Components of Forest Growth[1]

Thomas W. Beers

FOREST GROWTH undoubtedly represents the heart of sustained yield management. The increased use of Continuous Forest Inventory (C.F.I.) and intensified growth evaluation brought about by the application of punched-card data processing, demands that the components of growth be thoroughly understood and properly manipulated in summarization. The components of forest growth are commonly designated as survivor growth (growth on trees present at both terminals of the growth period), mortality, cut, and ingrowth. The manner in which these are handled in summarization to obtain such expressions as net growth, gross growth, and net increase is not as routine as might first appear.

The variant technique of growth summarization employed in most C.F.I. analyses (wherein the basic growth figures are obtained at the tree level) does not warrant the introduction of new terminology. Indeed, confusion can be avoided by the consistent and proper application of traditional terminology. The terminology used by H. Arthur Meyer[2] (also briefly presented in *Forestry Handbook*[3] and elsewhere) in early discussions of continuous forest inventory is more or less standard and deserves reiteration.

The purpose of this paper is to present again the terms described by Meyer and to point out some peculiarities of their calculation with reference to a C.F.I. remeasurement analysis, which employs a somewhat different technique of summarization.

Derivation and Description of Terms

Using the same example as Meyer, we can consider an even-aged stand which has been measured at two successive inventories, ten years apart (shown schematically in Fig. 1). If we "assume that during this period of 10 years the diameter growth of all trees was the same, namely two inches, and that *no trees died or were cut during the period* . . . the trees of any given diameter class must then have moved into the next higher class. The growth . . . is thus characterized by a displacement of the diameter distribution to the right."

Referring to Figure 1, the difference between the two inventory volumes[4] (ignoring for the time being the notations of ingrowth, mortality and cut) represents the mantle of wood laid down during the 10-year period on trees which

[4]Number of trees can be converted to volume by employing a local volume table (volume per tree by d.b.h. class).

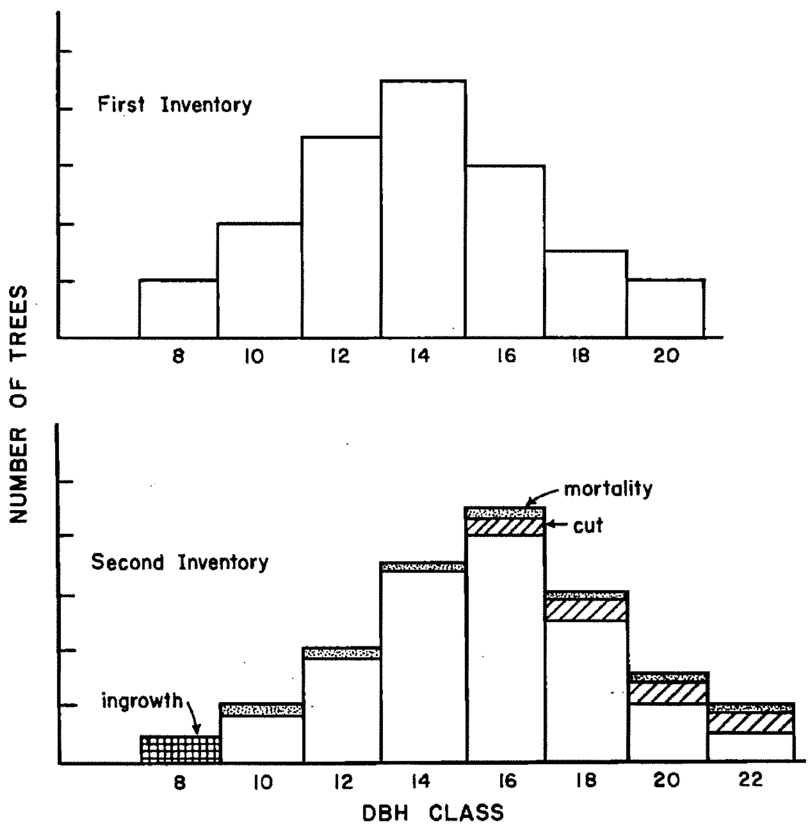

FIG. 1.—Schematic representation of the change in diameter distribution of an even-aged stand due to growth over a ten-year period.

THE AUTHOR is assistant professor of forestry, Department of Forestry and Conservation, Purdue University, Lafayette, Ind.

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[2]Meyer, H. Arthur. Forest mensuration. Penns Valley Publishers, Inc., State College, Pa. 357 pp. 1953.

[3]Forbes, R. D., and A. B. Meyer. Forestry handbook. The Ronald Press Company, New York. Section 1. 99 pp. 1955.

were present, were of merchantable size, and were measured at both inventories. In the absence of any cut or mortality this mantle of wood represents the gross growth of the volume present at the first inventory.

Realistically, however, we should have a certain amount of ingrowth, probably have some mortality, and may have some trees which were cut. Therefore, assuming a certain amount of ingrowth, mortality and cut, reference to Figure 1 should make it clear that in order to obtain the gross growth of the volume present at the first inventory, the second inventory must be augmented by the volume of trees which died or were cut, and reduced by the volume of ingrowth trees prior to the subtraction of the first inventory volume. That is, gross growth of initial volume

$$=(V_2+M+C-I)-V_1, \quad (1)$$

where $V_1 =$ the volume of trees measured at the first inventory,

$V_2 =$ the volume of trees measured at the second inventory,

$M =$ the initial[5] volume of trees dying during the period between inventories,[6]

$C =$ the initial[7] volume of trees which were cut during the period between inventories,

and $I =$ the volume of trees at the second inventory which were below merchantable size at the first inventory.

If we wish to include ingrowth in the gross figure, formula (1) becomes gross growth including ingrowth$=V_2+M+C-V_1$. (2) In common usage this is usually abbreviated to "gross growth"; the inclusion of ingrowth being implied.

The effect of mortality loss is considered in the net growth figure, obtained by subtracting mortality from the gross growth. When such is done to formula (2) we have net growth including ingrowth$=V_2+C-V_1$. (3) In common usage this is usually abbreviated to "net growth"; the inclusion of ingrowth being implied.

If the net growth of initial volume is desired the mortality is subtracted from growth of initial volume given by formula (1), obtaining net growth of initial volume $=V_2+C-I-V_1$. (4)

An expression of the actual change in growing stock is obtained if in addition to the mortality subtraction, the volume of cut trees is subtracted from formula (2). Thus is obtained the net increase in growing stock; net increase$=V_2-V_1$. (5) Obviously, if the amount of cut and mortality exceeds the gross growth including ingrowth (formula 2) then a negative net increase is possible, amounting to a net loss in growing stock.

As a general guide, Meyer points out the following relationships of gross growth, net growth, mortality, cut, and net increase: gross growth = net growth + mortality; net growth = net increase + cut; net increase = V_2-V_1 = the net change between inventories.

The status of the ingrowth component must be stated or clearly understood in the specific application of these terms. By definition net increase must include ingrowth. Although it is common practice to consider the ingrowth included in the terms gross growth and net growth, the use of the more definitive terms gross growth including ingrowth and net growth including ingrowth has the advantage of complete clarity. If it is desired to exclude the ingrowth component, the terms gross growth of initial volume and net growth of initial volume should be used.

All confusion in this matter of ingrowth can be avoided by the use of the complete terminology; however, it is frequently convenient to use the terms gross growth and net growth and imply that ingrowth is included. In order to do this the implication must be understood by the complete audience.

Application to Modern C. F. I.

The relationships described in the foregoing paragraphs, especially formulas (1) through (5), have applicability when dealing with groups of volume data; that is, when tree volumes at each terminal of the growth period are totaled with no attempt made to pair successive volumes of each individual tree. For instance, consider that on a given area (plot, stand, forest, etc.) the following totals have been obtained: volume at the first and second inventories, ingrowth volume, mortality volume,[8] and cut volume.[9] Figures for gross growth, net growth, etc., can be obtained for plot, stand, or forest without ever computing the volume growth per tree.

If, on the other hand, we begin at the tree level to obtain volume growth figures (the modern C.F.I. approach) in order to calculate the various summary growth expressions, the same formulas as cited above do not strictly apply. In this approach, successive tree volumes are paired to determine the growth contribution of each tree, which is then considered as a separate entity for subsequent growth summaries. By way of explanation, it is desirable to segment each inventory volume total into its individual tree components. That is, volume at the first inventory,

$$V_1 = V_{s_1} + M + C,$$

where $V_{s_1} =$ the initial volume of trees measured at both inventories, i.e., survivor trees,

$M =$ the initial volume of trees which were measured at the first inventory but died before the second inventory,

[5]In the traditional European continuous inventory approach, the *final* volume of mortality trees is used; however, in the usual modern C.F.I. analysis mortality is defined in terms of the initial volume.

[6]In addition, trees which become cull between the inventories are sometimes considered as mortality.

[7]In the European approach the volume *at the time of cutting* is used. In some modern C.F.I. analyses, cut is defined in this way; but many analyses still employ the initial volume of the cut trees in the definition.

[8]Subject to the same variation in meaning pointed out in footnote 5.

[9]Subject to the same variation in meaning pointed out in footnote 7.

and C = the initial volume of trees which were measured at the first inventory but were cut before the second inventory;

volume at the second inventory, $V_2 = V_{s_2} + I$,

where V_{s_2} = the final volume of trees measured both times, i.e., survivor trees,

and I = the final volume of trees which became measurable size by the second inventory.

The growth on trees which were measured at both inventories (survivor trees) is called survivor growth. In terms of the above symbols, survivor growth, designated by G_s, equals $V_{s_2} - V_{s_1}$.

It now remains to show the calculation of the items described by formulas (1) through (5), but this time in terms of individual tree growth data.

By definition it is clear that gross growth of initial volume = G_s, but also note that it can be obtained from formula (1), that is gross growth of initial volume

$$= V_2 + M + C - I - V_1$$
$$= (V_{s_2} + I) + M + C - I - (V_{s_1} + M + C)$$
$$= V_{s_2} - V_{s_1}$$
$$= G_s = \text{survivor growth.}$$

Proceeding in this way the following formulas can be derived:

gross growth (or gross growth ii[10]) $= G_s + I$

net growth (or net growth ii) $= G_s + I - M$

net growth of initial volume $= G_s - M$

net increase $= G_s + I - M - C$

The differences of procedure whether using volume totals or individual tree growth data are apparent in Table 1.

In order to emphasize the differences, take as an example the calculation of net growth on a plot having all types of trees represented: i.e., survivor, mortality, cut, and ingrowth. Typical data are shown in Table 2.

If the volume totals are first calculated, the net growth of the plot is obtained by adding the volume of cut trees to the second inventory volume and subtracting the first inventory volume (749.3+241.4−744.4=246.3 bd. ft.); neither the volume of ingrowth nor mortality enters into the calculation.

If, however, the growth per tree is first calculated, then the net growth of the plot is obtained by adding the growth of each survivor tree (tree numbers 4, 5, 6, 9, 10), adding the volume of each ingrowth tree (tree number 7), and subtracting the volume of each mortality tree (tree number 1); cut trees in this case are ignored and do not enter into the calculation, although they usually are retained as a separate total. Calculating the periodic net plot growth (246.3 bd. ft.) by adding (algebraically) the entries in the last column of Table 2, typifies the modern C.F.I. technique. Note that the cut trees (numbered 2, 3, and 8) are not entered in this column, indicating that they were "wired out" and kept from entering the total.

The calculation of the other expressions of growth defined by formula in Table 1 is shown in Table 3. It is clear that the same results can be obtained by the consistent use of either approach (using volume totals or using tree growth

TABLE 1.—FORMULAS FOR THE CALCULATION OF GROWTH

Type of growth	Formula if using: Volume totals	Formula if using: Individual tree growth figures
1. Gross growth of initial volume	$=V_2+M+C-I-V_1$	$=V_{s_2}+I+M+C-I-V_{s_1}-M-C$ $=V_{s_2}-V_{s_1}$ $=G_s$
2. Gross growth (or gross growth ii)	$=V_2+M+C-V_1$	$=V_{s_2}+I+M+C-V_{s_1}-M-C$ $=G_s+I$
3. Net growth (or net growth ii)	$=V_2+C-V_1$	$=V_{s_2}+I+C-V_{s_1}-M-C$ $=G_s+I-M$
4. Net growth of initial volume	$=V_2+C-I-V_1$	$=V_{s_2}+I+C-I-V_{s_1}-M-C$ $=G_s-M$
5. Net increase	$=V_2-V_1$	$=V_{s_2}+I-V_{s_1}-M-C$ $=G_s+I-M-C$

TABLE 2.—AN EXAMPLE OF GROWTH SUMMARIZATION. DATA FROM ONE PERMANENT SAMPLE PLOT (1/5 ACRE). LENGTH OF GROWTH PERIOD: TEN YEARS

Tree number	Tree status[1]	Sound volume: First inventory	Sound volume: Second inventory	Sound volume: Survivor	Sound periodic growth: Mortality	Sound periodic growth: Cut	Sound periodic growth: Ingrowth	Sound periodic growth: Net[2]
		Board feet						
1	20	62.1	----	----	−62.1	----	----	−62.1
2	24	81.3	----	----	----	−81.3	----	----
3	24	66.8	----	----	----	−66.8	----	----
4	22	42.4	62.3	19.9	----	----	----	19.9
5	22	63.3	122.5	59.2	----	----	----	59.2
6	22	106.0	163.8	57.8	----	----	----	57.8
7	12	----	34.6	----	----	----	34.6	34.6
8	24	93.3	----	----	----	−93.3	----	----
9	22	82.0	119.8	37.8	----	----	----	37.8
10	22	147.2	246.3	99.1	----	----	----	99.1
Plot totals		744.4	749.3	273.8	−62.1	−241.4	34.6	246.3
Symbol		V_1	V_2	G_s	M	C	I	

[1]Tree status as used here defines the class of tree from a growth-contribution standpoint. Status at each inventory is coded as follows: 0 = not present, 1 = pulpwood size, 2 = sawlog size, 3 = cull, 4 = cut. By combining the tree classes at successive inventories, then, 20 = sawlog mortality, 24 = sawlog cut, 22 = sawlog survivor tree, 12 = sawlog ingrowth from pulpwood size, etc. For an elaboration of this system refer to Stott, C. B. Forest control by continuous inventory. No. 75, U. S. Forest Service Region 9. 1960.

[2]Net growth including ingrowth.

[10]The abbreviation ii is used hereafter in this paper to mean including ingrowth.

TABLE 3.—SOLUTION OF FORMULAS SHOWN IN TABLE 1, USING DATA FROM TABLE 2

Type of growth	Solution if using	
	Volume totals	Individual tree growth figures
	Bd. ft.	*Bd. ft.*
1. Gross growth of initial volume	749.3+62.1+241.4 −34.6−744.4=273.8	273.8
2. Gross growth (or gross growth ii)	749.3+62.1+241.4 −744.4=308.4	273.8+34.6=308.4
3. Net growth (or net growth ii)	749.3+241.4−744.4 =246.3	273.8+34.6−62.1=246.3
4. Net growth of initial volume	749.3+241.4−34.6 −744.4=211.7	273.8−62.1=211.7
5. Net increase	749.3−744.4=4.9	273.8+34.6−62.1−241.4=4.9

figures). Applying the volume totals approach, where possible, to plot and higher order totals provides an excellent check on the arithmetic of the calculations.

Some Complications

Throughout the entire discussion of modern C.F.I. to this point, a certain amount of growth has been overlooked or ignored; the growth put on by cut and mortality trees prior to their removal from the growing stock. This practice tends to yield a conservative estimate of growth. It is easy to rationalize ignoring the growth of trees which subsequently die before the second measurement but the growth on cut trees is another matter. Lacking any intermediate measurements the amount of such growth must be grossly approximated; therefore, we may be justified in ignoring the inclusion of this growth and be satisfied that the resultant summary growth figure will tend to be on the conservative side. On the other hand, especially if the volume of trees cut is great, the total growth put on by these trees prior to felling becomes a component worthy of consideration. Many methods of approximating this growth component have undoubtedly been employed but few have been publicized. An exception is that mentioned by Berklund.[11]

It is on this very point (the inclusion of growth on cut trees) that much of the terminology presented by Gilbert [12] differs from that presented herein. Gilbert's "accretion" represents the growth on the initial trees and is further defined as including growth on trees that were cut during the measurement period, and as excluding growth on trees that died during the period. Perhaps the use of the term "accretion" should be reserved to describe growth when including growth on cut trees and excluding growth on mortality trees. On the other hand, it might seem less confusing to state whether given growth figures include or exclude these growth contributors and use the basic, definitive terminology as described in the present paper. If past practices are followed, the usual C.F.I. analysis will continue to ignore the growth on cut and mortality trees; however, this particular point is receiving more and more attention and is quite subject to refinement.

Another complexity in growth terminology warrants clarification. That is the dual use of the terms net and gross when referring to both stand growth and to tree growth (or volume). "Gross" tree growth (or volume) has been construed to mean the growth (or volume) of the tree irrespective of any defect which might be present. The application of a "soundness" factor to the "gross" tree growth (or to the successive periodic volumes involved in the growth calculation) leads to the so-called "net" tree growth. All the terms described earlier in this paper having to do with plot or stand growth can be applied to either "gross" or "net" tree growth. The confusion which can be caused by the ambiguous use of these terms should be obvious. Consider the possibilities; "gross" net growth, "net" gross growth, "gross" gross growth and "net" net growth.

Meyer has previously suggested a solution to the ambiguity by referring to "gross" tree volume as total volume (i.e., before a defect deduction) and using sound volume in place of "net" tree volume (i.e., after a defect deduction). Employing this terminology then, we can have gross and net plot, stand or forest growth in terms of either total or sound tree growth. Restricting the use of total and sound to the consideration of tree defect (a reduction applied to individual trees) and restricting the use of gross and net to the consideration of tree mortality (a reduction applied when making growth statements about groups of trees) should lessen or eliminate the confusion. Such terms as total or sound gross growth and total or sound net growth have definite meaning.

Summary

Recent intensive investigations of forest growth have emphasized the need for a thorough understanding of the components of forest growth and their manipulation. Such understanding can be facilitated by the consistent use of rather standard terminology. This terminology is re-presented and thoroughly described.

The application of modern Continuous Forest Inventory, by its very nature, to individual tree data precipitates some interesting variations in growth calculations. A discussion of the problem including calculation formulas and an example are presented to justify the computation of the various plot, stand, or forest growth employing the growth figures of individual trees.

Several further complications regarding growth terminology are discussed and suggestions for their clarification are presented.

[11]Berklund, B. L. Handling tree status. Proceedings Forest Management Control Conference. Purdue University. pp. 43-49. 1960.

[12]Gilbert, A. M. What is this thing called growth? Northeastern Forest Experiment Station Paper No. 71. 1954.