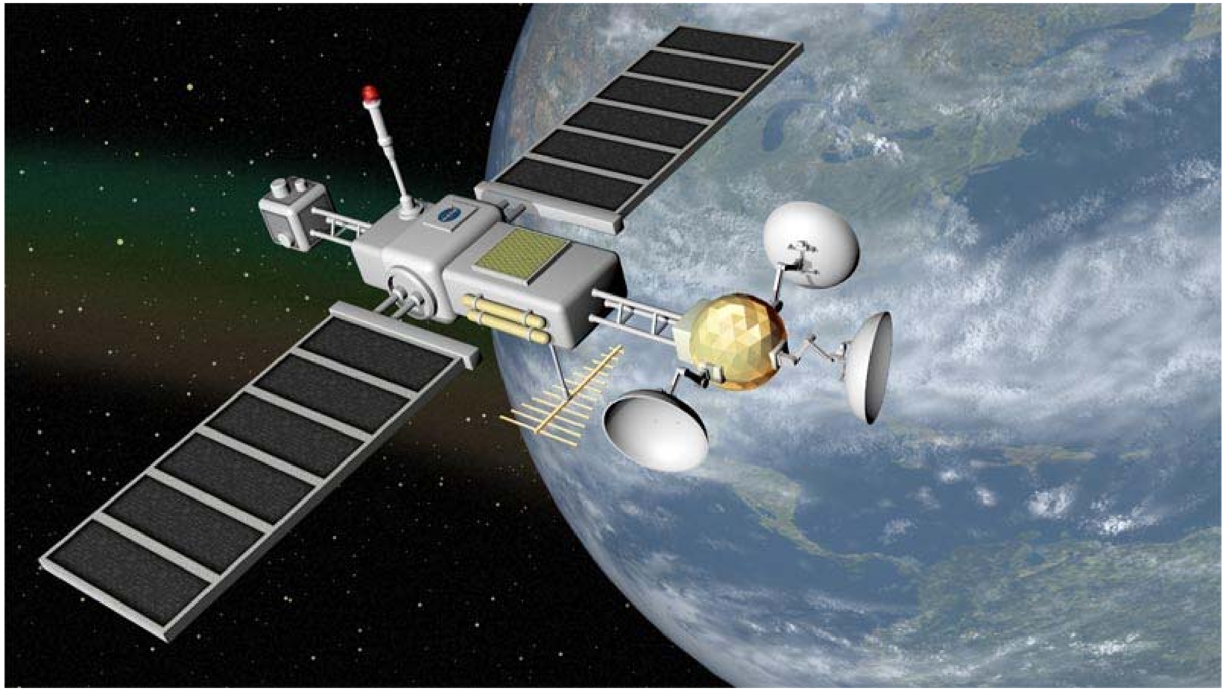
- **Planar kinetics of a rigid body: Impulse and Momentum**

Linear and <u>angular momentum</u>

Principle of impulse and momentum

Conservation of momentum

-19.1-19.3



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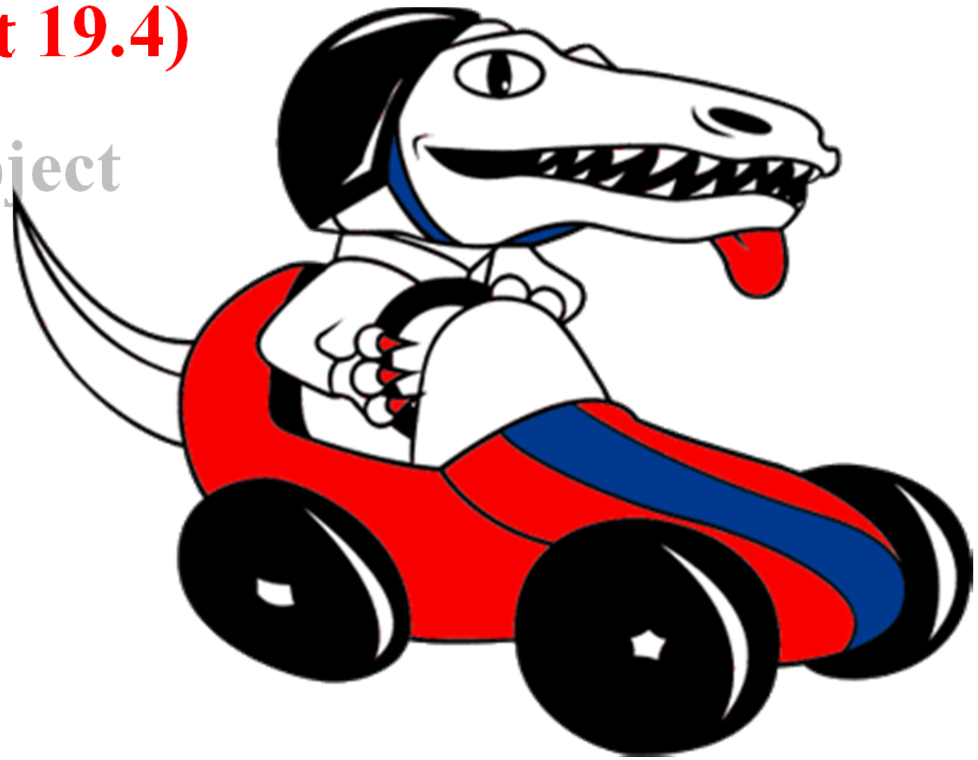
Material covered

- **Planar kinetics of a rigid body :Impulse and Momentum**

Whole of Chapter 19 (except 19.4)

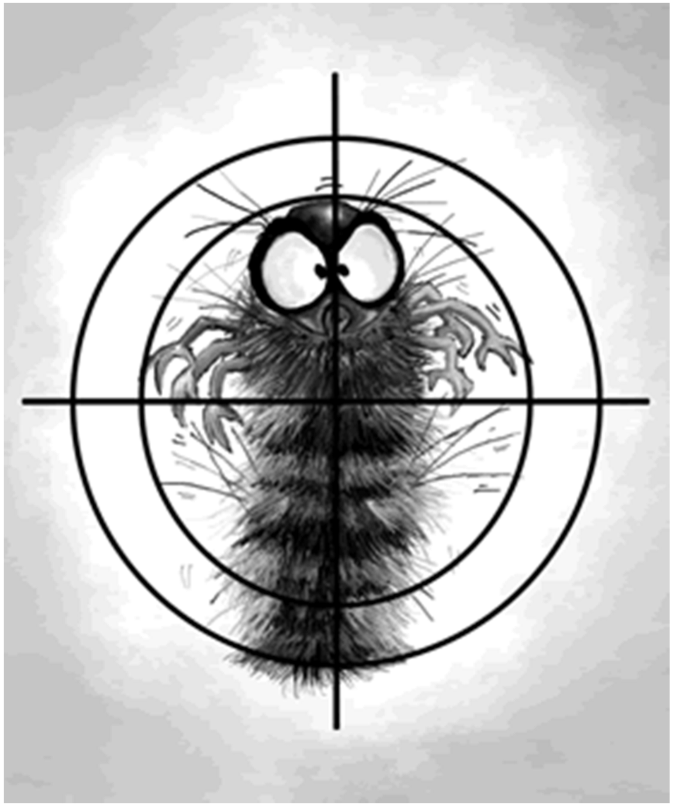
…Next lecture…Final project



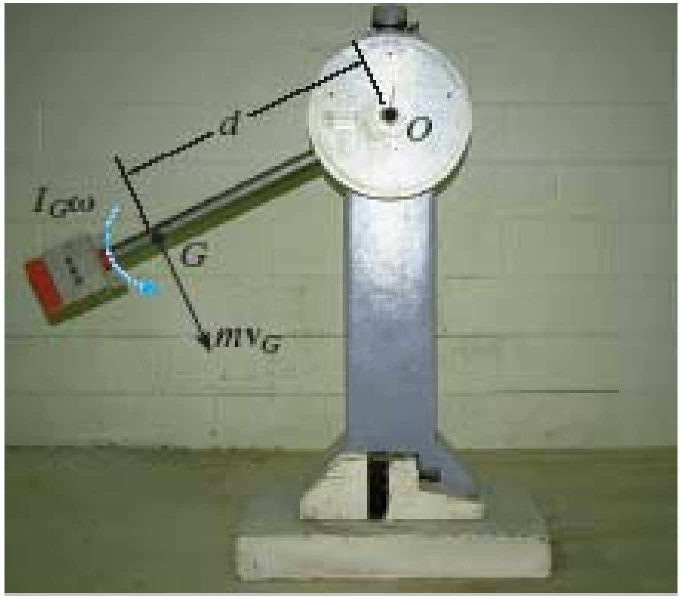
Today's Objectives

Students should be able to:

1. Develop formulations for the linear and angular momentum of a body.
2. Apply the principle of linear and angular impulse and momentum.
3. Understand the conditions for conservation of linear and angular momentum.
4. Use the condition of conservation of linear/ angular momentum.

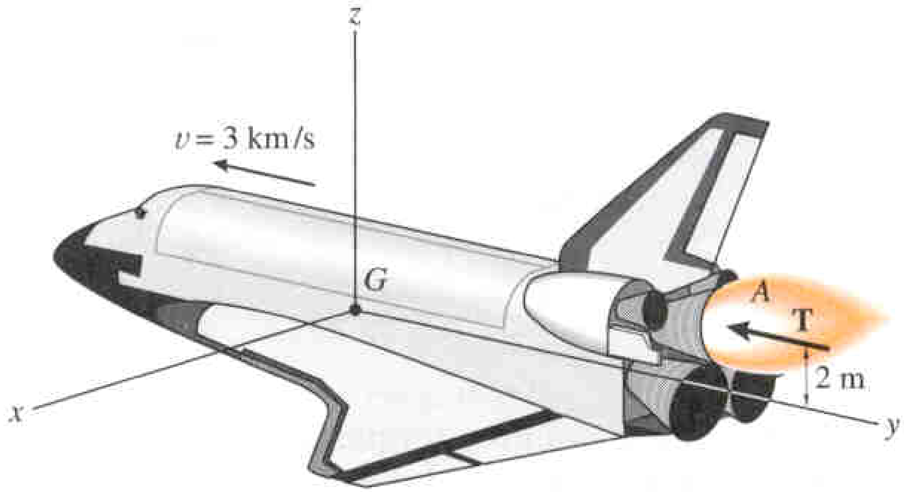


Applications



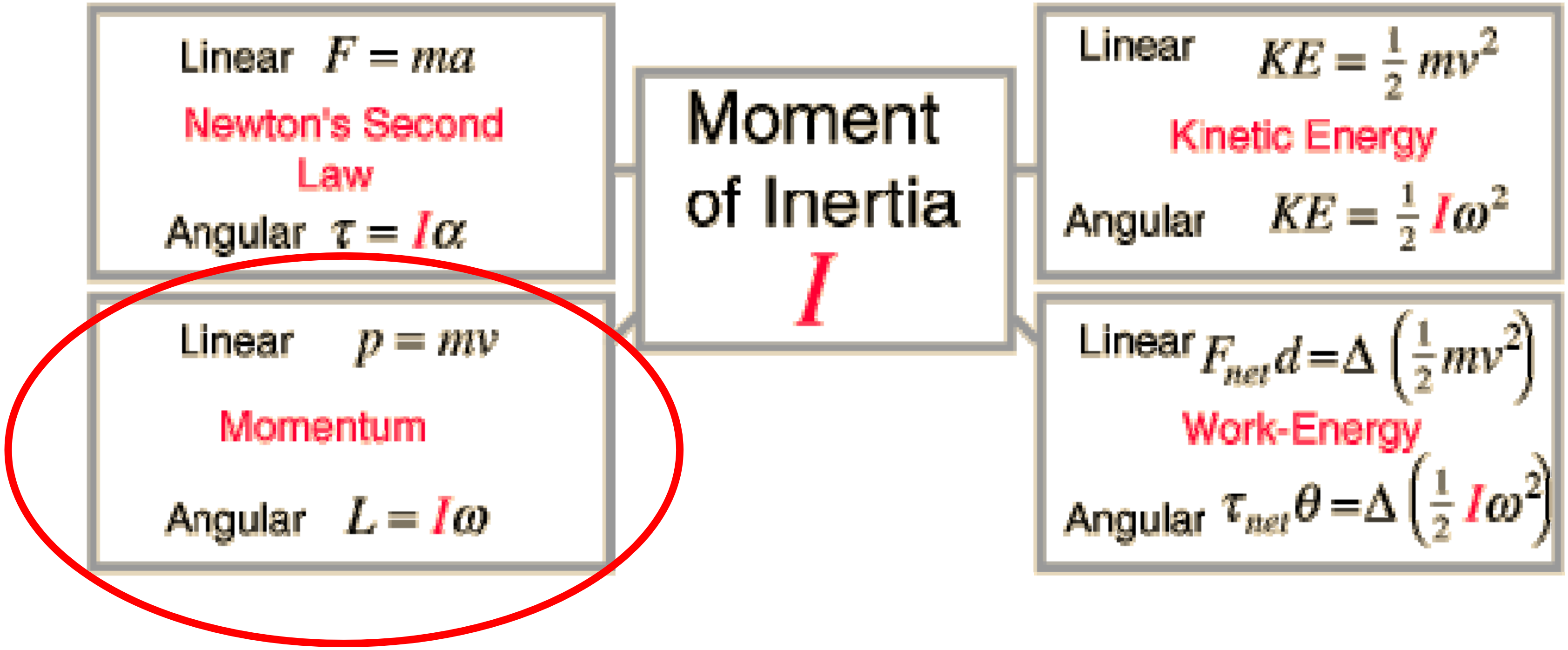
As the pendulum of the Charpy tester swings downward, its angular momentum and linear momentum both increase. By calculating its momenta in the vertical position, we can calculate the impulse the pendulum exerts when it hits the test specimen.

Applications continued



The space shuttle has several engines that exert thrust on the shuttle when they are fired. By firing different engines, the pilot can control the motion and direction of the shuttle.

Moment of Inertia



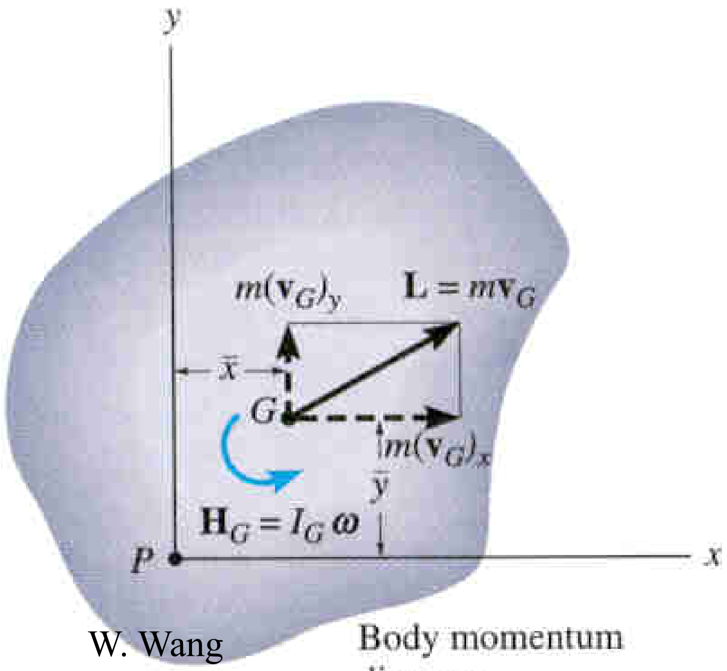
Moment of inertia is the name given to rotational inertia, the rotational analog of mass for linear motion. For a point mass the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis, $I = mr^2$.

Linear and angular momentum (19.1)

The linear momentum of a rigid body is defined as

$$\boldsymbol{L} = m\, \boldsymbol{v}_G$$

This equation states that the linear momentum vector $\boldsymbol{L}$ has a magnitude equal to (mv_G) and a direction defined by $\boldsymbol{v}_G$.



Body momentum diagram

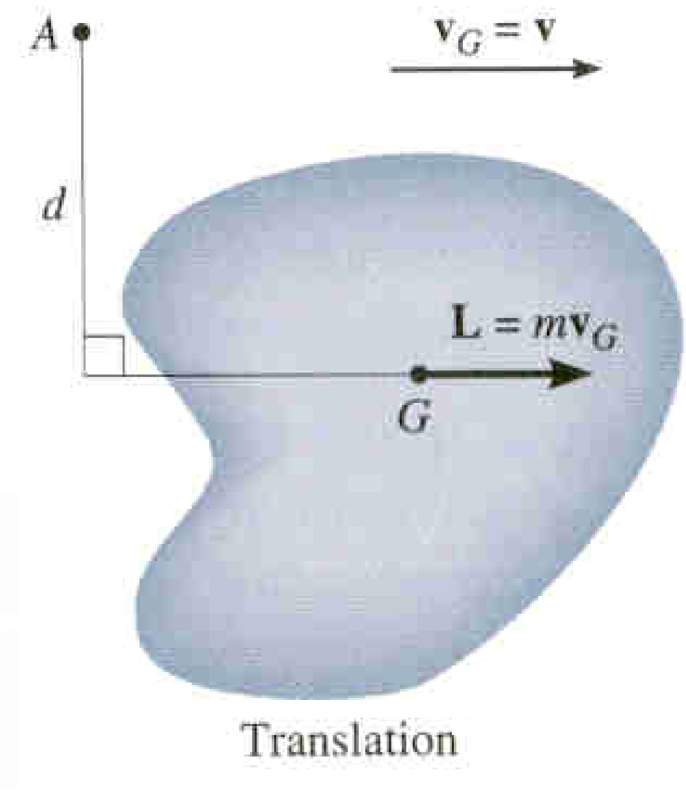
The angular momentum of a rigid body is defined as

$$\boldsymbol{H}_G = I_G\, \boldsymbol{\omega}$$

Remember that the direction of $\boldsymbol{H}_G$ is perpendicular to the plane of rotation.

Linear and angular momentum (19.1) continued

Translation.



When a rigid body undergoes rectilinear or curvilinear translation, its angular momentum is zero because ω = 0.

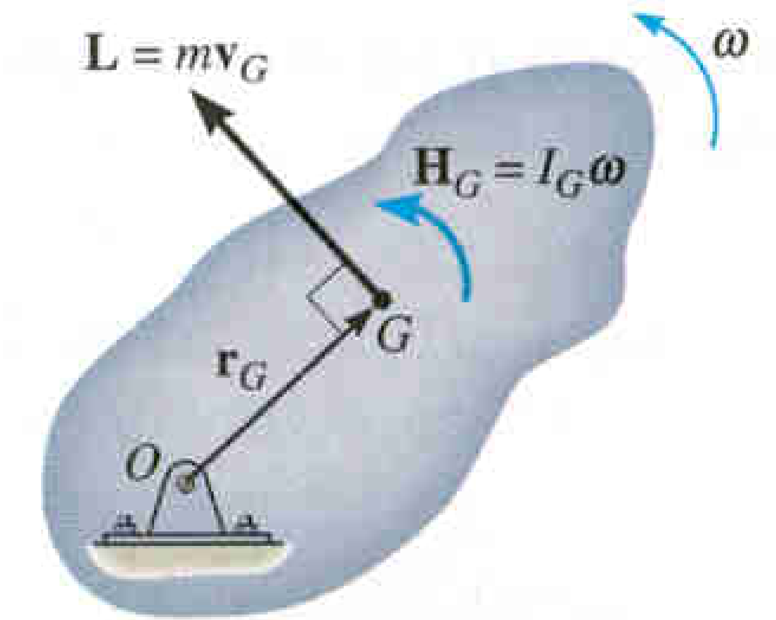
Therefore:

$$\boldsymbol{L} = m\,\boldsymbol{v}_G$$

$$\boldsymbol{H}_G = 0$$

Linear and angular momentum (19.1) continued

Rotation about a fixed axis.



Rotation about a fixed axis

When a rigid body is rotating about a fixed axis passing through point O, the body's linear momentum and angular momentum about G are:

$$\boldsymbol{L} = m\,\boldsymbol{v}_G$$

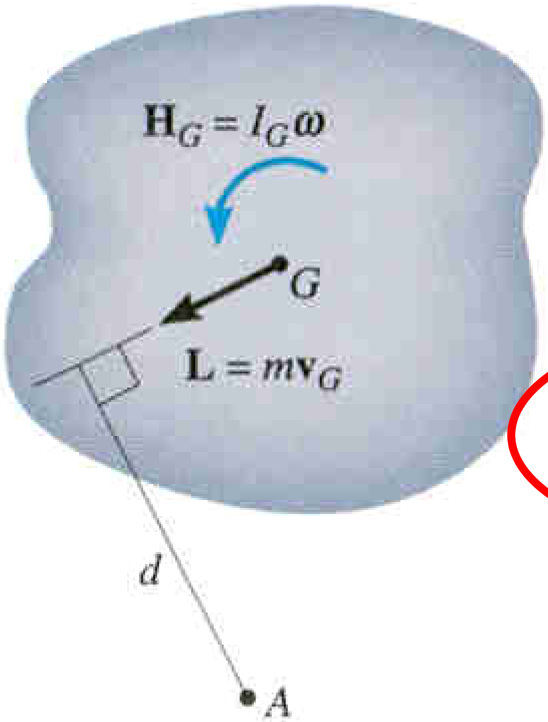
$$H_G = I_G\boldsymbol{\omega}$$

It is sometimes convenient to compute the angular momentum of the body about the center of rotation O.

$$\boldsymbol{H}_O = (\,\boldsymbol{r}_G \times m\boldsymbol{v}_G) + I_G\boldsymbol{\omega} = I_O\,\boldsymbol{\omega}$$

Linear and angular momentum (19.1) continued

General plane motion.



General plane motion

When a rigid body is subjected to general plane motion, both the linear momentum and the angular momentum computed about G are required.

$\boldsymbol{L} = m\,\boldsymbol{v}_G$

$H_G = I_G\boldsymbol{\omega}$

The angular momentum about point A is

$$H_A = I_G\omega + (d)mv_G$$

Principle of Impulse and Momentum (19.2)

As in the case of particle motion, the principle of impulse and momentum for a rigid body is developed by combining the equation of motion with kinematics. The resulting equations allow a direct solution to problems involving force, velocity, and time.

Linear impulse-linear momentum equation:

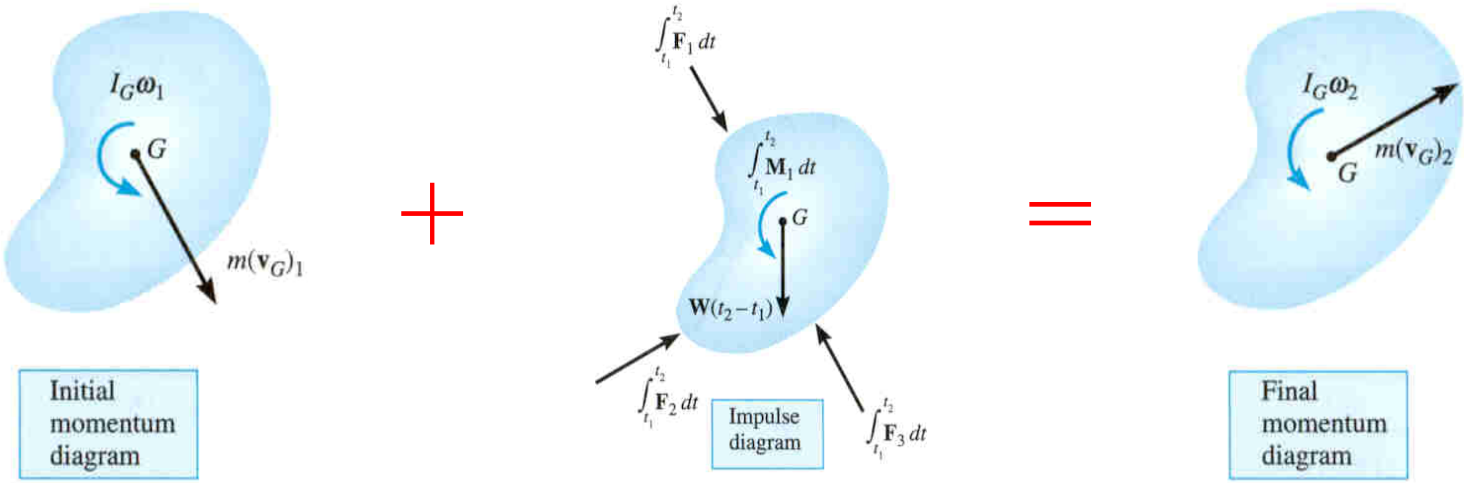
$$\boldsymbol{L}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\, dt = \boldsymbol{L}_2 \quad \underline{\text{or}} \quad (m\boldsymbol{v}_G)_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\, dt = (m\boldsymbol{v}_G)_2$$

Angular impulse-angular momentum equation:

$$(H_G)_1 + \sum \int_{t_1}^{t_2} M_G\, dt = (H_G)_2 \quad \underline{\text{or}} \quad I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2$$

Principle of Impulse and Momentum (19.2) continued

The previous relations can be represented graphically by drawing the impulse-momentum diagram.



To summarize, if motion is occurring in the x-y plane, the linear impulse-linear momentum relation can be applied to the x and y directions and the angular momentum-angular impulse relation is applied about a z-axis passing through any point (i.e., G). Therefore, the principle yields three scalar equations (eqs 19-14) describing the planar motion of the body.

$$m(v_{Gx})_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_{Gy})_2$$

$$I_G(\omega)_1 + \sum \int_{t_1}^{t_2} M_z \, dt = I_G(\omega)_2$$

(eqs 19-14)

Procedure of analysis

- Establish the x, y, z inertial frame of reference.
- Draw the impulse-momentum diagrams for the body.
- Compute I_G, as necessary.
- Apply the equations of impulse and momentum (one vector and one scalar or the three scalar equations 19-14).
- If more than three unknowns are involved, kinematic equations relating the velocity of the mass center G and the angular velocity ω should be used in order to have additional equations.

W. Wang

APPLICATIONS (section 19.3)

A skater spends a lot of time either spinning on the ice or rotating through the air. To spin fast, or for a long time, the skater must develop a large amount of angular momentum.

If the skater's angular momentum is constant, can the skater vary her rotational speed? How?

The skater spins faster when the arms are drawn in and slower when the arms are extended. Why?

APPLICATIONS (section 19.3)
(continued)

Conservation of angular momentum allows cats to land on their feet and divers to flip, twist, spiral and turn. It also helps teachers make their heads spin!

Conservation of angular momentum makes water circle the drain faster as it gets closer to the drain.

Conservation of momentum (section 19.3)

Recall that the linear impulse and momentum relationship is

$$\boldsymbol{L}_1 + \sum \int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{F}}\, dt = \boldsymbol{L}_2 \quad \text{or} \quad (m\,\boldsymbol{v}_G)_1 + \sum \int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{F}}\, dt = (m\,\boldsymbol{v}_G)_2$$

If the sum of all the linear impulses acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, the linear momentum for a rigid body (or system) is constant, or conserved. So $\boldsymbol{L}_1 = \boldsymbol{L}_2$.

This equation is referred to as the conservation of linear momentum. The conservation of linear momentum equation can be used if the linear impulses are small or non-impulsive.

Conservation of angular momentum (section 19.3)

The angular impulse-angular momentum relationship is:

$$(\boldsymbol{H}_G)_1 + \sum \cancelto{0}{\int_{t_1}^{t_2} \boldsymbol{M}_G \, dt} = (\boldsymbol{H}_G)_2 \quad \text{or} \quad I_G \boldsymbol{\omega}_1 + \sum \cancelto{0}{\int_{t_1}^{t_2} \boldsymbol{M}_G \, dt} = I_G \boldsymbol{\omega}_2$$

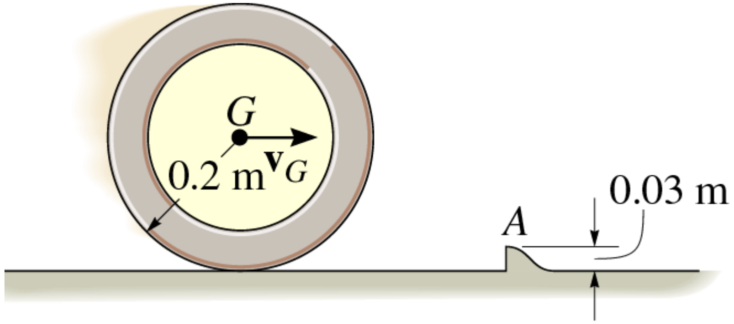
Similarly, if the sum of all the angular impulses due to external forces acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, angular momentum is conserved . The resulting equation is referred to as the conservation of angular momentum or $(\boldsymbol{H}_G)_1 = (\boldsymbol{H}_G)_2$.

If the initial condition of the rigid body (or system) is known, conservation of momentum is often used to determine the final linear or angular velocity of a body just after an event occurs.

Procedure of analysis (section 19.3)

- Establish the x, y, z inertial frame of reference and draw FBDs.
- Write the conservation of linear momentum equation.
- Write the conservation of angular momentum equation about a fixed point or at the mass center G.
- Solve the conservation of linear or angular momentum equations in the appropriate directions.
- If the motion is complicated, kinematic equations relating the velocity of the mass center G and the angular velocity ω may be necessary.

EXAMPLE



Given: A 10 kg wheel ($I_G = 0.156$ kg·m^2) rolls without slipping and does not rebound.

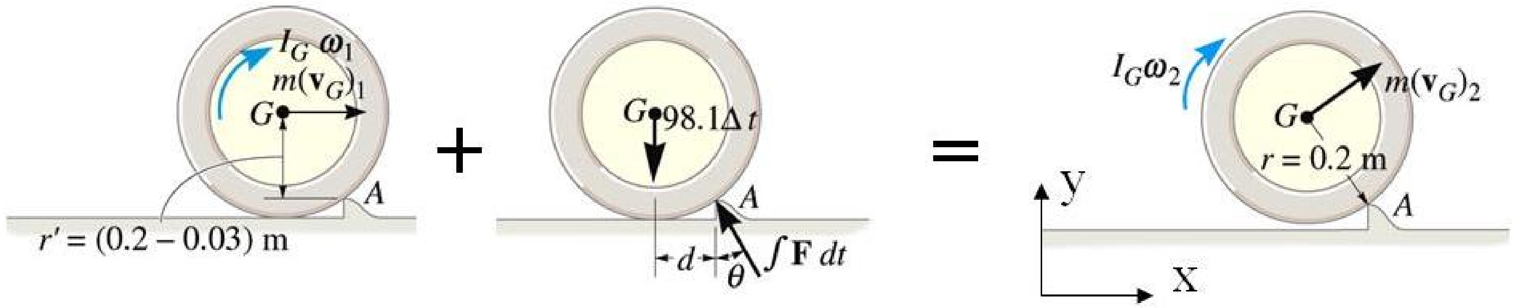
Find: The minimum velocity, $\boldsymbol{v}_G$, the wheel must have to just roll over the obstruction at A.

Plan: Since no slipping or rebounding occurs, the wheel pivots about point A. The force at A is much greater than the weight, and since the time of impact is very short, the weight can be considered non-impulsive. The reaction force at A is a problem as we don't know either its direction or magnitude. This force can be eliminated by applying the conservation of angular momentum equation about A.

EXAMPLE (continued)

Solution:

Impulse-momentum diagram:



Conservation of angular momentum:

Initial moment arm is 0.2-0.003 when just touching the bump

$$(H_A)_1 = (H_A)_2$$

$$r' \, m \, (v_G)_1 + I_G \, \omega_1 = r \, m \, (v_G)_2 + I_G \, \omega_2$$

$$(0.2 - 0.03) \; 10 \; (v_G)_1 + 0.156 \; \omega_1 = 0.2(10) \; (v_G)_2 + 0.156 \; \omega_2$$

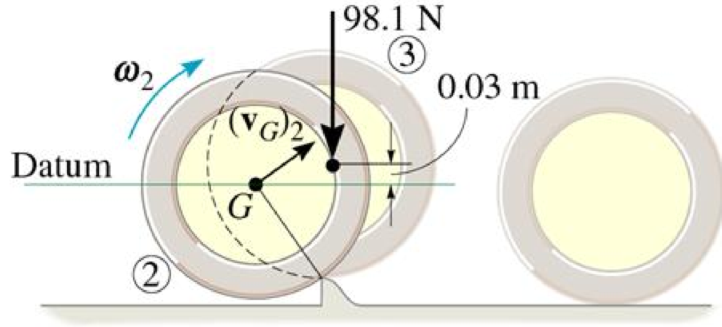
Kinematics: Since there is no slip, $\omega = v_G/r = v_G/0.2 = 5 \; v_G$.

Substituting and solving the momentum equation yields

$$(v_G)_2 = 0.892 \; (v_G)_1$$

EXAMPLE (continued)

To complete the solution, conservation of energy can be used. Since it cannot be used for the impact (why?), it is applied just after the impact. In order to roll over the bump, the wheel must go to position 3 from 2. When $(v_G)_2$ is a minimum, $(v_G)_3$ is zero. Why? Barely making over the bump



Energy conservation equation : $T_2 + V_2 = T_3 + V_3$

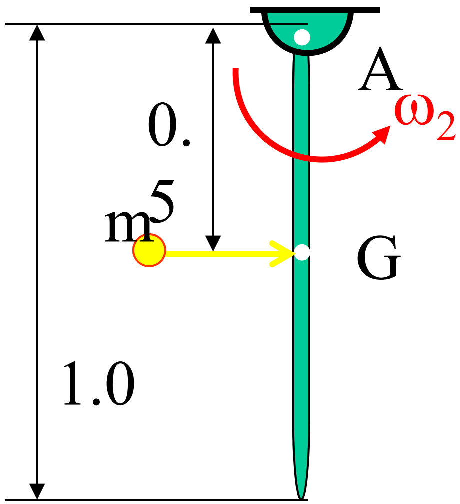
$$\{½\,(10)\,(v_G)_2^{\,2} + ½\,(0.156)\,\omega_2^{\,2}\} + 0 = 0 + 98.1\,(0.03)$$

Substituting $\omega_2 = 5\,(v_G)_2$ and $(v_G)_2 = 0.892\,(v_G)_1$ and solving yields

$$(v_G)_1 = 0.729 \text{ m/s}$$

CONCEPT QUIZ

1. A slender rod (mass = M) is at rest. If a bullet (mass = m) is fired with a velocity of v_b, the angular momentum of the bullet about A just before impact is ___________.



A) $0.5\ m\ v_b$ B) $m\ v_b$

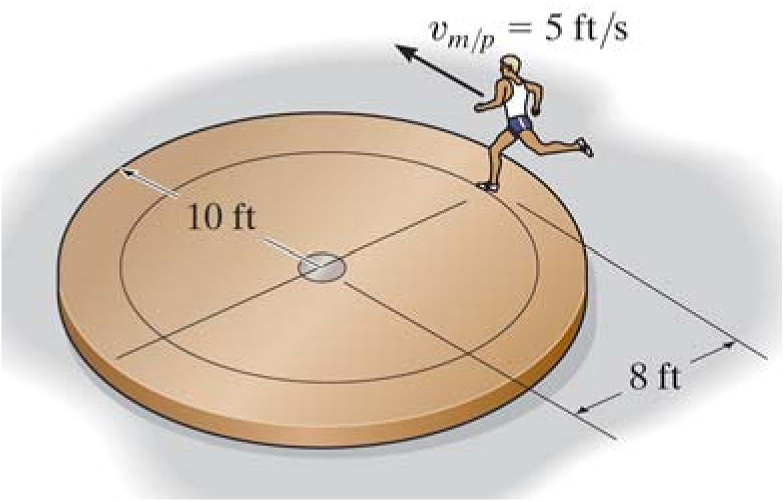
C) $0.5\ m\ v_b^2$ D) zero

2. For the rod in question 1, the angular momentum about A of the rod and bullet just after impact will be ___________.

A) $m\ v_b + M(0.5)\omega_2$ B) $m(0.5)^2\omega_2 + M(0.5)^2\omega_2$

C) $m(0.5)^2\omega_2 + M(0.5)^2\omega_2 + (1/12)\ M\ \omega_2$ D) zero

Example II



Given: A 150-lb man leaps off the circular platform with a velocity of $v_{m/p}$=5 ft/s, relative to the platform. Initially, the man and platform were at rest. The platform weighs 300 lb.

Find: The angular velocity of the platform afterwards.

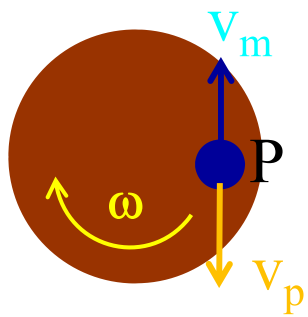
Plan: Apply the relative velocity equation to find the relationship between the velocities of the man and platform. Then, the conservation of angular momentum can be used to find the angular velocity of the platform.

Example II (continued)

Solution:

Kinematics: Since the platform rotates about a fixed axis, the speed of point P on the platform from which the man leaps is

$$v_p = \omega\, r = \omega\,(8)$$



Applying the relative velocity equation,

$$v_m = v_p + v_{m/p}$$

$$+\uparrow\ v_m = -\,\omega\,(8) + 5 \qquad (1)$$

Man running at 5ft/s relative to platform

Apply the conservation of angular momentum equation:

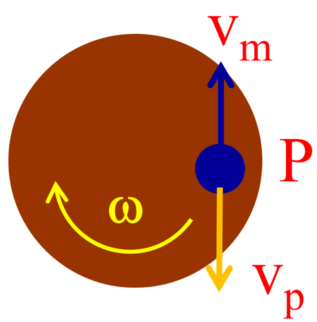
The impulse generated during the leap is internal to the system. Thus, angular momentum of the system is conserved about O.

Example II (continued)

The mass moment of inertia of the platform about this axis is

$$I_O = (1/2)\, m\, r^2 = (1/2)\,(300/32.2)\, 10^2$$

$$I_O = 465.8 \text{ slug ft}^2$$



Then, the conservation of angular momentum equation is

$$(H_O)_1 = (H_O)_2$$

$$+ \quad 0 = m_m v_m\,(8) - I_O\,\omega$$

$$0 = (150/32.2)\, v_m\,(8) - 465.8\,\omega$$

$$v_m = 12.5\,\omega \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$v_m = 3.05 \text{ ft/s}, \quad \omega = 3.05 \text{ rad/s}$$

Homework Assignment

Chapter19- 7, 11, 19, 20

Due next Wednesday !!!

W. Wang

Final Design Project

You will be given next two weeks to work on your final project in groups. Each team can only have up to four members.

W. Wang

Please form groups of 4

Final Project

- Design and construct a simple mechanical system and explain how it works using particle or planar kinematics and kinetics you learn in ME 230.

Rules

- Explain at least three components in your device using planar kinematics and kinetics (okay one component be particle)
- Construct the device and perform tests on those three components in your design
- Write up a report based on the format describe on the final report assignment.
- More interesting and challenging the design and analysis the more extra credit you will get.

Grade breakdown

- 10% proposal (follow the proposal format)
- 20% hardware (please upload a video of your machine in action to your Youtube account and provide a link in your final report or upload the video along with your report to the dropbox)
- 70% final report (Please follow the final report format and upload the file to the dropbox)

Additional Information

Scores are based on the complexity of your design, the correctness of the theory, calculation and report format you presented in the final report.

Design using planar kinetics and kinematics get higher grade than particle

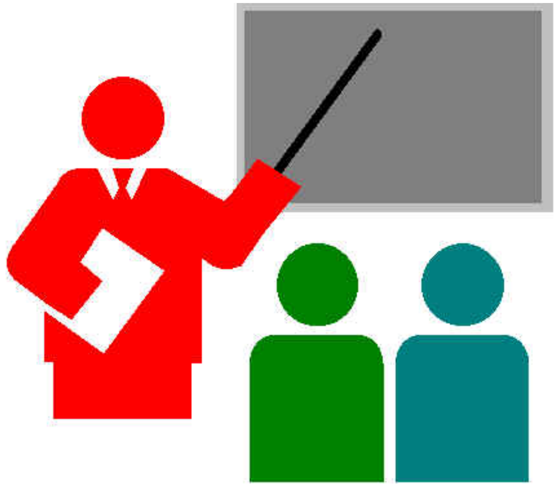
Extra credit on unique or more challenging concept or design.

Project Topics

- Mechanical Toy Design
- Mechanical System
- Motor, Engine design
- Gadgets
- Clock design
- Perpetual motion machine
- Mechanical Calculator (probably not)
- Amusement Park Rides
- Slot Machine, Pachinko Machine
- Shock absorber, bumper design
- Rude Goldberg Machine
- Eraser
- Rubber band propel airplane
- Variable damper (spring with different diameter)
- Electronic circuit (DC only)

Review

- Chapter 12: Kinematics of a particle
- Chapter 13: Kinetics of a particle: Force and acceleration
- Chapter 14: Kinetics of a particle: Work and energy
- Chapter 15: Kinetics of a particle: Impulse and momentum
- Chapter 16: Planar kinematics of a rigid body
- Chapter 17: Planar kinetics of a rigid body: Force and acceleration
- Chapter 18: Planar kinetics of a rigid body: Work and Energy
- Chapter 19: Planar kinetics of a rigid body: Impulse and Momentum



Chapter 12: Kinematics of a particle

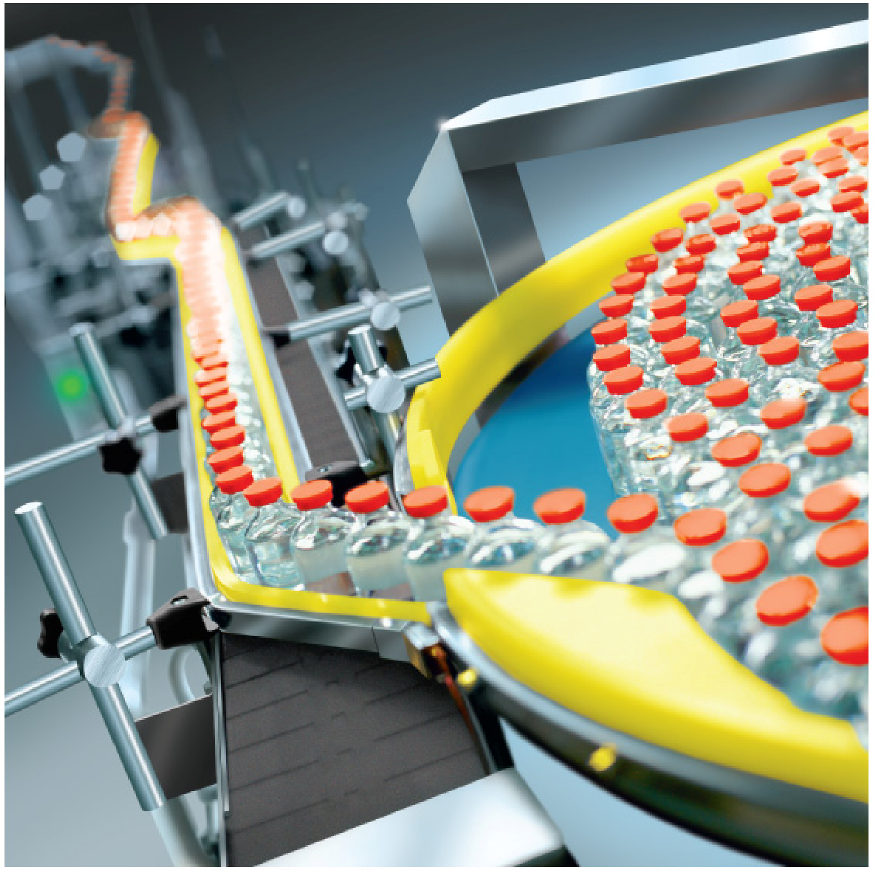
- Rectilinear kinematics
- Graphical solutions
- Curvilinear motion x,y,z
- Projectile motion
- Curvilinear motion n,t
- Curvilinear motion r, θ, z

- Absolute dependent motion of two particles
- Relative – motion analysis using translating axes

Chapter 13: Kinetics of a particle

Force and acceleration

- Kinetics
- Inertial coordinate systems



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Chapter 14: Kinetics of a particle

Work and energy

- Work of a force
- The principle of work and energy
- Power and efficiency
- Conservation of energy

Chapter 15: Kinetics of a particle

Impulse and momentum

- Principle of linear impulse and momentum for a particle
- Conservation of linear momentum for particles
- Mechanics of impact
- Concept of angular impulse and momentum

Chapter 16: Planar kinematics of a rigid body

- Types of rigid body planar motion
- Rigid body translation and motion about fixed axis
- Planar motion using absolute motion analysis
- Provide a relative motion analysis of velocity and acceleration using a translating frame of reference
- Instantaneous center of zero velocity
- Relative motion analysis of velocity and acceleration using a rotating frame of reference

Planar kinetics of a rigid body: Force and acceleration
Chapter 17

Chapter objectives

- Introduce the methods used to determine the mass moment of inertia of a body
- To develop the planar kinetic equations of motion for a symmetric rigid body
- To discuss applications of these equations to bodies undergoing translation, rotation about fixed axis, and general plane motion

Planar kinetics of a rigid body: Work and Energy
Chapter 18

Chapter objectives

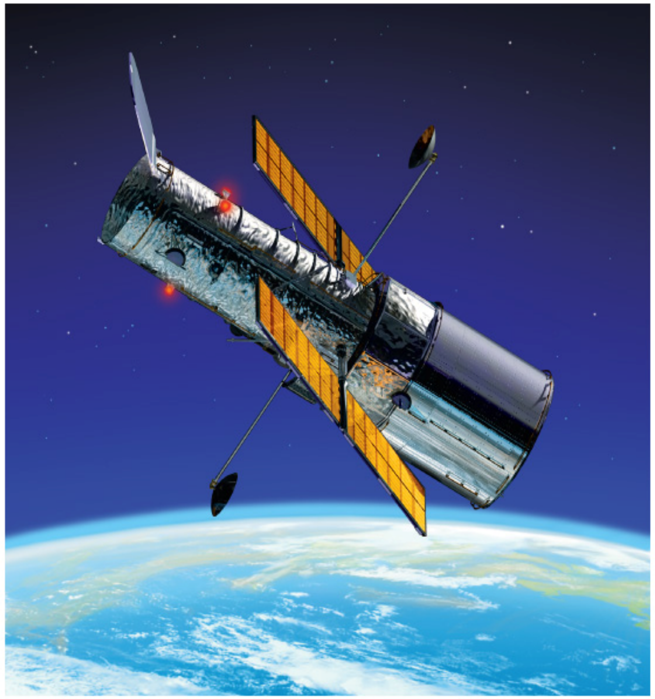
- Develop formulations for the kinetic energy of a body, and define the various ways a force and couple do work.
- Apply the principle of work and energy to solve rigid-body planar kinetic problems that involve force, velocity and displacement
- Show how the conservation of energy can be used to solve rigid-body planar kinetic problems

Planar kinetics of a rigid body: Impulse and Momentum

Chapter 19

Chapter objectives

- Develop formulations for the linear and angular momentum of a body
- Apply the principles of linear and angular impulse and momentum to solve rigid body planar kinetic problems that involve force, velocity and time
- To discuss the application of the conservation of momentum



Chapter reviews

Chapter 12: pages 101-105

Chapter 13: pages 166-167

Chapter 14: pages 217-219

Chapter 15: pages 295-297

Chapter 16: pages 391-393

Chapter 17: pages 452-453

Chapter 18: pages 490-493

Chapter 19: pages 531-533

Book chapter reviews give you a good but brief idea about each chapter…

Now I will put some music on…

…I will leave the room for a few minutes and…

…You will fill the evaluation forms

IBIZA SUNSET

By Alastair Ingason

W. Wang