6.3 Assessing Stands w/ Variable-area Plots (or Points)

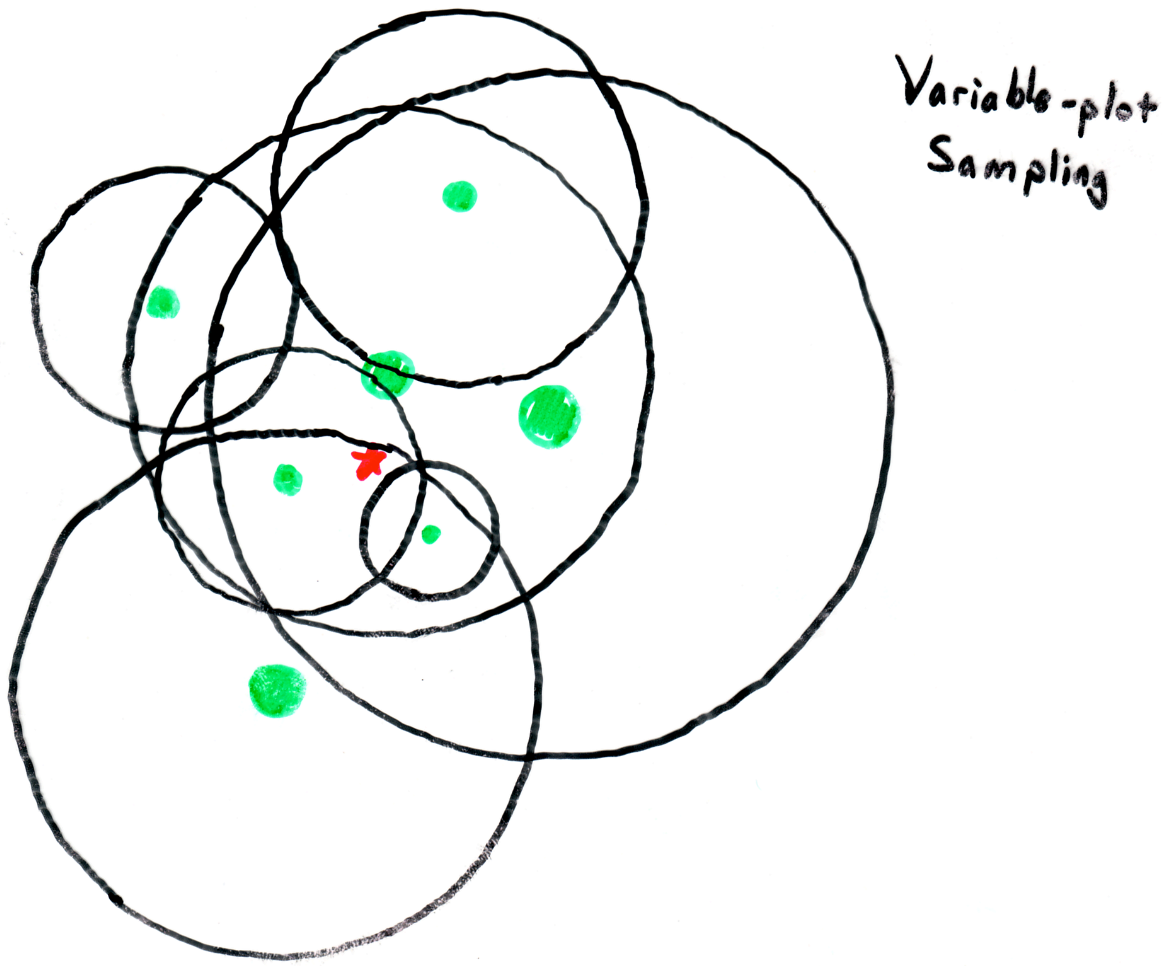
Use of variable-area plots as sample units (also known as variable-plot sampling, variable radius plot sampling, point sampling, plotless cruising, angle-count sampling, Bitterlich sampling, etc.) was developed by Prof. Dr. Walter Bitterlich in 1948

Lewis Grosenbaugh popularized the method in USA around 1952

Many features are similar to fixed-area plot sampling

- number and location of sample points is similar
- establish a plot center
- measure DBH and height the same way

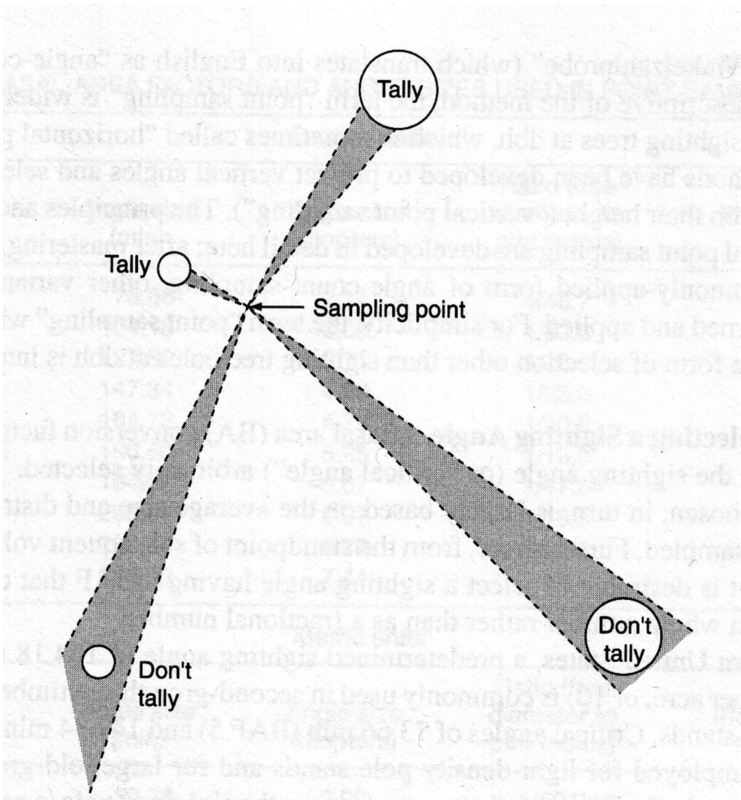
Differs in that a plot radius is not "flagged" because each tree has its own plot size:



Fortunately, the circles are conceptual (or "virtual") in the sense that we do not have to measure a plot radius for every tree

To determine which trees are IN the "plot," stand at the sample point using an instrument that projects a fixed horizontal angle to "sight in" a diameter for every tree at a fixed height, usually taken to be breast height – trees thicker than the angle are "IN"

"IN" trees are also called TALLY trees



Unique features of variable-area plots

- Stand Basal Area estimates are found by multiplying the number of tallied trees by the so-called *Basal Area Factor*, or *BAF*, which is directly tied to the size angle that is projected – no measurements needed !
- If any other estimates are desired, such as trees per acre, a stand table, or a stock table, etc., we have to measure DBH on the IN trees
- Tree Factor, TF is different for each individual tree and is given by

$$TF_i = \frac{BAF}{ba_i}$$

where TFi denotes the *Tree Factor* of i-th TALLY (IN) tree

BAF denotes the so-called *Basal Area Factor* in units of sq.ft./acre

ba_i denotes basal area of i-th TALLY or IN tree (in sq.ft., of course)

- Plot Radius, R (in feet), for any tree is given by

$$R_i = \left(\sqrt{\frac{75.625}{BAF}} \right) \cdot DBH_i$$

where 75.625 denotes a constant pertinent to American units

"R" also goes by "Horizontal Limiting Distance" (HLD)

Advantages of Variable-Area plots

- Not necessary to establish fixed-plot boundaries, leading to greater measurement speed
- Large, often high value, trees that make up the bulk of the overstory are sampled in greater proportions than smaller stems
- Basal area and volume per acre may be derived without direct measurement of stem diameter

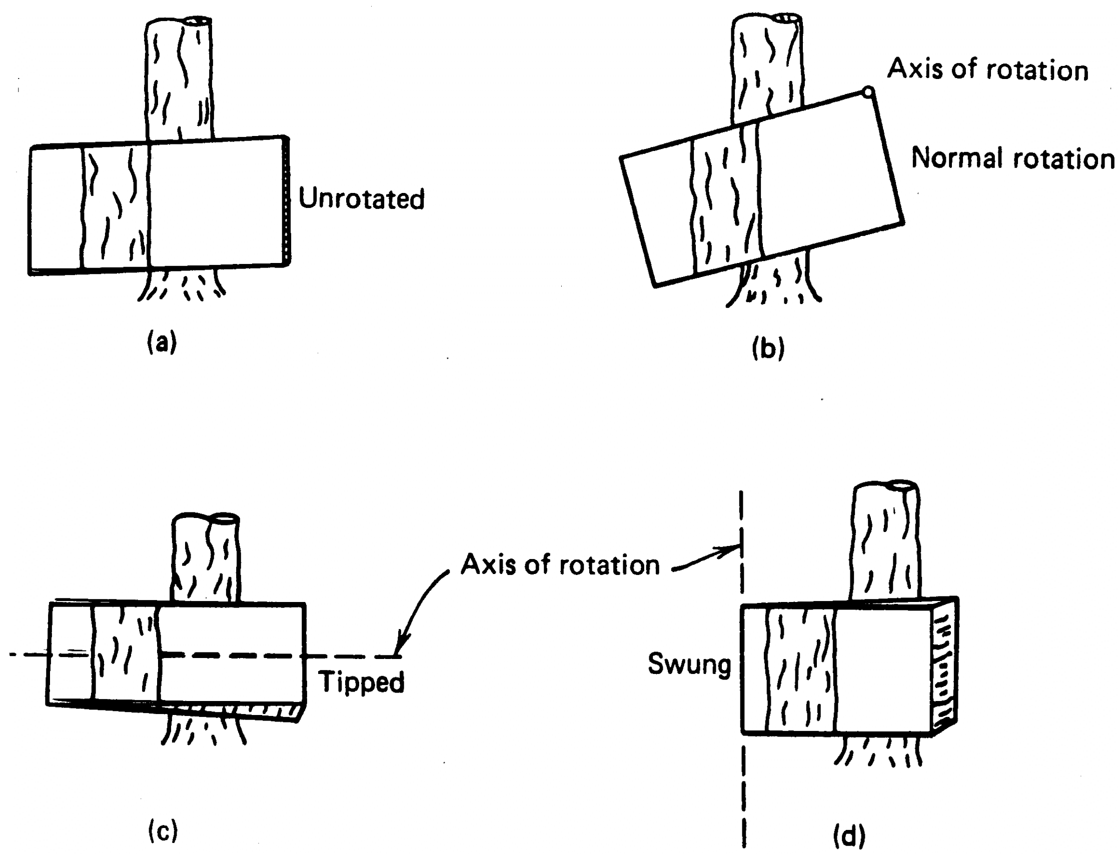
Disadvantages of Variable-Area plots

- Heavy underbrush reduces sighting visibility and measurement speed
- Small size of sampling unit (6 to 8 trees per point is most common) makes careful measurement & checking of borderline trees imperative – big relative errors result otherwise
- Slope compensation is important or large errors will result (this is same for fixed-area plots as well)

Instrumentation for projecting horizontal angles

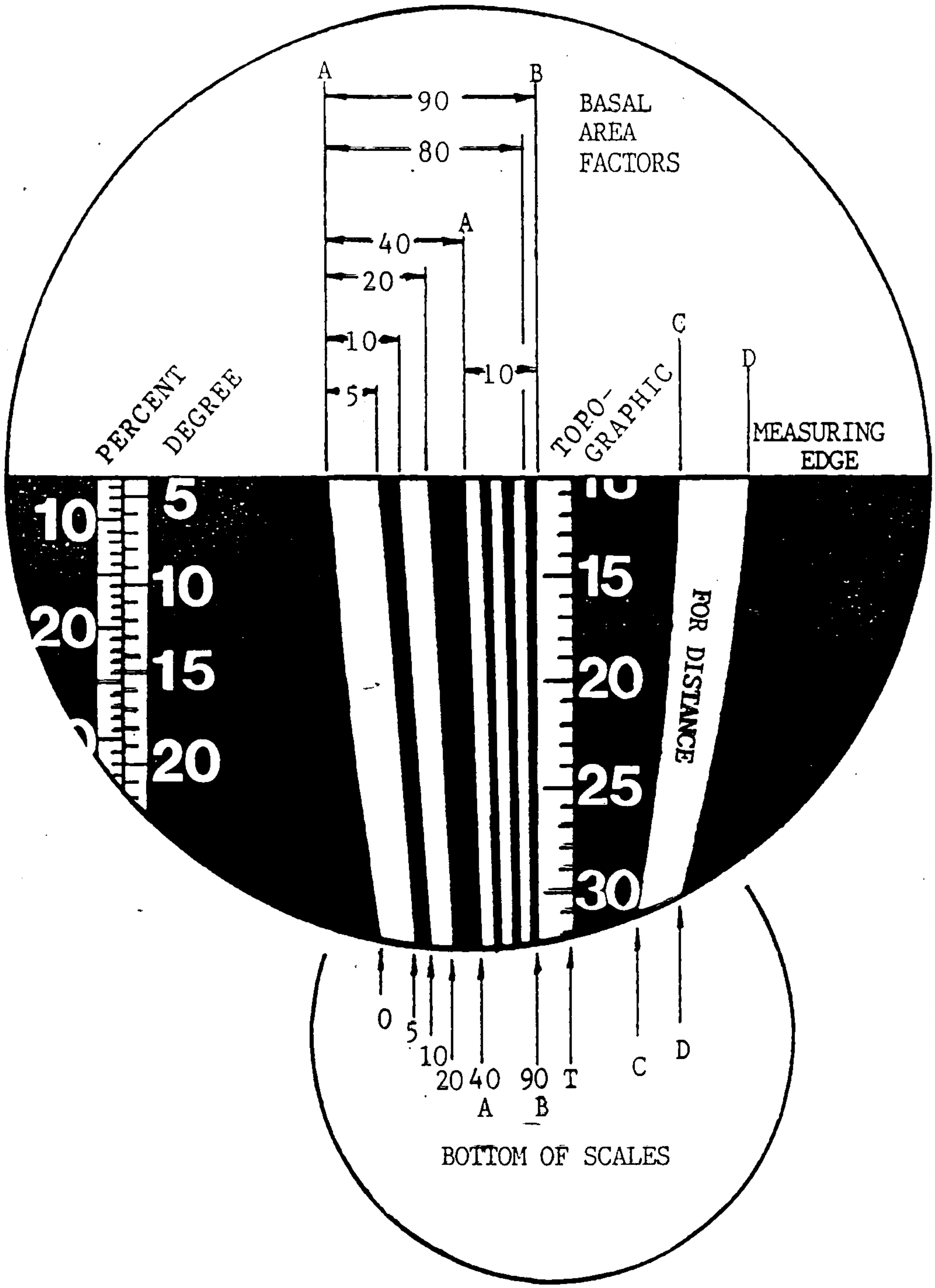
Wedge Prism

- hold <u>instrument</u> over sample point, sight on a tree, if image overlaps actual – tree is “IN” or is a “TALLY” tree
- slope adjustments are conducted manually



Relaskop

- hold your <u>eye</u> over sample point
- automatically adjusts for slope



Tree Zone (Plot) area Illustration & Derivation

Keep in mind the following:

- horizontal angle is fixed
- every tree size (diameter) determines its zone (plot) radius, i.e., the critical angle subtends diameters of different sizes as distance is changed
- radius of each trees' zone is NOT affected by spatial location
- at the "borderline" the ratio of tree radius to zone radius is constant

Example Illustration

$$DBH = 6\ in. \quad r = 3\ in. \quad ba = \pi\left(\frac{r}{12}\right)^2 = 0.19634\, ft^2$$

$$R = 16.5\, ft \quad A = \pi R^2 = 855.2986\, ft^2$$

$$TF = \frac{unit\ area}{sample\ area} = \frac{43560\, ft^2 / acre}{855.2986\, ft^2} = 50.9296 / acre$$

$$BA / acre = TF \times ba = 50.9296 / acre \times 0.19634\, ft^2 = 10\, ft^2 / acre$$

$$DBH = 12\ in. \quad r = 6\ in. \quad ba = \pi\left(\frac{r}{12}\right)^2 = 0.7854\, ft^2$$

$$R = 33\, ft \quad A = \pi R^2 = 3421.1944\, ft^2$$

$$TF = \frac{unit\ area}{sample\ area} = \frac{43560\, ft^2 / acre}{3421.1944\, ft^2} = 12.7324 / acre$$

$$BA / acre = TF \times ba = 12.7324 / acre \times 0.7854\, ft^2 = 10\, ft^2 / acre$$

Derivation

(In what follows, both r and R are expressed in feet – makes the math a bit easier!)

$$\frac{tree\ radius}{plot\ radius} = \frac{r}{R} = \sin\alpha \qquad \frac{tree\ basal\ area}{plot\ area} = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2} = \sin^2\alpha$$

Now, the ratio of tree basal area to plot area is the same when projected to a unit area basis,

$$\sin^2\alpha = \frac{tree\ basal\ area}{plot\ area} \times \frac{TF}{TF} = \frac{basal\ area / acre}{43560\, ft^2 / acre}$$

$$\frac{basal\ area/acre}{43560\,ft^2\ /\ acre} = \sin^2\alpha$$

$$basal\ area/acre\ =\ 43560\,ft^2/acre\left(\sin^2\alpha\right)$$

$$BAF\ =\ 43560\,ft^2/acre\left(\sin^2\alpha\right)$$

Now, let's say we wish to have each tree represent 10 sq. ft / acre

$$10\,ft^2/acre\ =\ 43560\,ft^2/acre\left(\sin^2\alpha\right)$$

$$\frac{10\,ft^2/ac}{43560\,ft^2/ac}\ =\ \sin^2\alpha$$

$$\sqrt{\frac{10\,ft^2/ac}{43560\,ft^2/ac}}\ =\ \sin\alpha$$

$$\sin^{-1}\left(\sqrt{\frac{10\,ft^2/ac}{43560\,ft^2/ac}}\right)\ =\ \alpha =\ 0.86815109^\circ$$

$$\phi\ =\ 2\alpha\ =\ 2\left(0.86815109^\circ\right)\ =\ 1.736^\circ$$

Derivation of Horizontal Limiting Distance formula

$$BAF\ =\ 43560\,ft^2/ac\left(\sin^2\alpha\right)$$

$$BAF\ =\ 43560\,ft^2/acre\left(\frac{r}{R}\right)^2$$

(Now back to the conventional units: r is in inches, R is in feet)

$$BAF\ =\ 43560\,ft^2/ac\left(\frac{DBH/2}{R}\bullet\frac{1\,ft}{12\,in.}\right)^2$$

$$BAF\ =\ \frac{43560\,ft^2/ac\bullet ft^2}{\left(12\bullet 2\right)^2}\frac{DBH^2}{R^2}$$

$$R^2 =\ \frac{43560\,ft^2}{\left(12\bullet 2\right)^2 BAF}DBH^2$$

$$R =\ \sqrt{\frac{75.625\,ft^2}{BAF}}DBH$$

Variable-area Point Summary

1/2

A particular forest was surveyed using a 10-factor angle-guage. An estimate of CV4 per acre with confidence interval and stand and stock tables are desired.
Tarif # for the stand is 35.5; all DF.

Point 1			Point 2			Point 3		
DBH	CV4	VBAR	DBH	CV4	VBAR	DBH	CV4	VBAR
16	51.0	36.53	14	38.2	35.73	18	65.4	37.01
12	27.2	34.63	12	27.2	34.63	10	17.9	32.82
14	38.2	35.73	16	51.0	36.53	12	27.2	34.63
18	65.4	37.01	10	17.9	32.82	14	38.2	35.73
10	17.9	32.82	18	65.4	37.01	16	51.0	36.53
			14	38.2	35.73	10	17.9	32.82
			12	27.2	34.63	16	51.0	36.53
ΣVBAR		176.72			247.08			246.07

Mean Tree Count (MTC) $= \frac{\Sigma(\text{"IN" trees})}{\text{no. points}} = \frac{19}{3} = 6.33\bar{3}$ trees/poin

Avg. VBAR $= \frac{\Sigma \text{ VBAR}}{\text{no. "in" trees}} = \frac{176.72 + \cdots + 246.07}{19} = 35.26$ ft^3/ft^2

Avg. Basal Area/acre $= \text{MTC(BAF)} = 6.33\bar{3}(10) = 63.\bar{3}$ ft^2/ac

Avg. Volume/acre $= (\text{Avg. VBAR})(\text{Avg. BA/ac})$

$= (35.26\ ft^3/ft^2)(63.\bar{3}\ ft^2/ac)$

$= 2233.r\ ft^3/ac$ to 4" top

Stand Table Calculation

Number of trees per acre in a D-class (TPA_D) is equal to the total BA in that class divided by ba of one such tree (ba_D), i.e.,

$$TPA_D = MTC_D\left(\frac{BAF}{ba_D}\right).$$

So, for example

Variable-area Point Summary (cont'd)

2/2

for the 18" class, there are three such trees on all three points ∴ $MTC_D = \frac{3}{3} = 1$.
The basal area of one 18" tree is 1.7671 ft²

Thus, $TPA_{18} = 1(10)/(1.7671) = 5.7$ trees/ac

Perform similar calculations for other classes to get:

DBH	TPA
10	24.4
12	17.0
14	12.5
16	9.5
18	5.7
Total	69.1

Stock Table calculations

Volume per acre in a D-class is found from the stand table by multiplying MTC_D by the average VBAR for that class ($VBAR_D$) and BAF

$VPA_D = MTC_D(BAF)(VBAR_D)$. So, for the 16" class there were 4 such trees on 3 points, ∴

$VPA_{16} = \left(\frac{4}{3}\right)(10) \cdot (36.53) = 487.1$ ft³/ac.

Other classes treated similarly

DBH	VPA
10	
12	462.4
14	477.4
16	487.1
18	372.8
Total	

Confidence Interval Construction

Let $Y = \sum \text{VBAR}$ per point. $n = 3$

$\sum y = 669.87, \quad \bar{y} = 223.29$

$\sum y^2 = 152828.93$

$$S_y^2 = \frac{152828.93 - (669.87)^2/3}{3-1} = 1626.83$$

$$S_y = \sqrt{S_y^2} = 40.33$$

Presuming very low sampling intensity, ignore f.p.c.

$$S_{\bar{y}} = \frac{S_y}{\sqrt{n}} = \frac{40.33}{\sqrt{3}} = 23.28$$

Assuming we desire a confidence coefficient of 80% we'll need $t_{.20,2} = 1.8856$

80% CI: $223.29 \pm 1.8856(23.28)$

$\Rightarrow (179.38, 267.20)$ $\sum$VBAR per acre.

VOLUME: BAF × $\sum$VBAR

$10(179.38, 267.20) \Rightarrow (1793.8, 2672.0)$

Unless we were unlucky, true CV4 per acre lies within these limits.