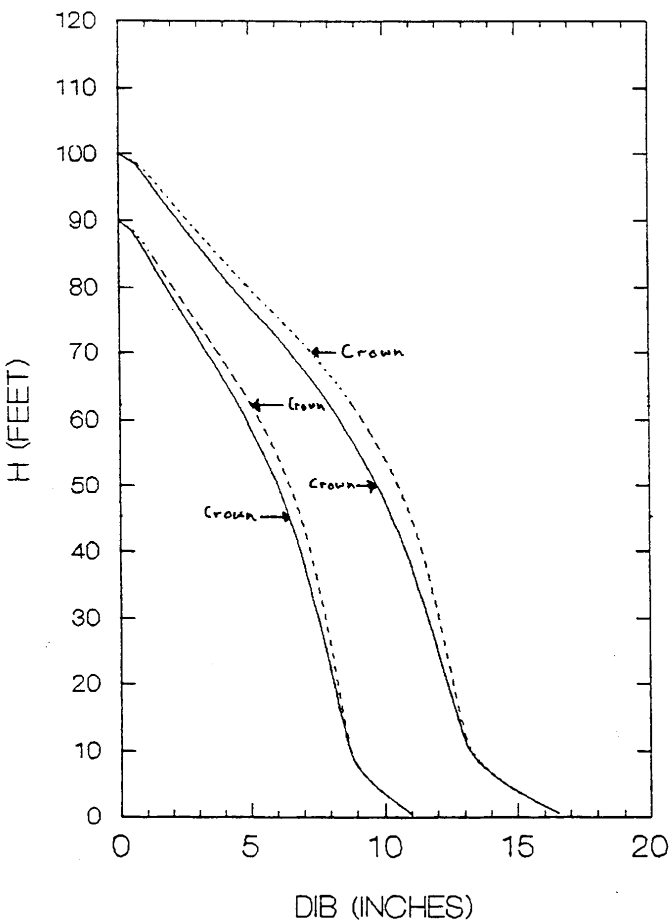
4.3 Stem Volume Distribution & Taper Equations

Knowing the distribution of volume over a tree stem can be used to make more specific volume (biomass) estimates, can be used as an aid in estimating volume losses from damage and defects, and can even be valuable information for assessing wildlife habitat quality

Stem form is affected by:

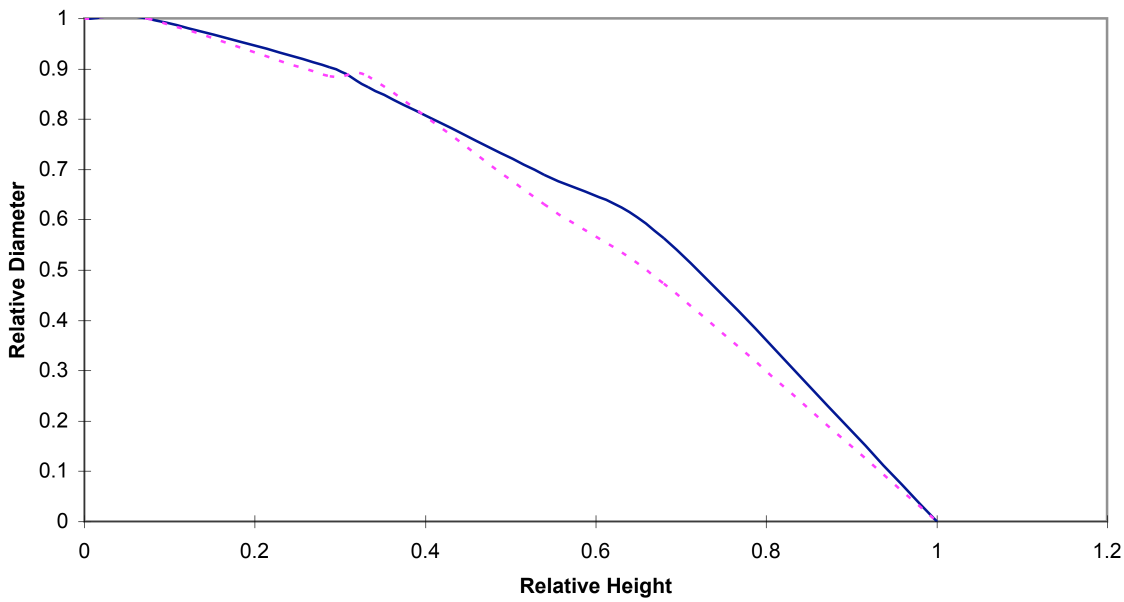
1. Crown length –
   Stem area growth increases downward within the tree crown, producing strongly tapered stems within the crown. Sapwood area at any point in crown is proportional to the amount of foliage above that point. Factors affecting crown length also affect stem form.



Predicted taper of representative 10- and 15-inch trees in control plots (- - -) and thinned plots (——). From Hoskins installation in LOGS study.

2. Relative rates of Height & DBH growth –
   A tree growing faster in height relative to DBH will have "better" stem form (less taper). Of course, tree growth rates are tied to the tree's crown length (or LCR), also tied to tree size, "social" position in stand, site quality, and stand density.

3. Genetic Inheritance –
   Genetic makeup can have a strong influence on stem form. Many differences are observed between species, too.
4. Treatment effects –
   Thinning generally increases taper, pruning decreases taper. These treatments chiefly affect crown length, hence taper.



Taper of representative size-matched trees in the control (unpruned) plots (- - -) and 60% pruned plots (——). From Ostrander Rd. installation in SMC Green Crown Removal Study

Volume Distribution Tables

In this approach, relationships between the percentage of volume present in a tree stem below a particular point on the stem are developed and tabulated against either a) log position or b) a fixed set of diameter and/or height ratios

a) Average distribution of tree volumes by logs according to log position

Usable length (16-ft logs)	Percent of Total Volume in each Log, by Position					
	1st	2nd	3rd	4th	5th	6th
1	100					
2	59	42				
3	42	33	25			
4	34	29	22	15		
5	29	25	21	15	10	
6	24	23	20	16	11	6

Source: Adapted from Mesavage and Girard. 1946. USDA Forest Service.

b) Proportions of Total Stem Volume for (Upper Height:Total height) & (Upper Diameter:DBH) ratios.

Height Ratio h/H	Volume Percentage	Diameter Ratio d_u/DBH	Volume Percentage
0.10	21.76	0.10	100.00
0.20	39.60	0.20	99.96
0.30	55.12	0.30	98.88
0.40	68.30	0.40	96.22
0.50	79.12	0.50	91.59
0.60	87.70	0.60	84.07
0.70	93.92	0.70	72.52
0.80	97.80	0.80	55.63
0.90	99.36	0.90	31.87

Source: Adapted form Honer, et al. 1983. Canadian Forestry Service.

Volume Ratio Systems

As with Comprehensive Tree Tarif System, consistency among different merchantability standards is maintained by utilizing the merchantability standard as an independent variable

For example, the volume to any merchantability limit of slash pine trees growing in the lower coastal plain of Georgia and Florida:

$$V_t = 0.00616 D^{2.05799} H^{0.74679}$$

$$V_m = V_t \left(1 - 0.61529 \frac{d^{3.66827}}{D^{3.47361}} \right)$$

where, V_t denotes total stem volume (cu. ft, o.b.); D denotes *DBH*; H denotes total Height; d denotes merchantable top diameter (o.b.), V_m denotes merchantable volume to a d-inch top diameter

Taper (Stem profile) Equations

- Taper is the term used to describe the decrease in tree stem diameter with increasing height (technically it is a *rate*, with units of (inches / ft)
- Taper equations (which actually describe stem profile) are most often used to estimate (predict) diameter or squared-diameter (i.e., cross-sectional area) of a tree stem at any height on the tree, either inside or outside bark.

- Most common form of equation: $d_i = f\left(D, H, h_i\right)$

  where d_i denotes diameter of stem, either inside or outside bark, at any height, h_i, above ground; D, denotes DBH; H denotes total height of tree; h_i denotes any distance from ground level

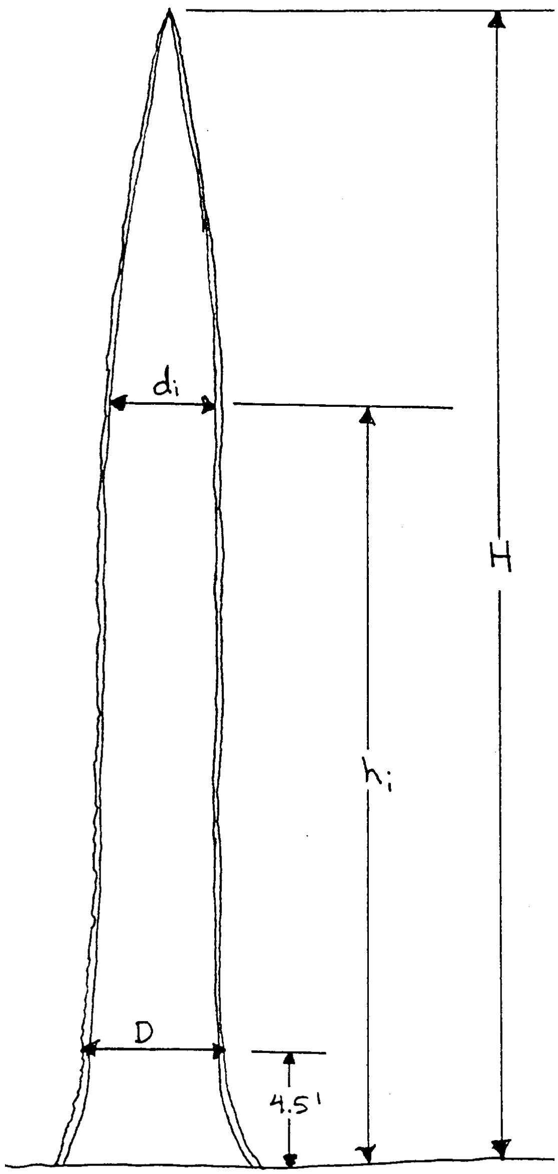


Illustration of variables typically used in taper equations.

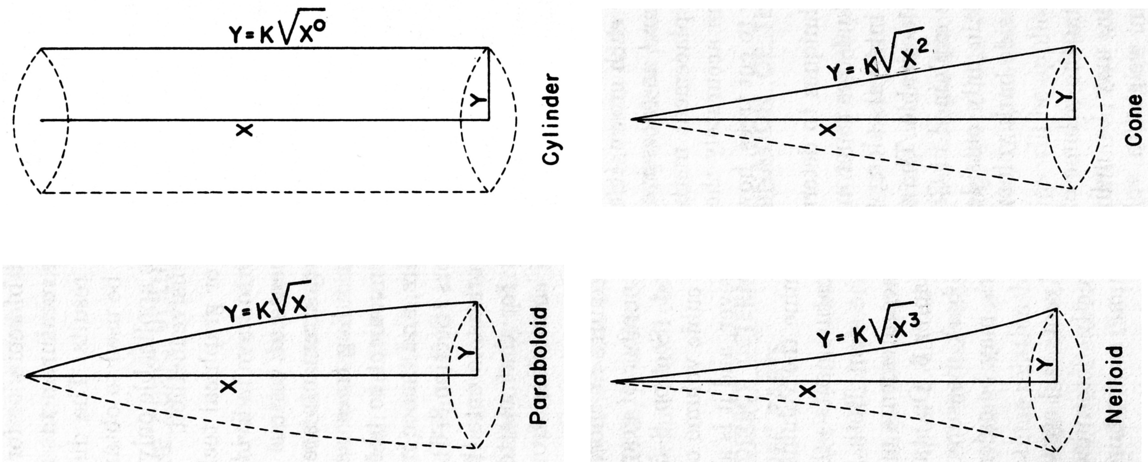
Example uses:

1. Predict cubic foot or board foot volume
2. Derive accurate biomass estimates
3. Finding optimal bucking pattern for desired product mix
4. Predict potential of tree for cavity nesting / dwelling animals
5. ?

Two approaches: Arbitrarily complex mathematical equations or Behre's hyperbola

I. Arbitrarily complex mathematical equation examples:

   0. Regular geometric solids (here, $Y = d_i$, $K = D$, $X = (H - h_i)$),



   1. For 19 tree species in British Columbia (Kozak, et al. 1969)

$$\left(\frac{d}{D}\right)^2 = c_0 + c_1\left(\frac{h}{H}\right) + c_2\left(\frac{h}{H}\right)^2, \quad \text{or} \quad d^2 = D^2\left[c_0 + c_1\left(\frac{h}{H}\right) + c_2\left(\frac{h}{H}\right)^2\right]$$

   For Douglas-fir in the PNW: c_0= 0.85458, c_1= -1.29771, c_2= 0.44313

   2. For six conifer species in California (Biging 1984)

$$\frac{d}{D} = b_0 + b_1 \ln\left\{1 - \left[1 - e^{\left(-b_0/b_1\right)}\right]\left[\left(\frac{h}{H}\right)^{1/3}\right]\right\}$$

   For Douglas-fir in northern, CA: b_0= 1.027763, b_1= 0.333721

II. The Behre hyperbola (Behre 1927, expanded by Bruce 1972)

   A very successful ways to describe tree form (stem profile) is with a hyperbola described by Behre: $D = \frac{L}{AL + B}$, where

   D = tree diameter at any point, expressed as a percentage of a defined basal diameter (usually, basal diam. is defined as diam. at top of first log, D_1, so $D = (d / D_1)$),

   L = length to the diameter point, measured downward from tree tip expressed as a percentage of the total length, and

   A, B are constants chosen to match a tree's general profile.

In practice, a “truncated” hyperbola is often used such that the tree profile is defined not from the tip, but from a truncation point such as 0.5 (one-half) the basal diameter, in which case the hyperbola becomes:

$$D = 0.5 + \frac{L}{0.9L + 1.1} \; ,$$ where D is the same as before, but

L is now the length to the diameter point, measured downward from the truncation point (in this case, 0.5 x basal diameter) to the point chosen for basal diameter measurement (usually either the top of the first 16- or 32-foot log), expressed as a % of total measured length (M) from truncation point to basal diameter.

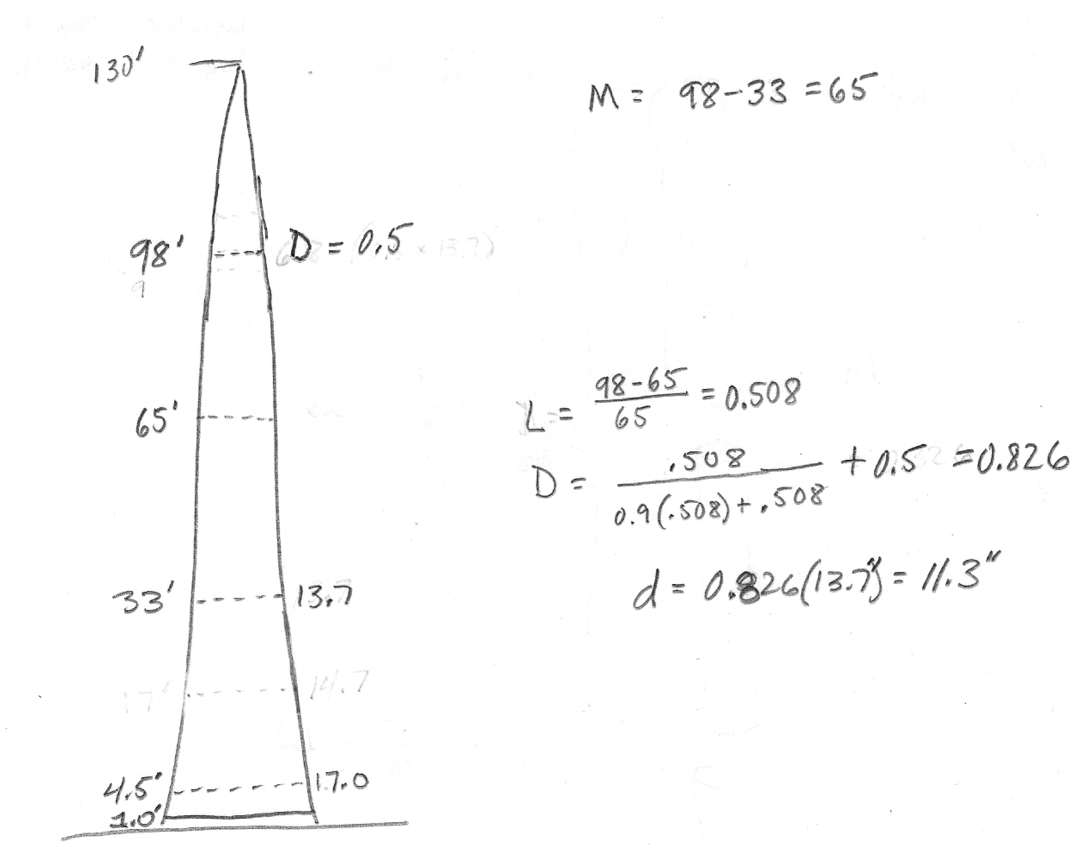
Example.

Basal diameter point: 33 ft. (top of first 32’ log, including stump ht.)

Basal diameter: 13.7 in.

Truncation point: 0.5 D (= 6.8”)

Desire diameter, d, at top of 2nd 32-ft log (all other log diam.’s follow similarly)



Estimating Volume from Taper Equations

Basically two ways to do this:

I. Use "The Calculus" (in some respects, the hard way) –

1. Total cubic foot volume (including top & stump, CVTS)

   Since stem profile is expressed as $d = f(D, H, h)$, then

$$CVTS = 0.005454\int_0^H \left(d^2\right)dh$$

$$CVTS = 0.005454\int_0^H \left[f\left(D, H, h\right)\right]^2 dh$$

   For example, Kozak, et al.'s (1969) quadratic taper equation

$$CVTS = 0.005454\int_0^H D^2\left[c_0 + c_1\left(\frac{h}{H}\right) + c_2\left(\frac{h}{H}\right)^2\right]dh$$

2. Merchantable cubic foot volume (between any two points)

   First solve for merchantable height, h_m corresponding to chosen merchantable diameter, d_m, by inverting taper equation

$$d_m = f\left(D, H, h_m\right)$$
$$f^{-1}\left(d_m\right) = f^{-1}\left[f\left(D, H, h_m\right)\right]$$

   Then, for CV8, for example,

$$CVTS = 0.005454\int_{h_s}^{h_8} \left[f\left(D, H, h\right)\right]^2 dh$$

II. Use Smalian's formula (in some respects, the easy way) –

   Since stem profile is expressed as $d = f(D, H, h)$,

   For every "log" of length L from 1 (the first, or butt log) to tip (H) or any merchantable top specification, say 4", compute diameters, d, corresponding to those logs, use a log rule (or Smalian's formula for cubic feet) on each L-ft. section, "L" being std. log length, then sum up the volumes in the sections.

Deduction Methods for Individual Trees

Causes for deducting tree volume

- Defect: burls, forks, crooks, fire scars
- Decay: conk rot, butt rot, top rot, root rot
- Breakage: mechanical damage that occurs anytime between the start of a harvesting operation and log scaling
- Missing parts

Example.
Consider a hypothetical tree that's 163-ft. tall has a 24-inch DBH. An applicable standard volume equation obtains 199.9 cu.ft for this tree. Field observation revealed this tree contains 4, 32-ft logs to an 8-inch top diameter, and that the 3rd log has crook, taking about 24 ft for the tree to straighten again.

First, find out what height corresponds to 4 logs, assuming a standard stump height of 1 ft., and a standard trim allowance of 0.3 ft. per log. So, 4 logs represents a height of 1 + 4 x 32.3 = 130.2 feet; a Height Ratio of 130.2 / 163 = 0.799 ~ 0.80.

From the table of volume proportions by d- and h-ratios above, this Height Ratio corresponds to a Volume Ratio = 97.80.

To deduct for the crook defect, 24-ft. is apparently not usable, thus 24 / 32 = 0.75, or 75% of 3rd log is cull

We can then construct the following cull tally for this tree.

CULL TALLY

Log	H Ratio	CumV%	Log%	Cull %
1	0.20	39.6	39.6	0
2	0.40	68.3	28.7	0
3	0.60	87.7	19.4	15
4	0.80	97.8	10.1	0
Totals:			97.8	15

Therefore, sound volume is 97.8 – 15 = 82.8% of total tree volume.

Sound volume = 0.828 x 199.9 = 165.5 cubic feet.

Summary

1. Four principal things affect taper of a tree stem: relative rates of DBH and Height growth, crown length, genetics, and treatments
2. There are three functional ways that tree stem volumes may be apportioned into different parts / segments / sections: Volume Distribution Tables, Volume Ratio Systems, and Stem Taper Equations / Systems
3. There are two main types of volume of interest in a tree: *total* and *sound*
4. Sound volume is derived from total by subtracting out known defects, decay, and missing parts