From: Husch, B.; T.W. Beers, and J.A. Kershaw, Jr. 2003, 4th Ed. Forest Mensuration. John Wiley and Sons, Inc., New York. 443 p.

11

SAMPLING UNITS FOR ESTIMATING PARAMETERS

To obtain information regarding parameters of a stand or forest area, the most common practice is to use sample plots. The sample plot is the unit for recording information and measurements. In most cases, this corresponds to the basic sampling unit. In cluster sampling, the basic sampling plot is subdivided into several subplots.

For many years, the most common sample plot has been a unit of fixed area. This results in a probabilistic procedure for selecting trees around a sample point; the probability of selection is constant and equal for all individuals that make up the population of the fixed-area plot. It is also possible to use a sampling procedure in which the probability of selecting a tree is variable and depends on some dimension of the tree. This is the case for sampling with *probability proportional to size*, commonly known as *PPS sampling*. Distance methods, which typically measure only the n closest trees, are another example of sampling procedures where the probability of selecting trees varies.

The choice of using plots of fixed or variable size depends on the practicality of their use in addition to theoretical considerations. In general, it is more efficient to use a plot where trees are selected with a probability proportional to the variable of interest. For example, if one is interested in the estimation of volume of a stand, it will be more efficient to use PPS sampling since the selection of sample trees is proportional to their basal area, which is closely related to volume. On the other hand, if one is interested in determining the number of trees in a stand, it will be more efficient to use fixed-area plots.

11-1. THE FACTOR CONCEPT

In most analyses of forest inventory data, measurements are summarized and expressed on a per unit area basis (per acre for the English system and per hectare for the metric system). Sample measurements are scaled to per unit area measurements using a ratio of unit area to associated sample area:

$$\mathrm{TF}_i = \frac{\text{unit area}}{\text{sample area}_i} \tag{11-1}$$

where TF_i = expansion factor of ith sample tree
unit area $= 43{,}560\,\mathrm{ft}^2$ (1 acre) or $10{,}000\,\mathrm{m}^2$ (1 ha)
sample area$_i$ = size of sampling unit (ft^2 or m^2) associated with ith tree

Each tree selected for measurement represents TF trees per unit area. For example, if 0.2-acre fixed-area plots are used, each tree sampled will represent $(1/0.2) = 5$ trees per acre as, illustrated in Fig. 11-1.

The expansion factor shown in eq. (11-1) is often referred to as the *tree factor* since it gives the number of trees per unit area that each sample tree represents. Depending on the type of sampling unit used, the tree factor may be constant or may vary by tree size or some other factor. The number of trees per unit area is obtained by summing the tree factors for each tree on the plot. If tree factor is constant for all sample trees, trees per unit area may be obtained by multiplying the constant tree factor by the number of trees per plot.

The expansion factors for other tree characteristics (XF_i), such as basal area or volume, are obtained by multiplying the value of the characteristic, X_i, by the tree factor:

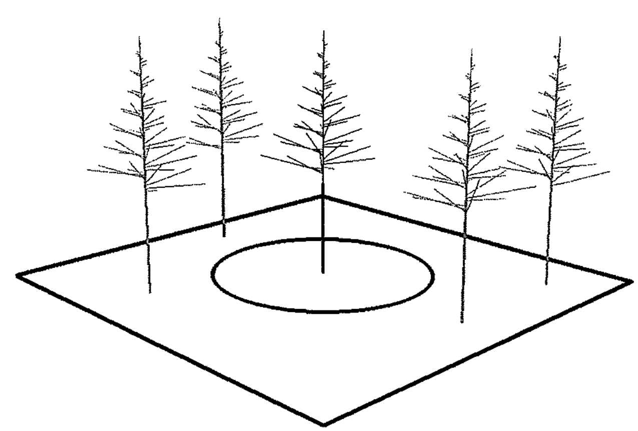


FIG. 11-1. The factor concept. Each tree on a 0.2-acre plot represents 5 trees per acre.

$$\mathrm{XF}_i = X_i \cdot \mathrm{TF}_i \tag{11-2}$$

For example, the basal area factor for an individual tree is

$$\mathrm{BAF}_i = \mathrm{BA}_i \cdot \mathrm{TF}_i = cD_i^2 \cdot \frac{\text{unit area}}{\text{sample area}}$$

where BAF_i = basal area factor of ith tree
BA_i = basal area of ith tree
TF_i = tree factor associated with ith tree [eq. (11-1)]
c = basal area conversion factor (0.005 454 in English system, 0.000 078 54 in metric)
D_i = dbh of ith tree

Per unit area estimates for a single plot are obtained by summing the factors:

$$\begin{aligned} X/\text{unit area} &= \mathrm{XF}_1 + \mathrm{XF}_2 + \mathrm{XF}_3 + \cdots + \mathrm{XF}_n \\ &= X_1(\mathrm{TF}_1) + X_2(\mathrm{TF}_2) + X_3(\mathrm{TF}_3) + \cdots + X_n(\mathrm{TF}_n) \\ &= \sum_{i=1}^{n} \mathrm{XF}_i = \sum_{i=1}^{n} X_i(\mathrm{TF}_i) \end{aligned} \tag{11-3}$$

For example, basal area per unit area is obtained by summing the BAF values for each sample tree. If TF is constant for each tree, basal area per unit area may be obtained by summing the basal area of each tree and multiplying that sum by the constant tree factor:

$$\text{BA per unit area} = \sum \mathrm{BAF}_i = \sum \mathrm{TF} \cdot \mathrm{BA}_i = \mathrm{TF} \sum \mathrm{BA}_i$$

Factors for other stand parameters are obtained in a similar manner.
The factors derived using this method lead to the following definitions:

- *Tree factor*: the number of trees per unit area represented by each tree tallied
- *Basal area factor*: the number of units of basal area per unit area represented by each tree tallied
- *Volume factor*: the number of units of volume per unit area represented by each tree tallied
- *Height factor*: the number of units of height per unit area represented by each tree tallied

The factor concept provides a simple and unified approach to forest inventory analysis. By focusing on the derivation of tree factor, all other factors are

simply obtained by multiplying the value of that parameter by the tree factor. In the following sections, various sampling units are described and the factors important to foresters derived for each type of sampling unit.

11-2. FIXED-AREA PLOTS

Fixed-area sampling units in a forest inventory or study are called *plots* or *strips*, depending on their dimensions. The term *plot* is loosely applied to sampling units of small areas of square, rectangular, circular, or triangular shape. The term *strip* is generally used to refer to a rectangular plot whose length is many times its width.

11-2.1 Circular Plots

Circular plots have been used widely since a single dimension, the radius, can be used to define the perimeter. The dimensions of commonly used circular sample plots are shown in Table 11-1. Circular plots have two advantages:

1. A circle has the minimum perimeter for a given area, which implies fewer decisions for trees near the plot boundary.
2. A circular plot has no predetermined orientation, which in some cases, especially in plantations, can be a cause of appreciable bias.

TABLE 11-1 Dimensions of Commonly Used Fixed-Area Plots

	English System			Metric System		
Plot Area as a Fraction of Per Unit Area	Plot Area (ft^2)	Circular Radius (ft)	Square Side (ft)	Plot Area (m^2)	Circular Radius (m)	Square Side (m)
1/1000	43.56	3.7	6.6	10	1.78	3.16
1/500	87.12	5.3	9.3	20	2.52	4.47
1/250	174.24	7.4	13.2	40	3.57	6.32
1/100	435.6	11.8	20.9	100	5.64	10.00
1/50	871.2	16.7	29.5	200	7.98	14.14
1/25	1,742.4	23.6	41.7	400	11.28	20.00
1/20	2,178	26.3	46.7	500	12.62	22.36
1/10	4,356	37.2	66.0	1,000	17.84	31.62
1/5	8,712	52.7	93.3	2,000	25.23	44.72
1/4	10,890	58.9	104.4	2,500	28.21	50.00
1/2	21,780	83.3	147.6	5,000	39.89	70.71
1	43,560	117.8	208.7	10,000	56.42	100.00

The main disadvantage of a circular plot is the error that may arise if the plot boundary is not observed carefully and if ocular estimations of limiting trees are not well done.

In general, small circular plots are more efficient than large ones. With large plots, the decision regarding trees near the plot boundary is often difficult. Consequently, it is advisable to use plots of a size that allows easier control of trees near plot limits. An experienced person standing at plot center if the radius of the plot is no greater than about 5 m can do this with confidence. Of course, one can determine more precisely if a tree near the perimeter is in the plot by measuring the distance from plot center to the tree using a tape or electronic distance measurement device (Chapter 4).

Circular plots on slopes can make decisions difficult regarding including a tree in a plot. To resolve this difficulty, a special technique described by Beers (1969) for horizontal point samples can be used to delimit fixed-area plots. With this technique one can establish a circular plot on a slope (which is elliptical in its horizontal projection) of an area equivalent to the preestablished size of the plot.

A circular plot of radius r on flat terrain has an area of $a = \pi r^2$. If the circular plot is located on terrain with a slope of α degrees, its horizontal projection will generate an elliptical plot with major radius $r_{\max}$ and minor radius $r_{\min}$. The area of the elliptical plot is less than the area of the circular plot in a horizontal plane.

Since the area of an ellipse is $a' = \pi r_{\max} r_{\min}$, one can calculate the area of the projected ellipse in a horizontal plane observing that the major radius is equivalent to the radius of the circle on the slope ($r_{\max} = r$), and the minor radius $r_{\min} = r \cos\alpha$. Thus the area of the ellipse projected in a horizontal plane a' is

$$a' = \pi r (r \cos\alpha) = \pi r^2 \cos\alpha$$

Since $a = \pi r^2$, then $a' = a \cos\alpha$.

Up to here we have shown the area of the horizontal projection of a plot on a slope. Since $\cos\alpha$ is always less than 1 (if $\alpha > 0$), the area of the horizontal projected ellipse is always less than the area of the circular plot on the slope. If we want the area of the circular plot on the slope to be always of the preestablished fixed area a, it will be necessary to adjust the radius of the circular plot on the slope. This radius r_c will vary depending on the slope. The constant area $a = \pi r_c^2 \cos\alpha$. The radius of a circular plot on a slope, which will give the constant area, is

$$r_c = \sqrt{\frac{a}{\pi \cos\alpha}} \qquad (11\text{-}4)$$

For example, if we are using circular plots of 0.05 ha (500 m^2), the radius of a circular plot on a slope of 20° to give a horizontal projected area of 500 m^2 would be

$$r_c = \sqrt{\frac{500}{\pi \cos 20^\circ}} = 13.01 \text{ m}$$

instead of the normal plot radius of 12.62 m.

11-2.2 Square and Rectangular Plots

The advantage of square plots is the somewhat greater ease of deciding if trees are in or out of the plot because plot boundaries are straight lines. The plot limits can easily be established by measuring diagonals at right angles from the plot center. Using a compass or right-angle prism to establish the direction of the diagonals, one measures and marks the distance from plot center to the corners of the square. The trees or plants are then measured systematically in the four triangles formed by the diagonals and periphery of the plot. Of course, the plot boundaries can be established by marking the corners using compass and tape directly. The dimensions for commonly used square plots are shown in Table 11-1.

In rectangular plots the width and length are not equal (the term *length* is generally applied to the longer dimension). Rectangular plots are usually established from the central axis of a given length. The width is measured from this axis and corners can be established. Rectangular plots are especially useful in natural forests with difficult topography and large altitudinal variation. In these situations the axis of the rectangular plot should be oriented to cross the maximum slope so that it may sample the maximum variability of the forest.

A strip is a type of rectangular plot whose length is many times its width. For recording purposes, the continuous strip may be subdivided into smaller recording units. It is important to remember, however, that the entire strip is still considered the ultimate sampling unit and is the basis for the number of degrees of freedom in subsequent statistical computations. [If interrupted strips are used (e.g., alternate lengths of a strip tallied), the individual units of the strip tallied should be treated like plots in subsequent calculations.]

An advantage of a continuous strip over plots is that a tally is taken for the entire strip traversed so that there is no unproductive walking time between sampling units. However, continuous strips have a disadvantage: The number of sampling units, and thus the degrees of freedom, is small. Comparing strip sampling to plot sampling of equal intensity, the size of the sampling unit is larger and the number of sampling units smaller. The larger sampling unit results in a reduction in the variability, but the smaller number of sampling units counteracts this advantage. For this reason, the sampling error of a strip sampling design is usually larger than for plot sampling, assuming the same sampling intensity.

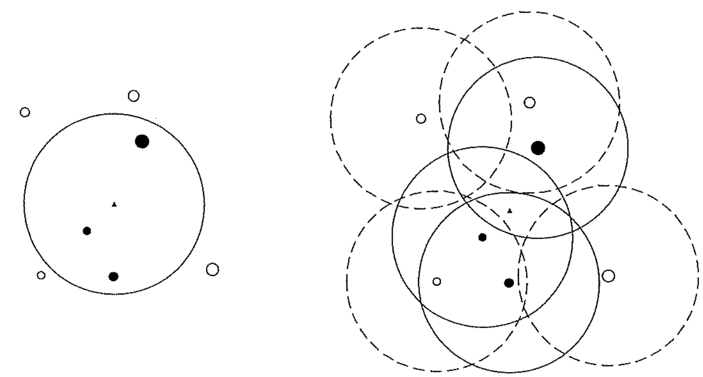
11-2.3 Subplots

With fixed-area plots such as circular plots, the different size classes of trees or plants (e.g., dbh or height) are sampled in proportion to their frequency in the population. Normally, in forest stands there are more small trees than large ones (this is typical in natural forests—not necessarily in plantations). Since each tree has the same probability of selection, there will be many small trees in the sample. This has the disadvantage that it will be necessary to measure many more small trees than large ones despite the fact that many times they will be of less importance, especially if volume is the parameter of interest. To remedy this situation, it is possible to modify the sampling plan so that more large trees and fewer small ones are measured in the sampling process. This modification consists of using different sizes of plots for different size classes or attributes of trees or vegetation. The common approach is to use a large plot for big trees and small plots for small trees and lesser vegetation. This can be accomplished by nesting or subdividing the large plot into subplots of different sizes. For example, one could establish a circular plot of 1000 m^2 (0.1 ha) with radius of 17.84 m, for measuring trees greater than 25 cm dbh. Within this plot, a concentric plot of 500 m^2 (0.05 ha) having a radius of 12.62 m can be established at the same center for trees with a dbh up to 25 cm. Meersschaut and Vandekerkhove (2000) used a plot design of four concentric circular sample plots with areas of 16, 64, 255, and 1018 m^2 to measure seedlings, shrubs, and trees (living and dead) of different size limits. They also established a 16 m × 16 m plot for measuring lesser vegetation and lying deadwood. Other systems of subplots can be established, such as small circular or square subplots established at some fixed design within a large circular plot or square plot. Subplots of different shapes and locations can be used with any plot shape.

11-2.4 Selection of Plots and Trees

To respect the laws of probability and obtain unbiased estimates of the parameters of a stand or forest, the sample plots should be selected at random from the total population of plots. All the trees or individual plants that are within the limits of the plot are measured to obtain information regarding the attributes of interest, such as dbh, height, or basal area.

With fixed-area plots, the tendency is to think of the tree selection process as the result of establishing a plot center and boundary, then measuring all the trees contained within that boundary, as illustrated for a circular plot in Fig. 11-2*a*. This view is often referred to as the *plot–centered approach* (Grosenbaugh, 1958; Beers and Miller, 1973; Oderwald, 1981a). It is also possible to view the selection process by visualizing an imaginary fixed-area



(*a*) (*b*)

FIG. 11-2. Selection of trees in fixed-area plot sampling: (*a*) plot-centered approach; (*b*) tree-centered approach.

plot around each tree (Fig. 11-2*b*). A plot center is established, and trees are included in the sample if this plot center is included in the area of their imaginary plot. This view is often referred to as the *tree–centered approach* (Grosenbaugh, 1958; Beers and Miller, 1973; Oderwald, 1981a).

The tree-centered approach aids in understanding the probability of a tree being selected for inclusion in a plot. A tree is included in a plot if its imaginary plot contains the established plot center (Fig. 11-2*b*). If random sampling is used, the probability of selection P_j is equal to the plot area divided by the area of the stand:

$$P_j = \frac{\text{plot area}}{\text{stand area}} \tag{11-5}$$

Because the plot area is the same for all trees, the probability of being selected is constant for all trees.

11-2.5 Stand and Stock Tables

Plot measures are scaled to per unit area measures using the expansion factors described in Section 11-1. Average per unit area values and total values are obtained using the methods described in Chapters 3 and 13. While estimates of the averages per unit area and total values for various stand parameters are important, foresters often desire more detailed summaries. Typically, foresters will summarize data into stand and stock tables. A *stand table* gives the number of trees by species and dbh class. A *stock table* gives volumes (or weights) by similar classifications. In many cases, basal area and average height are included. Occasionally, tables may be subdivided by height class as well as dbh class. A *combined stand and stock table* gives density and volume by species and dbh class. Values within stand and stock tables are normally expressed on a per unit area basis; however, total values for the stand may be given.

The construction of stand and stock tables from fixed-area plot data will be illustrated using the following example. Four 20 m × 20 m plots were measured and the following total tally obtained:

Dbh Class	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42+
No. Tallied	9	6	17	8	9	4	8	7	5	6	2	5	3	1	3	0	2

Table 11-2 shows the combined stand and stock table. Column 1 shows the dbh classes corresponding to the tally. The number of trees tallied are in column 5. Columns 2, 3, and 4 contain the average per-tree values for height, basal area, and volume. Total height was measured on all trees shown in the tally above, and average height (column 2) was calculated for each dbh class. The basal area per tree (column 3) is found by calculating the cross-sectional area of a circle corresponding to the diameter class:

$$\begin{aligned} BA_{tree} &= \pi\left(\frac{D}{2 \cdot 100}\right)^2 \\ &= \frac{\pi}{40{,}000}D^2 \\ &= 0.000\,078\,54D^2 \end{aligned}$$

Volume per tree (column 4) can be estimated using a variety of methods, including a local volume table, a standard volume, or a volume equation. For this example, trees were assumed to be paraboloids [eq. (6-2)] and volume per tree is obtained by multiplying form (0.5) by basal area per tree (column 3) by average height (column 2). For example, the volume per tree for the 10-cm dbh class is

$$\begin{aligned} Vol_{10} &= \tfrac{1}{2}(A_b H) \\ &= \tfrac{1}{2}(BA \cdot H) \\ &= \tfrac{1}{2}(0.0079)(11.0) \\ &= 0.043\ m^3 \end{aligned}$$

TABLE 11-2. Combined Stand and Stock Table for a Northern Hardwood Stand Located in Central New Brunswick[a]

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
	Per Tree Averages				Factors[b]			Per Hectare Averages		
Dbh Class (cm)	Height (m)	BA (m^2)	Volume (m^3)	Number of Trees Tallied	Tree (number)	BA (m^2)	Volume (m^3)	Trees (number)	BA (m^2)	Volume (m^3)
10	11.0	0.0079	0.043	9	25	0.1964	1.080	56.25	0.44	2.4
12	11.7	0.0113	0.066	6	25	0.2827	1.654	37.50	0.42	2.5
14	12.9	0.0154	0.099	17	25	0.3848	2.482	106.25	1.64	10.5
16	13.8	0.0201	0.139	8	25	0.5027	3.468	50.00	1.01	6.9
18	13.9	0.0254	0.177	9	25	0.6362	4.421	56.25	1.43	9.9
20	16.5	0.0314	0.259	4	25	0.7854	6.480	25.00	0.79	6.5
22	16.8	0.0380	0.319	8	25	0.9503	7.983	50.00	1.90	16.0
24	17.3	0.0452	0.391	7	25	1.1310	9.783	43.75	1.98	17.1
26	19.8	0.0531	0.526	5	25	1.3273	13.141	31.25	1.66	16.4
28	18.0	0.0616	0.554	6	25	1.5394	13.854	37.50	2.31	20.8
30	20.0	0.0707	0.707	2	25	1.7672	17.672	12.50	0.88	8.8
32	18.2	0.0804	0.732	5	25	2.0106	18.297	31.25	2.51	22.9
34	21.0	0.0908	0.953	3	25	2.2698	23.833	18.75	1.70	17.9
36	19.0	0.1018	0.967	1	25	2.5447	24.175	6.25	0.64	6.0
38	19.7	0.1134	1.117	3	25	2.8353	27.928	18.75	2.13	20.9
40	20.0	0.1257	1.257	0	25	3.1416	31.416	0.00	0.00	0.0
42	20.0	0.1385	1.385	2	25	3.4636	34.636	12.50	1.73	17.3
Total				95				593.75	23.17	203.0

[a]Based on four 20 m × 20 m fixed-area plots.
[b]Factors are the value per unit area represented by each tree tallied.

Tree factor (column 6) is constant for fixed-area plots and is obtained using eq. (11-1):

$$\begin{aligned} \mathrm{TF} &= \frac{\text{unit area}}{\text{plot area}} \\ &= \frac{10{,}000}{20(20)} \\ &= 25 \end{aligned}$$

The basal area factor (column 7) and the volume factor (column 8) are obtained by multiplying the tree factor (column 6) by the corresponding per tree value [eq. (11-2)]. For the 10-cm class, the basal area factor is

$$\begin{aligned} \mathrm{BAF}_{10} &= \mathrm{BA}_{10} \cdot \mathrm{TF}_{10} \\ &= (\text{column 3})(\text{column 6}) \\ &= 0.0079(25) \\ &= 0.1964 \end{aligned}$$

and the volume factor is

$$\begin{aligned} \mathrm{VF}_{10} &= \mathrm{Vol}_{10} \cdot \mathrm{TF}_{10} \\ &= (\text{column 4})(\text{column 6}) \\ &= 0.043(25) \\ &= 1.080 \end{aligned}$$

Average per unit area values for each parameter is obtained by multiplying the corresponding factor by the number of trees tallied and dividing by the number of plots. Tree per hectare (column 9) is then obtained by multiplying the tree factor (column 6) by the number of trees tallied (column 5) and dividing by the number of plots:

$$\text{trees/ha} = \frac{\mathrm{TF} \cdot (\text{no. tallied})}{\text{no. plots}}$$

For example, there were 9 trees tallied in the 10-cm class; therefore, the trees/ha for the 10-cm class becomes

$$\begin{aligned} \text{trees/ha} &= \frac{\mathrm{TF} \cdot (\text{no. tallied})}{\text{no. plots}} \\ \text{column 9} &= \frac{(\text{column 6})(\text{column 5})}{\text{no. plots}} \\ &= \frac{25(9)}{4} \\ &= 56.25 \end{aligned}$$

Similarly, basal area per hectare (column 10) is

$$\begin{aligned} \text{BA/ha} &= \frac{\mathrm{BAF} \cdot (\text{no. tallied})}{\text{no. plots}} \\ \text{column 10} &= \frac{(\text{column 7})(\text{column 5})}{\text{no. plots}} \\ &= \frac{0.1964(9)}{4} \\ &= 0.44 \end{aligned}$$

and volume per hectare (column 11) is

$$\begin{aligned} \text{volume/ha} &= \frac{\mathrm{VF} \cdot (\text{no. tallied})}{\text{no. plots}} \\ \text{column 11} &= \frac{(\text{column 8})(\text{column 5})}{\text{no. plots}} \\ &= \frac{1.080(9)}{4} \\ &= 2.4 \end{aligned}$$

11-2.6 Plots near Stand Borders

Single Fixed-Area Plots. When a forest area is bounded by nonforest or other land-use classes, the trees near the border will usually be somewhat different than those in the interior of the forest. If the perimeter proportion of a forest area is large, it will be necessary to take precautions that this zone is adequately represented in the sample. In addition, very frequently, sample plots are located near the border of a forest area so that a portion of the plot falls outside the limits of the forest or stand or portions of the plot fall in different forest conditions or types.

To assure adequate representation of the border areas and take into consideration the part of the plot that may be partially outside the forest or in