Earth's Energy Budget and Heat Flow

Reviewed the collection of rocks
The green ones were "ultramafic"

- Peridotites
- high iron and magnesium relative to silica (SiO_2)
- from the mantle
- lower concentration of U, Th, K

The dark rocks were "mafic"

- Basalts and Gabbros
- more silica
- typical of oceanic crust
- chemically fractionated from peridotites
- more "incompatibles" like U, Th, K

The lighter "salt and pepper" rocks were higher still in silica

- more representative of continental crust
- more chemically fractionated than mafics
- granite and andesite for large crystal sizes
- rhyolite dacite for smaller crystal size (extrusive volcanic)
- even greater concentrations of "incompatibles"

Noted the relationship associated with partial melting and chemical fractionation – both for major elements and minor (trace) radiogenic elements.

Geothermal Flux:

42×10^{12} W (80 mW/m^2)

compare to:

1. Commercial power plant ($O(10^9)$ W)
2. Sum of all earthquakes (10^{11} W)
3. Solar irradiance (2×10^{17} W - 400 W/m^2)

Sources:

Heat production from radiogenic elements U^{235} U^{238} Th^{232} K^{40}
Secular cooling (difference between heat production and heat loss)

Heat Production (in units of 10^{-10} W/kg)

	Peridotite	"Basalt"	"Granite"	
U	.03	.1-.8	4	
Th	.03	.1-.7	4	
K.	.01	.4	1.3	
totals (mass)	.07	.3 – 2	9	$\times 10^{-10}$ W/kg
(volume)	.03	.1-.5	2.5	$\mu W/m^3$

Mass of mantle 4×10^{24} kg -> 28×10^{12} W (28 TW)
Mass of crust 2.8×10^{21} kg -> 5.5×10^{12} W (13% of total from 0.5% of mass)

Sum is about 80% of total -> 20% secular cooling?

Significant assumptions in this analysis – one finds a range of estimates for this

Heat Transport

Conduction
Convection
Advection
Radiative

Conduction: flux proportional to temperature gradient

$$Q = -k\frac{dT}{dz}$$

Metals –	10^2	W/m/°C	
Rocks –	1 to 10	W/m/°C	(2-4 most common)

For average values : $\frac{dT}{dz}$ is 30 – 40 °C/km

(note absurd results for even modest extrapolation - 4000 °C in 100 km)

conduction leads to diffusion equation: $\frac{dT}{dt} = \frac{k}{\rho C_p}\frac{d^2T}{dz^2}$

Diffusivity => $\kappa = \frac{k}{\rho C_p}$

Note units: m^2/s and typical value $O(10^{-6}\ m^2/s)$

Although solving the diffusion equation can be mathematically challenging, one finds that any solution will have a relationship between time and length scales of the form:

$$\tau = l^2/\kappa$$

Using a nominal value for κ, one can calculate the following time required for a temperature change to "propagate" a characteristic length:

time	**length**
hours	10 cm
11 days	1 m
30,000 years	1 km
300 Ma	100 km
30 Ga	1000 km

Cooling of the Oceanic Lithosphere by conduction:

One important case where a solution of the diffusion equation for time dependent conductive cooling is described as the **instantaneous cooling of a half-space**:

Initial condition and boundary conditions: $T = T_m$ for z>0, $T = T_s$ at z=0, T(z=infinity)=Tm for all time. The solution is given in terms of the "error function" which is a function coded into many mathematics packages.

$$T = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

note graph of erf(x)

Cooling of oceanic lithosphere:

Geotherms, topography, heat flow, hydrothermal circulation

t = distance/plate velocity

heat flow is vertical

$$T = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

$$Q = \frac{kT_m}{\sqrt{\pi\kappa t}}$$

$$W = const\sqrt{\kappa t}$$

Continental Geotherm

Linear relationship between surface heat production and flux is conventional idea

Note -> oceanic (80 mW/m^2) and continental (60 mW/m^2) heat flow
different contributions- radiogenics in relatively old
continental lithosphere vs cooling in younger oceanic lithosphere

Advection and the Adiabatic gradient:

Homogeneous compression causes self-heating

Gradient is 0.3 to 0.5 °C/km – O(1500°C) temperature difference in mantle

Earth Geotherm

The change of temperature with depth

- draw the curves
- understand what the slope of the curves implies
- note definition of lithosphere in terms of geotherm,
- note where partial melting occurs to produce mid ocean ridge basalts
- note age dependence of oceanic lithosphere geotherms
- note how continental geotherms differ