Fluid Mechanics Primer

Fluids and Solids: Fundamentals

We normally recognize three states of matter: **solid; liquid and gas**.

However, liquid and gas are both **fluids**: in contrast to solids they lack the ability to resist deformation.

Because a fluid cannot resist deformation force, it moves, or *flows* under the action of the force. Its shape will change continuously as long as the force is applied.

A solid can resist a deformation force while at rest. While a force may cause some displacement, the solid does not move indefinitely.

Introduction to Fluid Mechanics

- Fluid Mechanics is the branch of science that studies the dynamic properties (e.g. motion) of fluids
- A fluid is any substance (gas or liquid) which changes shape uniformly in response to external forces
- The motion of fluids can be characterized by a continuum description (differential eqns.)
- Fluid movement transfers mass, momentum and energy in the flow. The motion of fluids can be described by conservation equations for these quantities: the Navier-Stokes equations.

Some Characteristics of fluids

Pressure: P = force/unit area

Temperature: T = kinetic energy of molecules

Mass: M=the quantity of matter

Molecular Wt: M_w = mass/mole

Density: ρ = mass/unit volume

Specific Volume: $v = 1/\rho$

Dynamic viscosity: μ = mass/(length•time)

-Dynamic viscosity represents the "stickiness" of the fluid

Important fluid properties -1

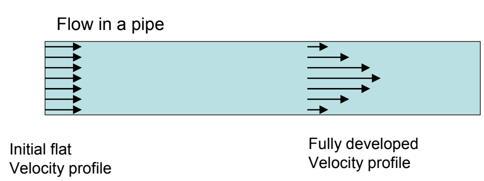
- A fluid does not care how much it is deformed; it is oblivious to its shape
- A fluid does care how fast it is deformed; its resistance to motion depends on the rate of deformation
- The property of a fluid which indicates how much it resists the rate of deformation is the dynamic viscosity

Important fluid properties -2

- If one element of a fluid moves, it tends to carry other elements with it… that is, a fluid tends to stick to itself.
- Dynamic viscosity represents the rate at which motion or momentum can be transferred through the flow.
- Fluids can not have an abrupt discontinuity in velocity. There is always a transition region where the velocity changes continuously.
- Fluids do not slip with respect to solids. They tend to stick to objects such as the walls of an enclosure, so the velocity of the fluid at a solid interface is the same as the velocity of the solid.

Boundary layer

- A consequence of this no-slip condition is the formation of velocity gradients and a boundary layer near a solid interface.



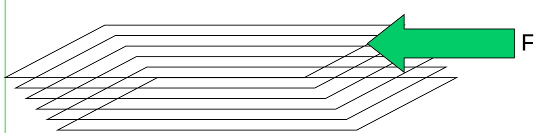
- The existence of a boundary layer helps explain why dust and scale can build up on pipes, because of the low velocity region near the walls

Boundary layer

- The Boundary layer is a consequence of the stickiness of the fluid, so it is always a region where viscous effects dominate the flow.
- The thickness of the boundary layer depends on how strong the viscous effects are relative to the inertial effects working on the flow.

Viscosity

- Consider a stack of copy paper laying on a flat surface. Push horizontally near the top and it will resist your push.



Viscosity

- Think of a fluid as being composed of layers like the individual sheets of paper. When one layer moves relative to another, there is a resisting force.
- This frictional resistance to a shear force and to flow is called viscosity. It is greater for oil, for example, than water.

Typical values

Property	Water	Air
Density ρ (kg/m^3)	1000	1.23
Bulk modulus K (N/m^2)	2×10^9	--------
Viscosity μ (kg/ms)	1.14×10^{-3}	1.78×10^{-5}

Shearing of a solid (a) and a fluid (b)

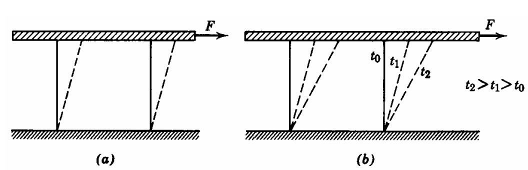


Fig. 1.1 Behavior of (a) solid and (b) fluid, under the action of a constant shear force.

The crosshatching represents (a) solid plates or planes bonded to the solid being sheared and (b) two parallel plates bounding the fluid in (b). The fluid might be a thick oil or glycerin, for example.

Shearing of a solid and a fluid

- Within the elastic limit of the solid, the shear stress τ = F/A where A is the area of the surface in contact with the solid plate.
- However, for the fluid, the top plate does not stop. It continues to move as time t goes on and the fluid continues to deform.

Shearing of a fluid

- Consider a block or plane sliding at constant velocity δu over a well-oiled surface under the influence of a constant force δF_x.
- The oil next to the block sticks to the block and moves at velocity δu. The surface beneath the oil is stationary and the oil there sticks to that surface and has velocity zero.
- **No-slip boundary condition--**The condition of zero velocity at a boundary is known in fluid mechanics as the "no-slip" boundary condition.

Shearing of a fluid (Couette flow model)

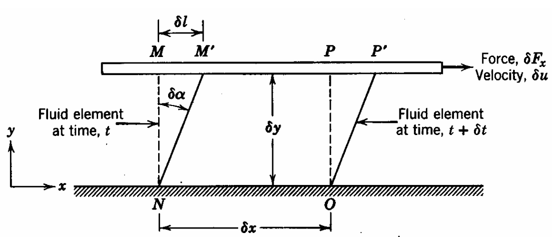


Fig. 2.8 Deformation of a fluid element.

Shearing of a fluid

- It can be shown that the shear stress τ is given by

$$\tau = \mu \frac{du}{dy}$$

- The term du/dy is known as the velocity gradient and as the rate of shear strain.
- The coefficient is the coefficient of **dynamic** viscosity, μ. (kg/m•s)

Shearing of a fluid

- And we see that for the simple case of two plates separated by distance d, one plate stationary, and the other moving at constant speed *V*
- Newton's law of viscosity:

$$\tau = \mu \frac{\mathrm{du}}{\mathrm{dy}} = \mu \frac{V}{h} = \frac{F_u}{Area}$$

Coefficient of dynamic viscosity

- Intensive property of the fluid.
- Dependent upon both temperature and pressure for a single phase of a pure substance.
- Pressure dependence is usually weak and temperature dependence is important.
- Typical symbol is μ. (mu) in units of: mass length^{-1} time^{-1} (kg/m•s or lbm/ft•s)

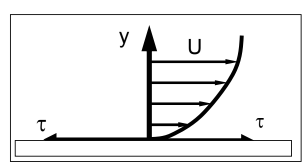
Shearing of a fluid

- Fluids are broadly classified in terms of the relation between the shear stress and the rate of deformation of the fluid.
- Fluids for which the shear stress is directly proportional to the rate of deformation are know as *Newtonian* fluids.
- Engineering fluids are mostly Newtonian. Examples are water, refrigerants and hydrocarbon fluids (e.g., propane).
- Examples of non-Newtonian fluids include toothpaste, ketchup, and some paints.

Shear stress in moving fluids

Newtonian fluid

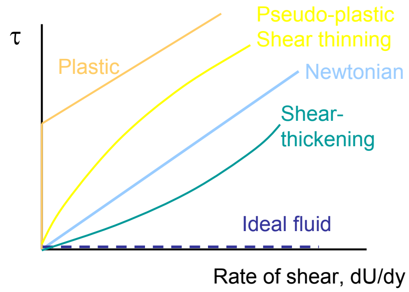
$$\tau = \mu \frac{dU}{dy}$$



μ = (mu)= viscosity (or dynamic viscosity) kg/m s

ν = (nu)= kinematic viscosity m^2/s $\boxed{\nu = \mu / \rho}$

Non-Newtonian Fluids



Viscous forces: F_u

$$\tau = \frac{F_u}{Unit\ Area} = \mu \frac{V}{h}$$

$$\tau \approx \frac{F_u}{h^2} = \mu \frac{V}{h}$$

$$F_u = \mu V h$$

Reynolds number derivation

Inertial force = F_I

$$F_I = \rho V^2 h^2$$

Viscous force = F_u

$$F_u = \mu V h$$

$$\mathrm{Re} = \frac{F_I}{F_u} = \frac{\rho V^2 h^2}{\mu V h} = \frac{\rho V h}{\mu}$$

$$\boxed{\mathrm{Re} = \frac{\rho V h}{\mu}}$$

Re indicates when inertial forces for the fluid flow are large compared to the viscous forces. It is one of the most important non-dimensional numbers in fluid mechanics. Geometrically similar flows with similar Re will have similar boundary layers and other flow structures.

Reynolds number (2)

- Kinematic viscosity = dynamic viscosity/density

$$\boxed{\nu \equiv \frac{\mu}{\rho}}$$

- So Reynolds number becomes:

$$\boxed{\mathrm{Re} = \frac{Vh}{\nu} = \frac{Velocity \bullet Length}{Kinematic\ Viscosity}}$$

- Re described by a velocity, length, and viscosity

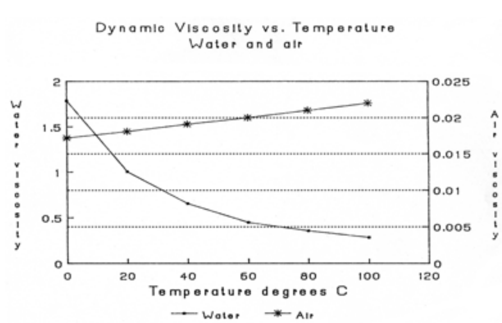
Application of Reynolds number

- The Re is useful to describe when the inertial of the fluid is important relative to the viscosity
  - Inertial forces → keeps things moving
  - Viscous forces → makes things stop
- Re also tells when the flow is smooth (laminar) or chaotic (turbulent)
  - High Inertial forces → large Re → turbulent flow
  - High viscous forces → small Re → laminar flow
- Laminar flow generally for Re < 1000
- Turbulent flow generally for Re > 10,000

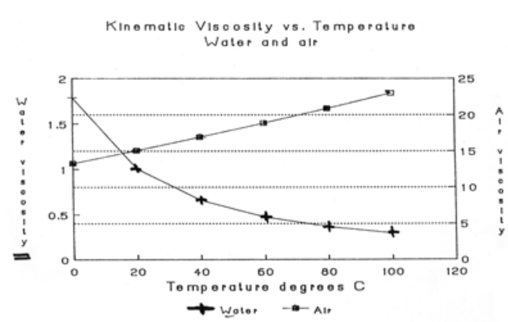
Viscosity changes with Temp

- Fluid properties depend on T (and P somewhat) because of molecular interactions
  - For a liquid, as T increases viscosity decreases
  - For a gas, as T increases viscosity increases
- Gases also change density significantly with T, so the kinematic viscosity increases more rapidly than the dynamic viscosity

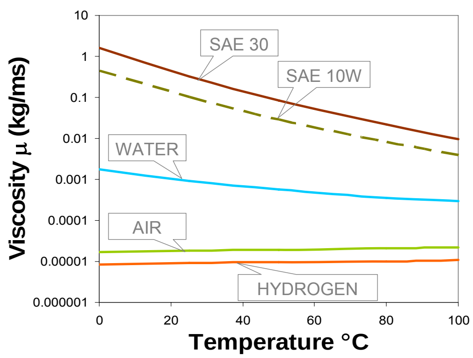
Dynamic viscosity



Kinematic Viscosity

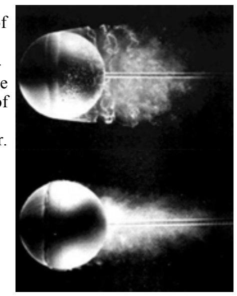


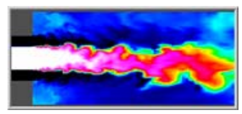
Variation of Fluid Viscosity with Temperature



Laminar and Turbulent flow

- UPPER IMAGE: Flow past a sphere at Re =15,000. Boundary layer separating ahead of the equator and remaining laminar ~ one radius, then becomes unstable and turbulent.
- BOTTOM IMAGE: Flow past a sphere at Re =30,000 with a trip wire. Wire hoop ahead of the equator trips the boundary layer, so it separates farther back than if it were laminar. The overall drag is dramatically reduced. This occurs naturally on a smooth sphere only at a Re numbers 10 times as great.
- Flow from a turbulent jet [car exhaust]





Jet: Werle, 1980 (ONERA) Photos from Album of Fluid Motion," by Van Dyke

Flow in a pipe for different Re

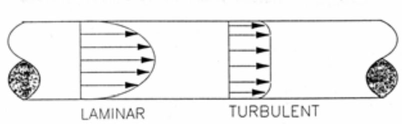
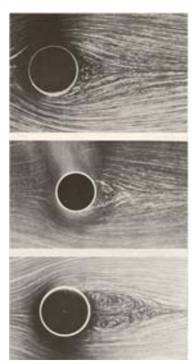


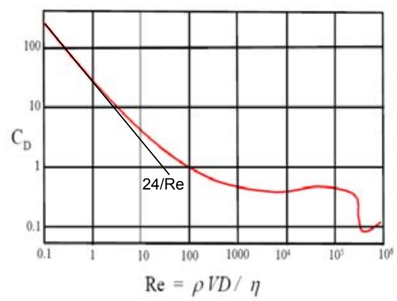
Figure 2.6 Flow-velocity profiles of laminar (Re < 2000) and turbulent (Re > 4000) flow regimes. The turbulent flow profile is nearly uniform across the radius, whereas the laminar profile is more parabolic.

Flow around a sphere



- Re ~ 1
- Re ~ 10
- Re ~ 100

Coefficient of Drag for a Sphere



$$F_D = \left(\frac{1}{2}\rho V^2\right) C_D A_D$$

$$A_D = \pi D^2 / 4$$

Variation of Log (Cd) vs Log(Re) for a smooth sphere

Line shows Cd for stokes region (Re<1)

END HERE Part I

- For smooth spheres Newton's drag law:

$$F_D = C_D \frac{\pi}{8} \rho_G d^2 V^2$$

- Re < 1, Cd = 24/Re
- 1< Re<1000

$$C_D = \frac{24}{Re}\left(1 + \frac{Re^{\frac{2}{3}}}{6}\right)$$

- $1000 < Re < 10^5$
  Cd ≈ 0.44 Re

PART II

- Some applications of fluid mechanics

Fluid Mechanics – Pressure

- Pressure = F/A
- Units: Newton's per square meter, Nm^{-2}, $kgm^{-1}\ s^{-2}$
- The same unit is also known as a Pascal, *Pa*, i.e. 1*Pa* = 1 Nm^{-2})
- Also frequently used is the alternative SI unit the *bar,* where 1 bar = 10^5 Nm^{-2}
- Dimensions: M L^{-1} T^{-2}

Fluid Mechanics – Pressure

- Gauge pressure:

$$p_{gauge} = \rho gh$$

- Absolute Pressure:

$$p_{absolute} = \rho gh + p_{atmospheric}$$

- Head (h) is the vertical height of fluid for constant gravity (g):

$$h = p/\rho g$$

- When pressure is quoted in head, density (ρ) must also be given.

Fluid Mechanics – Specific Gravity

- Density (r): mass per unit volume. Units are M L^{-3}, (slug ft^{-3}, kg m^{-3})
- Specific weight (SW): wt per unit volume. Units are F L^{-3}, (lbf ft^{-3}, N m^{-3})
- sw = rg
- Specific gravity (s): ratio of a fluid's density to the density of water at 4° C

$$s = r/r_w$$

- r_w = 1.94 slug ft^{-3}, 1000 kg m^{-3}

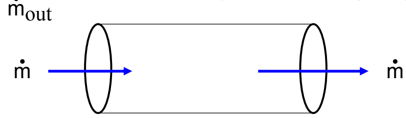
Fluid Mechanics – Continuity and Conservation of Matter

- Mass flow rate ($\dot{m}$) = Mass of fluid flowing through a control surface per unity time (kg s^{-1})
- Volume flow rate, or Q = volume of fluid flowing through a control surface per unit time (m^3 s^{-1})
- Mean flow velocity (V_m):

$$V_m = Q/A$$

Continuity and Conservation of Mass

- Flow through a pipe:
- Conservation of mass for steady state (no storage) says $\dot{m}_{in} = \dot{m}_{out}$



$$\rho_1 A_1 V_{m1} = \rho_2 A_2 V_{m2}$$

- For incompressible fluids, density does not changes ($\rho_1 = \rho_2$) so $A_1 V_{m1} = A_2 V_{m2} = Q$

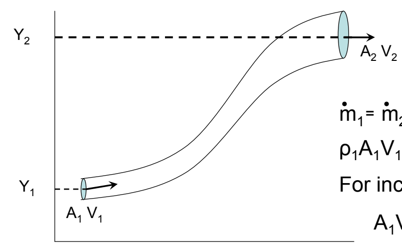
Fluid Mechanics – Continuity Equation

- The equation of continuity states that for an incompressible fluid flowing in a tube of varying cross-sectional area (A), the mass flow rate is the same everywhere in the tube:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- Generally, the density stays constant and then it's simply the flow rate (Av) that is constant.

Bernoulli's equation



$\dot{m}_1 = \dot{m}_2$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

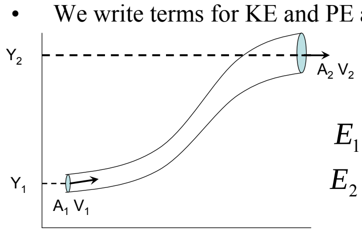
For incompressible flow

$A_1 V_1 = A_2 V_2$

Assume steady flow, V parallel to streamlines & no viscosity

Bernoulli Equation – energy

- Consider energy terms for steady flow:
- We write terms for KE and PE at each point



$E_i = KE_i + PE_i$

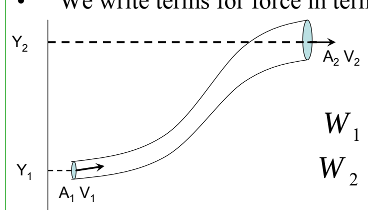
$$E_1 = \tfrac{1}{2}\dot{m}_1 V_1^2 + g\dot{m}_1 y_1$$

$$E_2 = \tfrac{1}{2}\dot{m}_2 V_2^2 + g\dot{m}_2 y_2$$

As the fluid moves, work is being done by the external forces to keep the flow moving. For steady flow, the work done must equal the change in mechanical energy.

Bernoulli Equation – work

- Consider work done on the system is Force x distance
- We write terms for force in terms of Pressure and area



$W_i = F_i V_i \, dt = P_i V_i A_i \, dt$

Note $V_i A_i \, dt = \dot{m}_i / \rho_i$

$$W_1 = P_1 \dot{m}_1 / \rho_1$$

$$W_2 = -P_2 \dot{m}_2 / \rho_2$$

Now we set up an energy balance on the system. Conservation of energy requires that the change in energy equals the work done on the system.

Bernoulli equation- energy balance

Energy accumulation = ΔEnergy – Total work

0 = (E_2-E_1) – (W_1+W_2)   i.e. no accumulation at steady state

Or W_1+W_2 = E_2-E_1 Subs terms gives:

$$\frac{P_1\dot{m}_1}{\rho_1} - \frac{P_2\dot{m}_2}{\rho_2} = (\tfrac{1}{2}\dot{m}_2V_2^2 + g\dot{m}_2y_2) - (\tfrac{1}{2}\dot{m}_1V_1^2 + g\dot{m}_1y_1)$$

$$\frac{P_1\dot{m}_1}{\rho_1} + \tfrac{1}{2}\dot{m}_1V_1^2 + g\dot{m}_1y_1 = \frac{P_2\dot{m}_2}{\rho_2} + \tfrac{1}{2}\dot{m}_2V_2^2 + g\dot{m}_2y_2$$

For incompressible steady flow $\dot{m}_1 = \dot{m}_2 \; and \; \rho_1 = \rho_2 = \rho$

$$\boxed{P_1 + \tfrac{1}{2}\rho V_1^2 + g\rho y_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + g\rho y_2}$$

Forms of the Bernoulli equation

- Most common forms:

$$P_1 + \tfrac{1}{2}\rho V_1^2 = P_2 + \tfrac{1}{2}\rho V_2^2 + g\rho\Delta h$$

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + \Delta P_{ht}$$

The above forms assume no losses within the volume…

If losses occur we can write:

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses \quad + \Delta P_{ht}$$

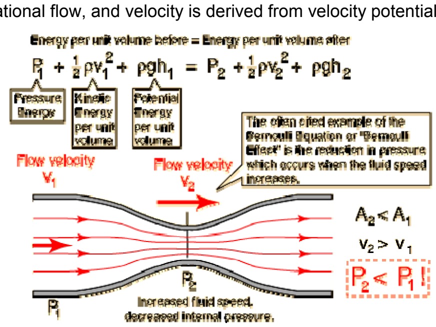
And if we can ignore changes in height:

$$\boxed{P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses}$$

Key eqn

Application of Bernoulli Equation

- Daniel Bernoulli developed the most important equation in fluid hydraulics in 1738. this equation assumes constant density, irrotational flow, and velocity is derived from velocity potential:



Bernoulli Equation for a venturi

- A venturi measures flow rate in a duct using a pressure difference. Starting with the Bernoulli eqn from before:

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses \quad + \Delta P_{ht}$$

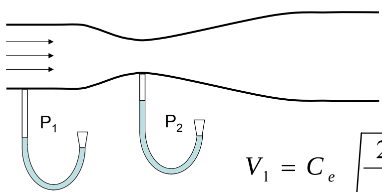
- Because there is no change in height and a well designed venturi will have small losses (<~2%) We can simplify this to:

$$P_{S1} + P_{S2} = P_{V2} + P_{V1} \quad or \quad -\Delta P_S = \Delta P_V$$

- Applying the continuity condition (incompressible flow) to get:

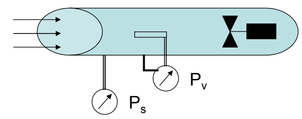
$$V_1 = \sqrt{\frac{2(P_{S1} - P_{S2})}{\rho\left(1 - \frac{A_2^2}{A_1^2}\right)}}$$

Venturi Meter



$$V_1 = C_e \sqrt{\frac{2(P_{S1} - P_{S2})}{\rho\left(1 - A_2^2 / A_1^2\right)}}$$

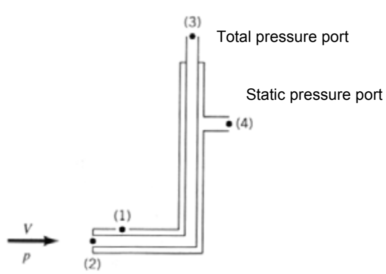
- Discharge Coefficient C_e corrects for losses = $f(R_e)$



- $P_T = P_S + P_V$ ← i.e. an energy balance
- Static pressure – potential energy
- Velocity pressure – KE
- Total pressure – total energy
- Recall KE is ½ mV^2 so KE term is proportional to V^2
- At NTP, $P_V = (V/4005)^2$ for V in ft/min, P_V in inches H_2O
  - Or $V = 4005\sqrt{P_V}$ {at non std conditions, $V = 4005\sqrt{(P_V/d)}$}

Pitot tube

The static and Pitot tube are often combined into the one-piece Pitot-static tube.



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