Comprehensive Final Exam (120 Points Total)

- The space provided below each question should be sufficient for your answer. If you need additional space, use additional paper.
- You are allowed to use a calculator, but only the basic functions. Use of advanced formulas (e.g., if your calculator does present value) or of material that you have programmed into your calculator is not allowed and will be considered cheating.
- You are encouraged to show your work for partial credit. It is very difficult to give partial credit if the only thing on your page is “x = 3”.
- **Expected value** is given by summing likelihood times value over all possible outcomes:
  \[
  \text{Expected Value} = \sum_{\text{Outcomes } i} \text{Probability}(i) \cdot \text{Value}(i).
  \]
- A **fair bet** is a bet with an expected value of zero.
- The **future value of a lump sum payment** of \$x\ invested for \(n\) years at interest rate \(s\) is \(FV = x(1 + s)^n\). The **present value of a lump sum payment** of \$x\ after \(n\) years at interest rate \(s\) is \(PV = \frac{x}{(1 + s)^n}\). (Note that this formula also works for values of \(n\) that are negative or zero.)
- The present value of an **annuity** paying \$x\ at the end of each year for \(n\) year at interest rate \(s\) is
  \[
  PV = x \left[ \frac{1}{(1 + s)^n} \right].
  \]
  The present value of the related **perpetuity** (with annual payments forever) is
  \[
  PV = \frac{x}{s}.
  \]
- The **inflation rate**, \(i\), is the rate at which prices rise. The **nominal interest rate**, \(n\), is the interest rate in terms of dollars. The **real interest rate**, \(r\), is the interest rate in terms of purchasing power. These are related by
  \[
  1 + r = \frac{1 + n}{1 + i}.
  \]
  When the inflation rate is small, we can approximate this as
  \[
  r \approx n - i.
  \]
• A **Pareto efficient** (or **Pareto optimal**) allocation or outcome is one in which it is not possible find a different allocation or outcome in which nobody is worse off and at least one person is better off. An allocation or outcome B is a **Pareto improvement over A** if nobody is worse off with B than with A and at least one person is better off.

• A (strictly) **dominant strategy** is a strategy which yields higher payoffs than any other strategy regardless of the other players’ strategies.

• In an **ascending price auction**, the price starts out at a low value and the bidders raise each other’s bids until nobody else wants to bid. In a **descending price auction**, the price starts out at a high value and the auctioneer lowers it until somebody calls out, “Mine.” In a **first-price sealed-bid auction**, the bidders submit bids in sealed envelopes; the bidder with the highest bid wins, and pays an amount equal to his or her bid (i.e., the highest bid). In a **second-price sealed-bid auction**, the bidders submit bids in sealed envelopes; the bidder with the highest bid wins, but pays an amount equal to the **second-highest** bid.

• **Total revenue** is price times quantity: \( TR = pq \).

• The **price elasticity of demand at point A** measures the percentage change in quantity demanded (relative to the quantity demanded at point A) resulting from a 1% increase in the price (relative to the price at point A). The formula is

\[
\varepsilon(A) = \frac{\text{% change in } q}{\text{% change in } p} = \frac{\frac{\Delta q}{q_A}}{\frac{\Delta p}{p_A}} = \frac{\Delta q}{\Delta p} \cdot \frac{p_A}{q_A} = \frac{q_B - q_A}{p_B - p_A} \cdot \frac{p_A}{q_A}
\]

In **English** If, at point A, a small change in price causes the quantity demanded to increase by a lot, demand at point A is elastic; if quantity demanded only changes by a little then demand at point A is inelastic; and if quantity demanded changes by a proportional amount then demand at point A has unit elasticity.

In **math** If, at point A, the price elasticity of demand is less than \(-1\) (e.g., \(-2\)), then demand at point A is elastic; if the elasticity is greater than \(-1\) (e.g., \(-\frac{1}{2}\)), then demand at point A is inelastic; if the elasticity is equal to \(-1\) then demand at point A has unit elasticity.
1. For each item, indicate the likely impact on the supply and demand for apples. Then indicate the effect on the equilibrium price and quantity. If you use a graph, all you need to have is an arrow indicating which curve(s) shift which way.

(a) (5 points) News reports suggest that an apple a day really does keep the doctor away.

(b) (5 points) Worms destroy a large part of the apple crop.

(c) (5 points) New farming methods make apple orchards more productive.

(d) (5 points) The price of oranges falls. (Assume that apples and oranges are substitutes, like tea and coffee or Coke and Pepsi.)

2. (5 points) Explain, as if to a non-economist, why the intersection of the market supply curve and the market demand curve identifies the market equilibrium.
3. Below is a hypothetical demand curve for oranges.

\[
\begin{array}{c|c}
\text{P ($/pound)} & \text{Q (millions of pounds per day)} \\
\hline
$1.60 & \text{X} \\
$1.40 & \\
$1.20 & \\
$1.00 & \\
$0.80 & \\
$0.60 & \\
$0.40 & \\
$0.20 & \\
\end{array}
\]

(a) (5 points) During normal years, the supply curve is such that point Y is the equilibrium. Of the other two points, one is the equilibrium during “bad” years (when frost damages the orange crop), and one is the equilibrium during “good” years (when the orange crop thrives). Which one is point X? Circle one: X = bad good

(b) (5 points) What is the total revenue at point X? At point Y? At point Z? (Use correct units! And note that the formula is in the cheat sheet . . . )

(c) (5 points) The orange growers’ profit is total revenue minus total costs. If total costs are the same in all years, do the growers have higher profits in “bad” years or “good” years? (Circle one.)
4. Below is a hypothetical market for oranges.

Suppose that the government decides to impose a per-unit tax of $0.80 on the buyers of oranges.

(a) (5 points) Show the impact of this tax on the supply and demand curves above.

(b) (5 points) Explain why the tax shifts the curves the way it does. Your answer here must be quantitative, i.e., must explain not only the direction of the curve shift(s) but also the amount of the curve shift(s).

(c) (5 points) Calculate the economic incidence of the tax, i.e., the amount of the tax burden borne by the buyers ($T_B = p_2 + t - p_1$) and the amount borne by the sellers ($T_S = p_2 - p_1$). Then calculate their ratio $\frac{T_B}{T_S}$. 
(d) (5 points) Calculate the price elasticity of supply, $\varepsilon_S$, at the original (pre-tax) equilibrium. Then calculate the price elasticity of demand, $\varepsilon_D$, at the original (pre-tax) equilibrium. Then calculate their ratio, $\frac{\varepsilon_S}{\varepsilon_D}$. How does this ratio compare to the ratio of the tax burdens?

5. (5 points) How would the economic incidence of the tax change if the legal incidence of the tax were shifted from the buyers to the sellers?

6. (5 points) Show the result if the government had instead imposed an $\.80 per-unit tax on the sellers. (No need to explain.)
7. Imagine that you own some land and you decide to manage it as a tree farm: you plant some trees, and then you cut them down and sell the lumber. Your objective is to make as much money as possible, i.e., to maximize the present value of the lumber. (Assume for simplicity that replanting or other land uses are not possible.)

(a) (5 points) Is the interest rate at the bank going to affect your decision about when to cut down the trees? Circle one (Yes No) and explain briefly.

(b) (5 points) Is the amount it cost you to plant the trees going to affect your decision about when to cut them down? Circle one (Yes No) and explain briefly.

8. Narrowly defined, a “Prisoners’ Dilemma” situation involves the following: (1) a symmetric, simultaneous-move game featuring two players; (2) the existence of a dominant strategy for each player; and (3) a predicted outcome that is Pareto inefficient.

(a) (5 points) Draw a payoff matrix that describes such a situation. (It may help to remember the following conventions about payoff matrices: player 1 chooses the row, player 2 chooses the column, and an outcome of \((x, y)\) indicates that player 1 gets \(x\) and player 2 gets \(y\).) You do not need to write any explanation, but if you cannot draw a payoff matrix then some words might get you some partial credit.
(b) (5 points) A slightly broader definition of “Prisoners’ Dilemma” would include situations featuring more than two players. Provide an example of one such situation—you can describe one we’ve discussed in class, or make up your own—and briefly explain what the strategies are, what the predicted outcome is, and what would be a Pareto improvement over that predicted outcome.

9. Consider the following game featuring 4 ounces of cake and two kids, each of whom has as his or her sole objective the desire for as much cake as possible: Player 1 splits the cake by offering Player 2 either 1, 2, 3 ounces of cake; Player 2 then either accepts the offer (in which case they split the cake accordingly) or rejects the offer (in which case each player gets 1.5 ounces of cake).

(a) (5 points) Draw a game tree that represents this game.

(b) (5 points) Identify (with a star on the game tree, or in words if you couldn’t draw a game tree) the predicted outcome of this game. Then circle all of the Pareto efficient outcomes in the following list, and identify a Pareto improvement for any outcome that is not Pareto efficient: (3, 1), (2, 2), (1, 3), (1.5, 1.5).
10. The Intergovernmental Panel on Climate Change reports that human activity (especially the burning of fossil fuels such as coal, oil, and natural gas) is warming the earth. (Note: With the exception of this fact, all of the numbers &etc in this question are completely made up.)

(a) (5 points) Assume that global warming will raise sea levels and increase the frequency of hurricanes, leading to damages of $1 trillion ($= 10^{12} = 1,000,000,000,000$) at the end of each year for the next seven years. What is the present value of that damage if the relevant interest rate is 4%? [Note: If all the zeroes confuse you or your calculator, use $1,000,000$ or $1,000$ instead.] Also: is the relevant interest rate nominal or real? (Circle one.)

(b) (5 points) Next, assume that the full damages you’ve calculated above will only occur with probability 1/3. With probability 1/3 the damages will be only half as big, and with probability 1/3 the damages will be zero. What is the expected value of the damage caused by global warming? [Note: If you didn’t answer part 10a above, just assume for this part that the total damage is $1,000,000$.]

(c) (5 points) Next, assume that the hurricanes &etc won’t happen for 100 years. So: take the expected damages you calculated in part 10b and compute the present value of having that amount of damage occur 100 years in the future if the relevant interest rate is 4%. [Note: If you didn’t answer part 10b, assume for this part that the total damage is $1,000,000$.]