Abbreviated Information on m_map Projections

Two useful properties for map projections are (1) the ability the preserve angles for small regions, (*conformal)* and (2) the ability to preserve area (*equal-area)*. No projection can be both conformal and equal area. Many projections (especially global projections) are neither. Instead an attempt is made to aesthetically balance the errors in both conditions.

Azimuthal projections are those in which points on the globe are projected onto a flat tangent plane. Maps using these projections have the property that direction or azimuth from the center point to all other points is shown correctly. Great circle routes passing through the central point appear as straight lines (although great circles not passing through the central point may appear curved). These maps are usually drawn with circular boundaries.

1. **Stereographic:** The stereographic projection is conformal, but not equal-area. This projection is often used for polar regions.

2. **Gnomonic:** This projection is neither equal-area nor conformal, but all straight lines on the map (not just those through the center) are great circle routes. There is, however, a great degree of distortion at the edges of the map, so maximum radii should be kept fairly small - 20 or 30 degrees at most.

3. **Satellite:** This is a perspective view of the earth, as seen by a satellite at a specified altitude. This is the common projection for Google Earth.

Cylindrical Projections are formed by projecting points onto a plane wrapped around the globe, touching only along some great circle. These are very useful projections for showing regions of great lateral extent, and are also commonly used for global maps of mid-latitude regions only.

1. **Mercator:** This is a conformal map, based on a tangent cylinder wrapped around the equator. Straight lines on this projection are rhumb lines (i.e. the track followed by a course of constant bearing).

2. **Miller Cylindrical:** This projection is neither equal-area nor conformal, but "looks nice" for world maps. Properties are the same as for the Mercator, above.

3. **Oblique Mercator:** The oblique mercator arises when the great circle of tangency is arbitrary. This is a useful projection for, e.g., long coastlines or other awkwardly shaped or aligned regions. It is conformal but not equal area.

Notes on Mercator projections

In a Mercator Projection, the following formulae describe the relationship between position on the map (Cartesian X, Y points) and Longitude and Latitude:

λ = Latitude (+ North) θ = Longitude (+ East)

It is necessary for you work in radians: ($2\pi = 360°$). Cartesian coordinates map into latitude and longitude as:

$$X = \boldsymbol{c}\,(\theta - \theta_o) \qquad Y = \boldsymbol{c}\; ln\,\{\tan(\pi/4 + \lambda/2)\}$$

where X, Y are relative to θ_o (the prime meridian) and the equator. Y is a non-linear function of latitude and the constant c is the same for X and Y. These equations can be inverted to give latitude and longitude as a function of X and Y coordinates:

$$(\theta - \theta_o) = X/\boldsymbol{c} \qquad \lambda = 2 \tan^{-1}(exp(Y/c)) - \pi/2$$

Notes on Spherical Trigonometry

Several relationships are useful in working with angles and distances on a sphere. In the figure below a, b, and c are arc sections on a sphere that intersect to form a spherical equivalent to a triangle. These lengths are characterized by the angles between the center of the sphere and the associated corners. The physical length is then determined from the radius and the formula for the circumference of a sphere. The angles between the arcs on the sphere surface are labeled as A, B, and C such that the arc length is opposite the angle of the same letter.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$