Gravity

"Nothing yet. ...How about you, Newton?"

Newton's Law of Gravity (1665)

$F = G\ (m_1 m_2) / r^2$

F = force of gravitational attraction

m_1 and m_2 = mass of 2 attracting objects

r = distance between the two objects

G -- ?

Earth dimensions:

$r_{earth} = 6.378139 \times 10^3$ km (equator)

$m_{earth} = 5.976 \times 10^{27}$ g

$F = G\ (m_1 m_2) / r^2)$

$F = m_{test}\ (G\ m_{Earth} / r_{Earth}{}^2)$

Newton's Second Law: $F = ma$

implies that the acceleration at the surface of the Earth is:

$(G\ m_{Earth} / r_{Earth}{}^2)$

$= \sim 981$ cm/sec^2 **= 981 "Gals"**

variations are on the order of (0.1 mgal)

Precision requires 2 spatially based corrections

must account for 2 facts about the Earth

it is ***not*** round (*.... it's flattened at the poles)*

it is ***not*** stationary (*.... it's spinning)*

Earth is an ellipsoid

first detected by Newton in 1687

clocks 2 min/day slower at equator than in England

concluded regional change in "g" controlling pendulum

translated this 2 min into different values of Earth radii

$\text{radius}_{equator} > \text{radius}_{pole}$

$r_{equator} / r_{pole} = 6378/6357$ km = flattening of ~1/298

two consequences

equator farther from center than poles

hence gravity is 6.6 Gals LESS at the equator

equator has more mass near it than do the poles

hence gravity is 4.8 Gals MORE at the equator

result

gravity is 1.8 Gals LESS at the equator

Earth is a rotating object

circumference at equator ~ 40,000 km; at poles ➔ 0 km
all parts of planet revolve about axis once in 24 hrs
hence equator spins at $\frac{40,000}{24}$ =1667 km/hr; at poles ➔ 0 km/hr
outward centrifugal force at equator; at poles ➔ 0

result .....

value of gravity is 3.4 Gals LESS at equator
gravity vector points away from <u>rotational axis</u>

Combine previous two considerations ...

correction as function of latitude to '**reference ellipsoid**' :

$$g_n = 978.03185\ (1 + A\sin^2\phi - B\sin^2 2\phi)$$

g_n = 'standard gravity'
ϕ = latitude
978.03185 = g measured at equator / sea level
A, B = fitted to observed data in 1967 agreement

$$g_n = 978.03185 * (1 + 0.005278895 * \sin^2\phi - 0.000023462 * \sin^4\phi)\ \ cm/s^2$$

g_n = what one expects to observe at latitude ϕ
$g_{observed} - g_n$ = gravity anomaly

Measuring Gravity

Gravimeters:

(1) pendulum
(2) vibrating fiber

(3) spring stretched by test mass
(4) falling mass in vacuum

Basic Stretched Spring Gravimeter

most common type

$kx = - mg$
amount of stretch is function of test mass
$g = - x (k/m)$
measure value of x
cannot determine k accurately enough, however

used for **RELATIVE** values of g, compare x from place to place

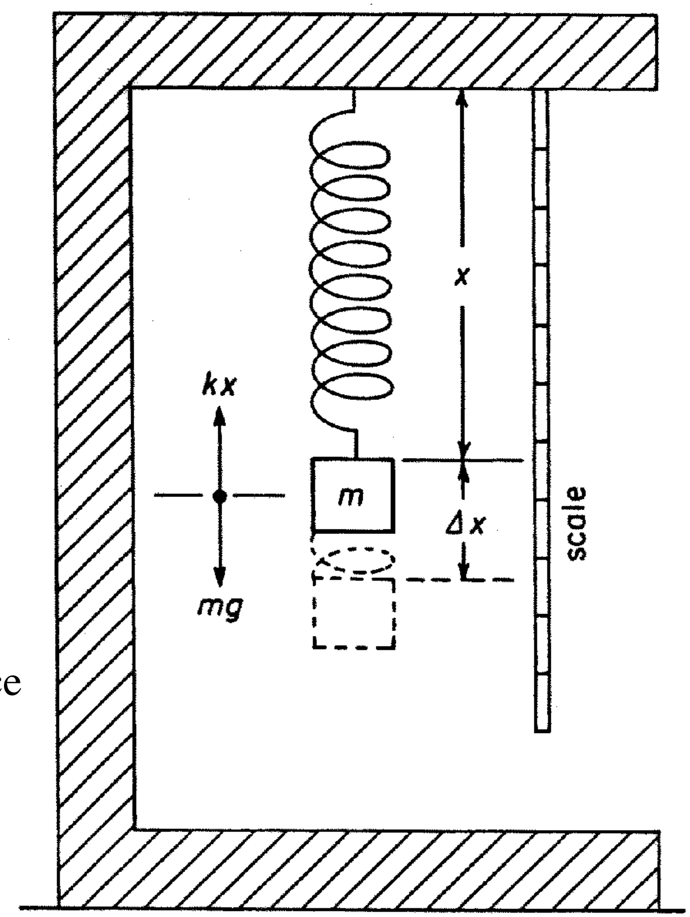


Figure 7–10
Mass supported by a vertical spring. A change in gravity will produce a change in the distension of the spring.

Gravimeter Drift

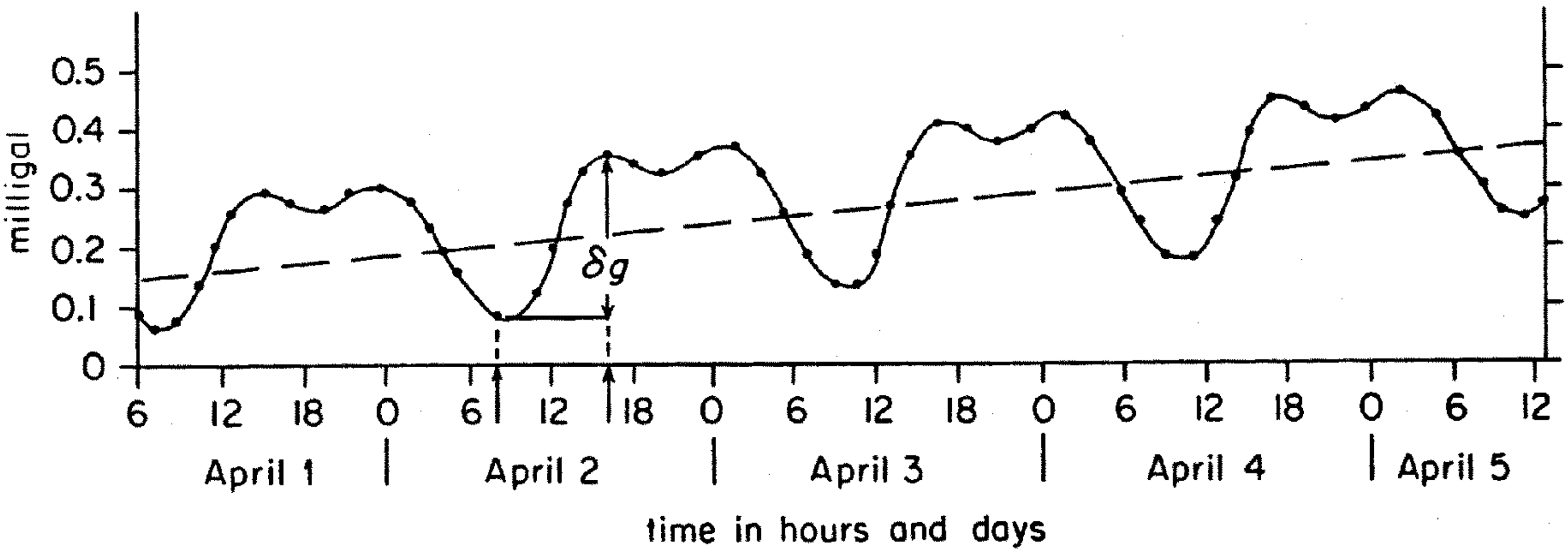


Figure 8–1
Graph showing how gravimeter readings (dots) change with time because of cyclic tidal variations and noncyclic instrument drift. On April 2, the change occurring between the times of 800 and 1600 hr is δg.

from Robinson + Coruh

Free-Fall Gravimeter

measure time for test mass to fall known distance in vacuum
only one that can be used for <u>ABSOLUTE</u> measurements

but is complex, lengthy to set-up

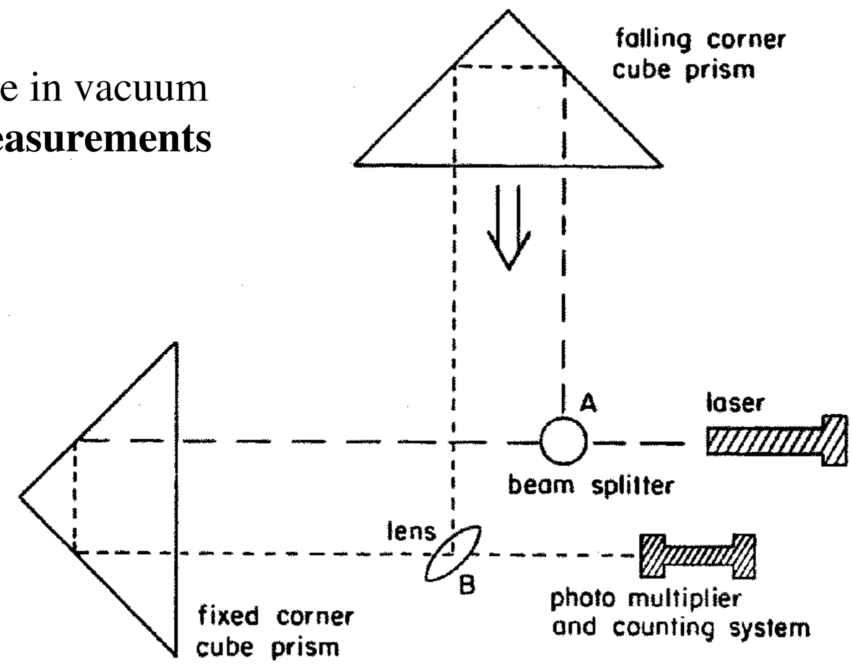


Figure 7–6
The free-falling-mass experiment for measuring the acceleration of gravity using corner cube prisms. The interference of light beams from the two prisms is used to measure the time required for the upper prism to fall a specific distance.

Temporal-based corrections

- instrument drift ~ 0.1 mgal/day due to
  - springs, bars, etc. stretch inelastically
  - temperature changes
- daily up+down earth tide ±0.2 mgal/12 hrs
  - sun+moon raise/lower Earth's surface by a few cm
  - = change in distance to center of earth

Spatial-based corrections

- latitude
- site elevation
- local topography

Gravity Corrections

Elevation, or Free-Air Correction

Recall that on the surface of spherical, non-rotating Earth:

$$g_E = Gm_E / r_E^2$$

$G = 6.6732 \times 10^{-11}\ Nm^2/kg^2$
$m_E = 5.976 \times 10^{24}\ kg$
avg $r_E = 6.370 \times 10^6\ m$

$$g_E = 982{,}801.6236 \text{ mgals}$$

Now increase r by 1 meter

$$g_{E+1} = (6.6732 \times 10^{-11}) \times (5.976 \times 10^{24}) / (6.370001 \times 10^6)^2$$
$$= 982{,}801.3150 \text{ mgals}$$

982,801.6236 mgals (at sea level)
\- 982,801.3150 mgals (1 m above sea level)
0.3086 mgals

hence, 1 m increase in elevation = 0.3086 mgal decrease in g_E

Relative to g at **reference ellipsoid**, each msmt is too low by 0.3086*h
To compare with 'standard gravity' g_n, must add 0.3086*h to msrd value

+ 0.3086*h ← **free-air correction**

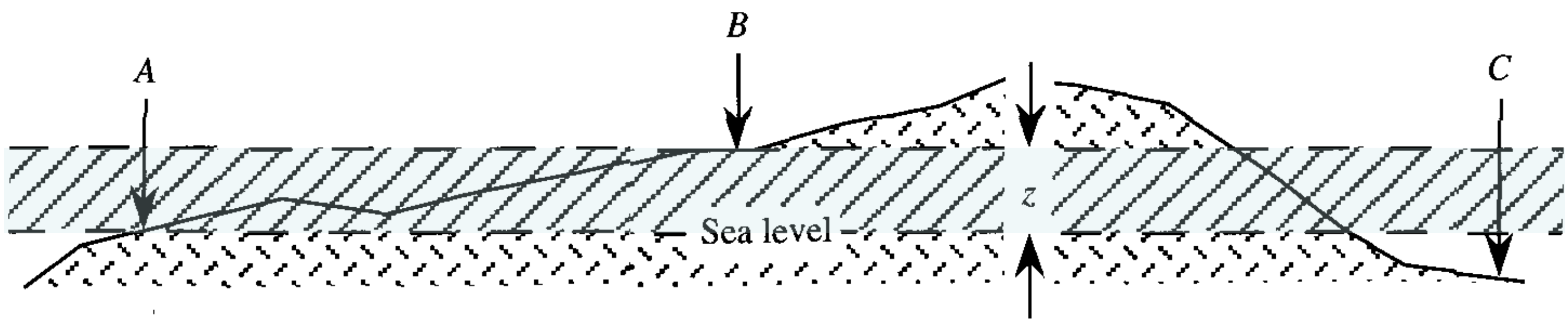
Diff between a 'corrected' value + 'standard gravity' is the **"free-air anomaly"**

$$g_{fa} = (g_{obs} - g_n) + 0.3086*h$$

Excess Mass, or Bouguer Correction

Assume 'space' from reference ellipsoid and measurement point is filled with a horizontal slab

- uniform density = ρ
- thickness = z (site elevation above sea level)
- infinite in lateral extent



$$\text{gravity effect of a plate} = (2\pi G)^*\rho z$$
$$= (0.04193)^*\rho z \quad \text{(in mgals)}$$

This will <u>INCREASE</u> g_{obs} above the ellipsoid value at that latitude

Relative to g_n, each measurement is <u>too large</u> by $0.04193*\rho z$

To compare with 'standard gravity', must <u>subtract</u> $0.04193*\rho z$ from free-air value

$$-\ 0.04193*\rho z \quad \leftarrow \textbf{Bouguer correction}$$

Residual after free-air correction and Bouguer correction = **"Bouguer anomaly"**

$$\begin{aligned} g_b &= g_{fa} - 0.04193*\rho z \\ &= (g_{obs} - g_n + 0.3086*h) - 0.04193*\rho z \end{aligned}$$

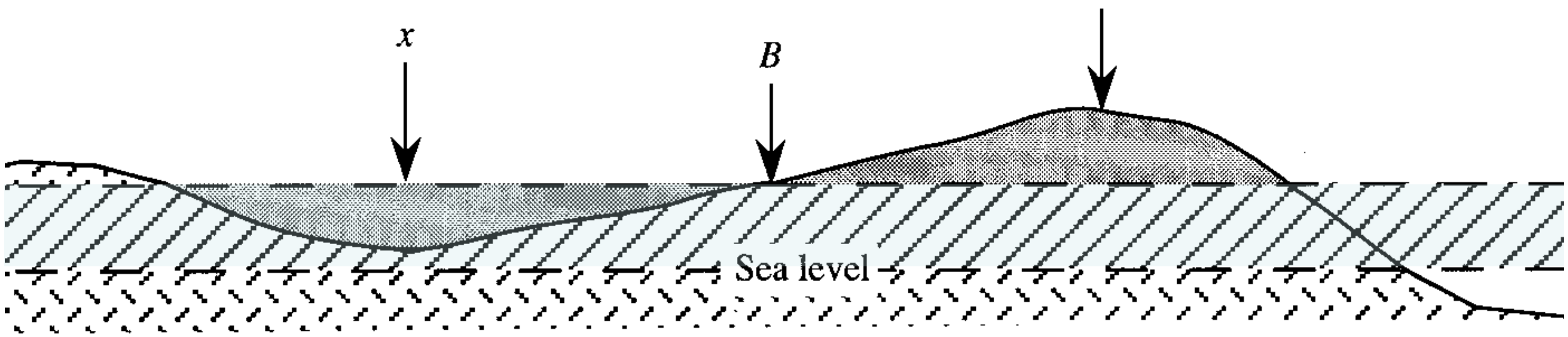
<u>note</u>: corrections are very sensitive to ρ
often one chooses avg crustal value of 2.67 g/cc

Terrain Corrrection

Recall that the Bouguer correction tries to get close to actual 'standard gravity'

$$g_{obs} + 0.3086*h - 0.04193*\rho z \sim g_n$$

Consider hills + valleys on this Bouguer 'plate'



hills = contribute upward component of gravitational attraction
= must ADD effects to g_{obs} to compare with ellipsoid value

valleys = areas where Bouguer slab was too thick
= must ADD effects to compensate for excess Bouguer corr

Diff between this 'corrected' value + 'standard gravity' = **"full Bouguer anomaly"**

$$g_B = g_b + TC$$

$$= (g_{obs} - g_n + 0.3086*h - 0.04193*\rho z) + TC$$

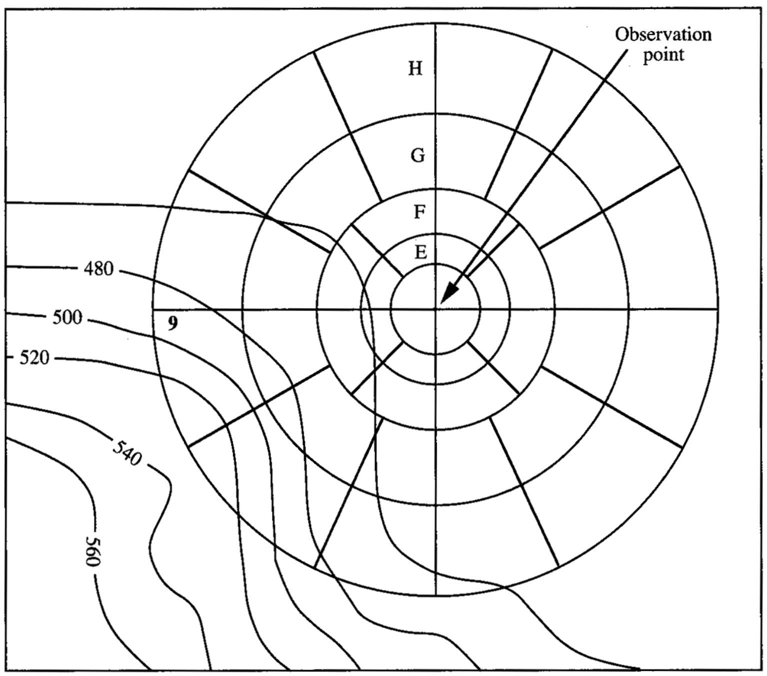
So how is 'TC' calculated? ….. ***very tediously!***

Can be automated if topography available at fine-scale digital values
But technique developed in 1930's (by Sigmund Hammer) is still widely used

Based on approximating local topography as sectors of a 3-D doughnut
-- dimension of sectors depends on terrain complexity + survey goals

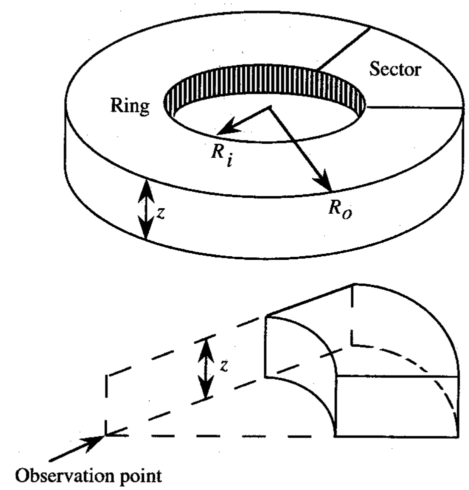
1. place tracing on topo map centered on measurement site
2. record the avg elevation within each sector
3. subtract elevation of site from each sector avg (sign doesn't matter)
4. calculate gravity effect of elevation <u>contrast</u> of each sector according to:

$$g_{ter} = 2\pi G\rho \left[R_o - R_i + (R_i^2 + H^2)^{1/2} - (R_o^2 + H^2)^{1/2} \right]$$



each sector has the following dimensions:

R_o = outside radius
R_i = inside radius
H = avg sector elevation - site elevation



5. add up the combined effect of <u>all</u> sectors; this is the **terrain correction**

So how accurately can we measure gravity?

instrument precision	~ 0.05 mgals
….with good drift correction	~ 0.01 mgals

What is the sensitivity of gravity vs. position?

latitude (for g_n)	~ 0-1 mgals / km (max at mid-latitudes)
elevation (for g_{fa} and g_B)	~ 0.2 mgals / m elevation

Hence, to match good instrumentation, must determine

latitude to ~ 10-50 meters
elevation to ~ 5-25 cm

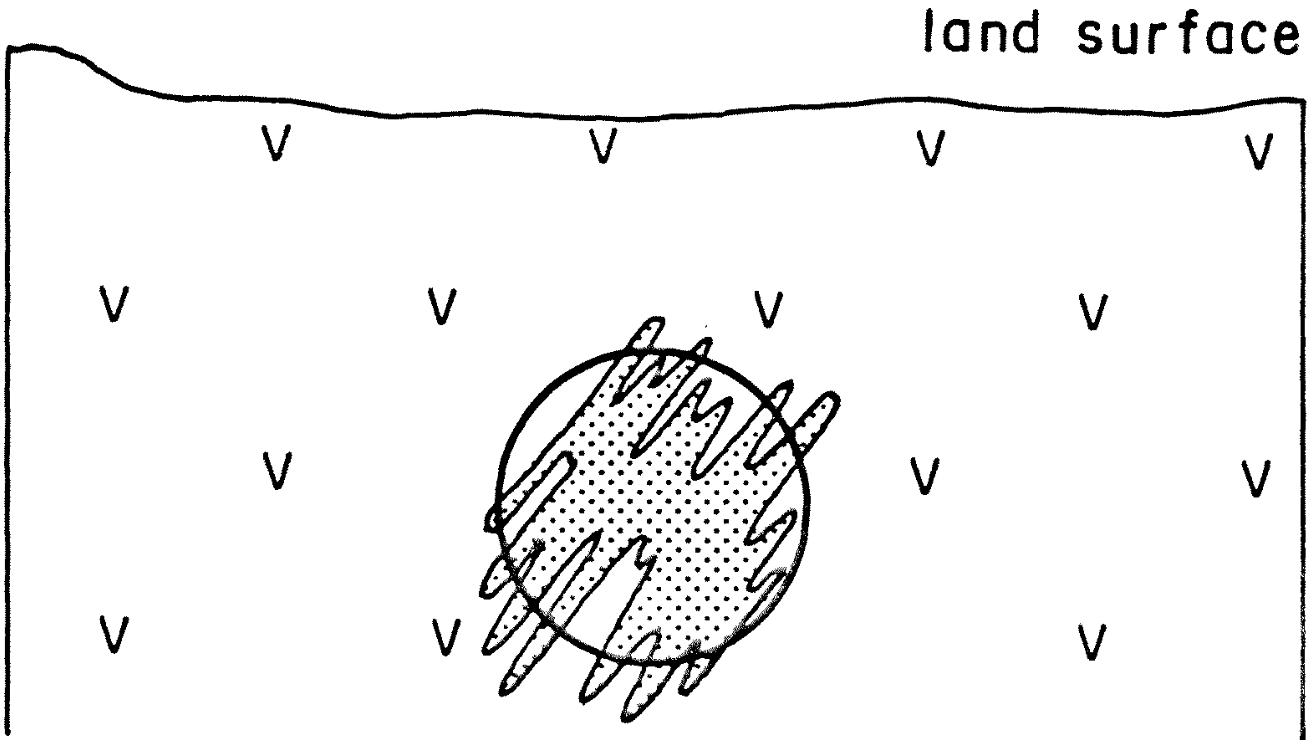


Figure 9–10
A sphere representing an irregular mass for purposes of gravity anomaly analysis.

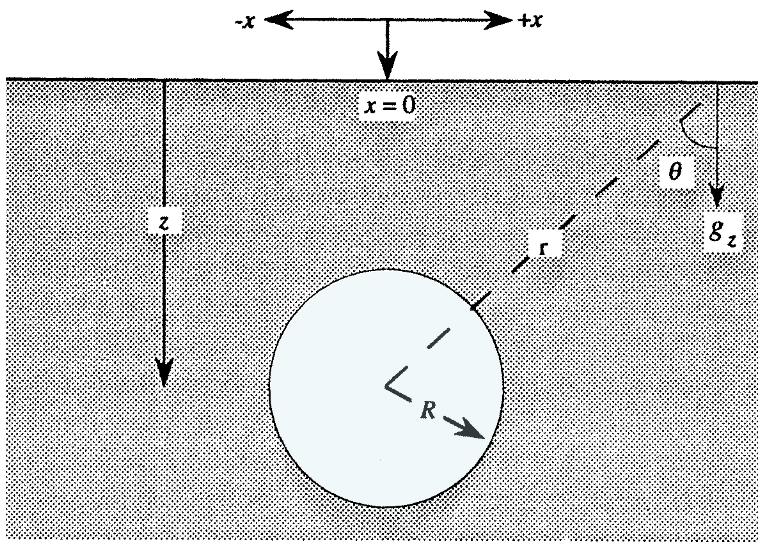
from Robinson + Coruh

The Gravitational Effects of Simple Shapes

Sphere

~ buried ore body
mass can be thought to be concentrated at center of sphere

- **Fig. 6-13**



R = radius of sphere
z = depth to center of sphere
x = distance from msrmt to directly above the sphere
ρ_c = density contrast between sphere and country rock
g_s = total gravity due to the sphere
g_z = vertical component of gravity due to the sphere

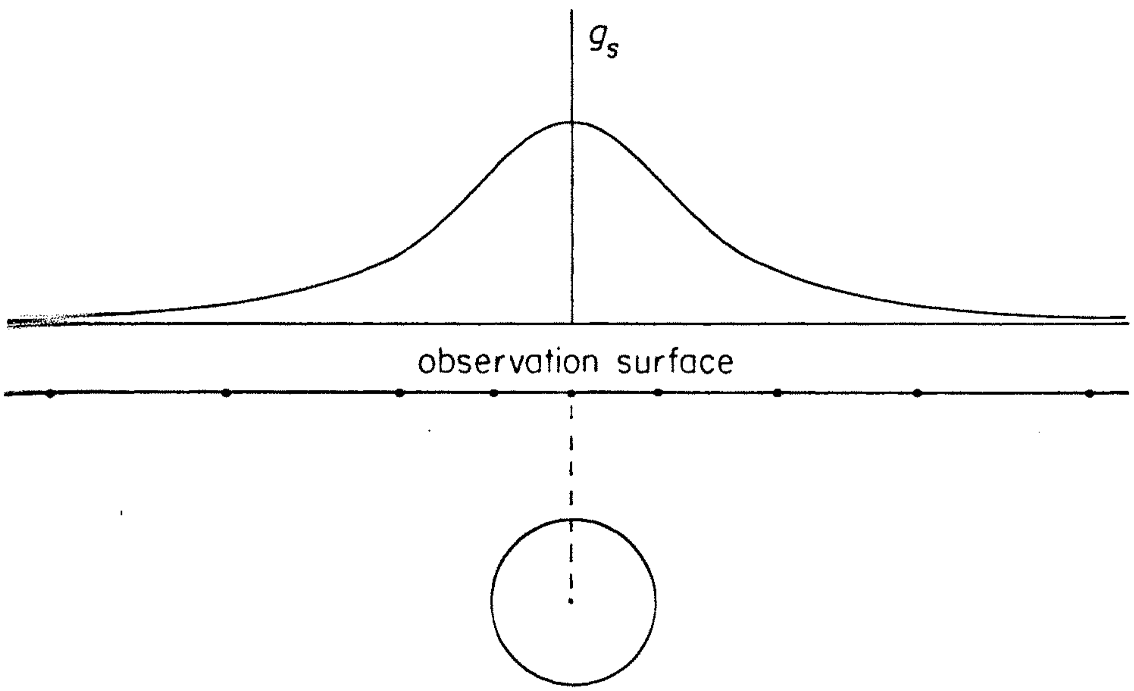


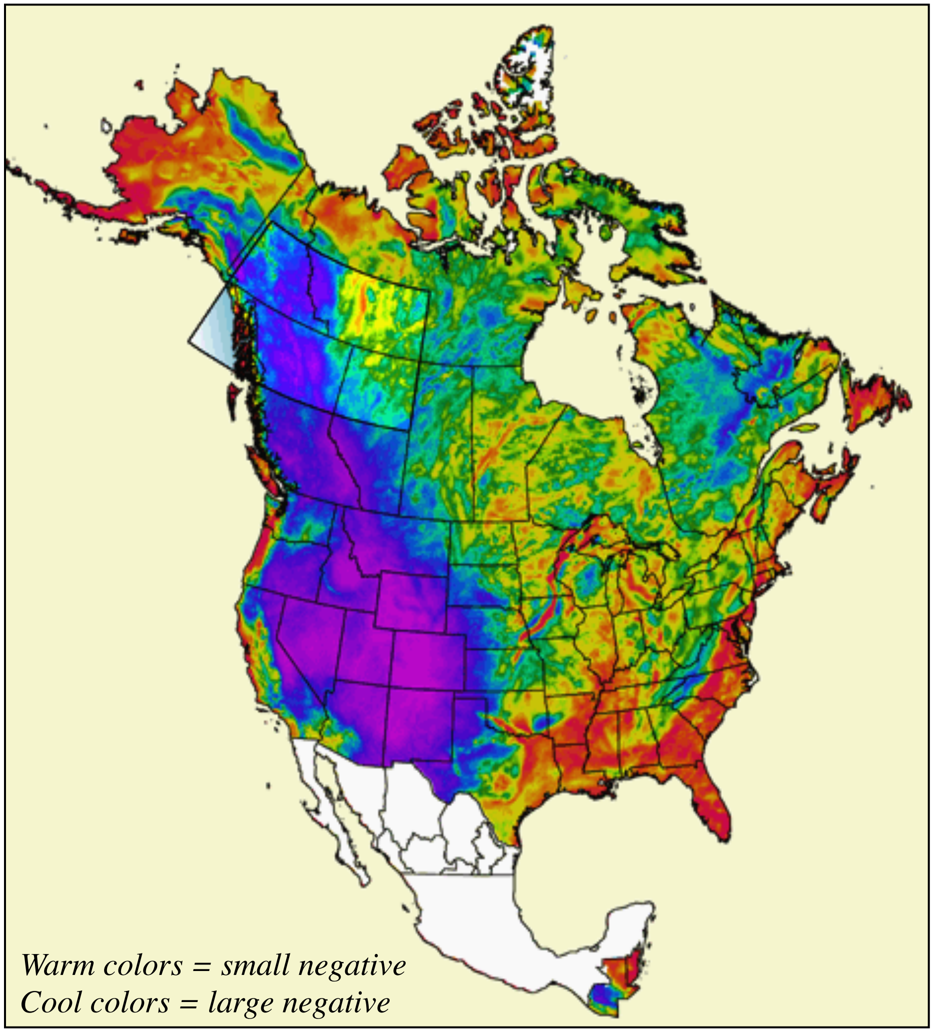
Figure 9–12

Gravity anomaly profile over a buried sphere. In this example, a sphere with a 400-meter radius is centered at a depth of 1000 meters, and the density contrast $\Delta\rho$ is 0.5 g/cm^3. Using these values in Equations 9-3 and 9-4a yields g_s = 0.894 mgal directly over the sphere.

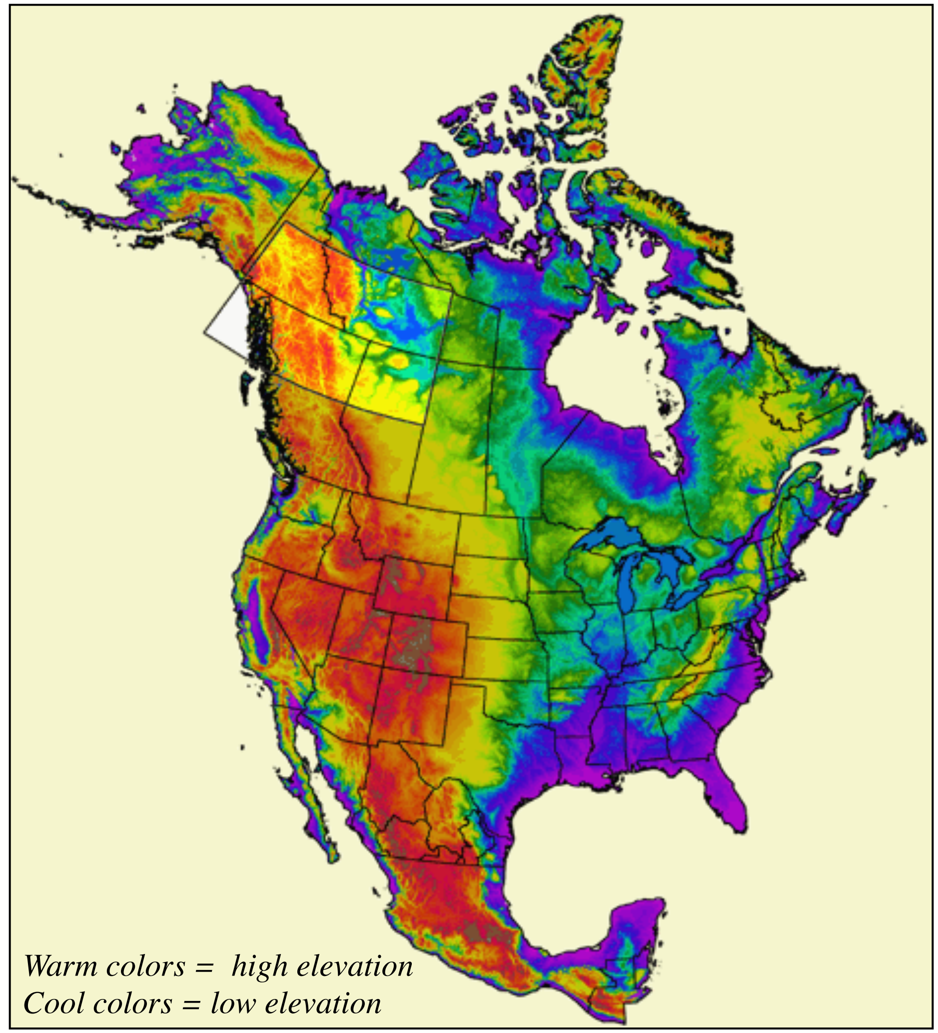
from Robinson + Coruh

Global Bouguer Gravity Anomalies

Bouguer Anomalies



Elevation



Bouguer anomalies begin near zero at shoreline and decrease to -100’s mgals over mountains

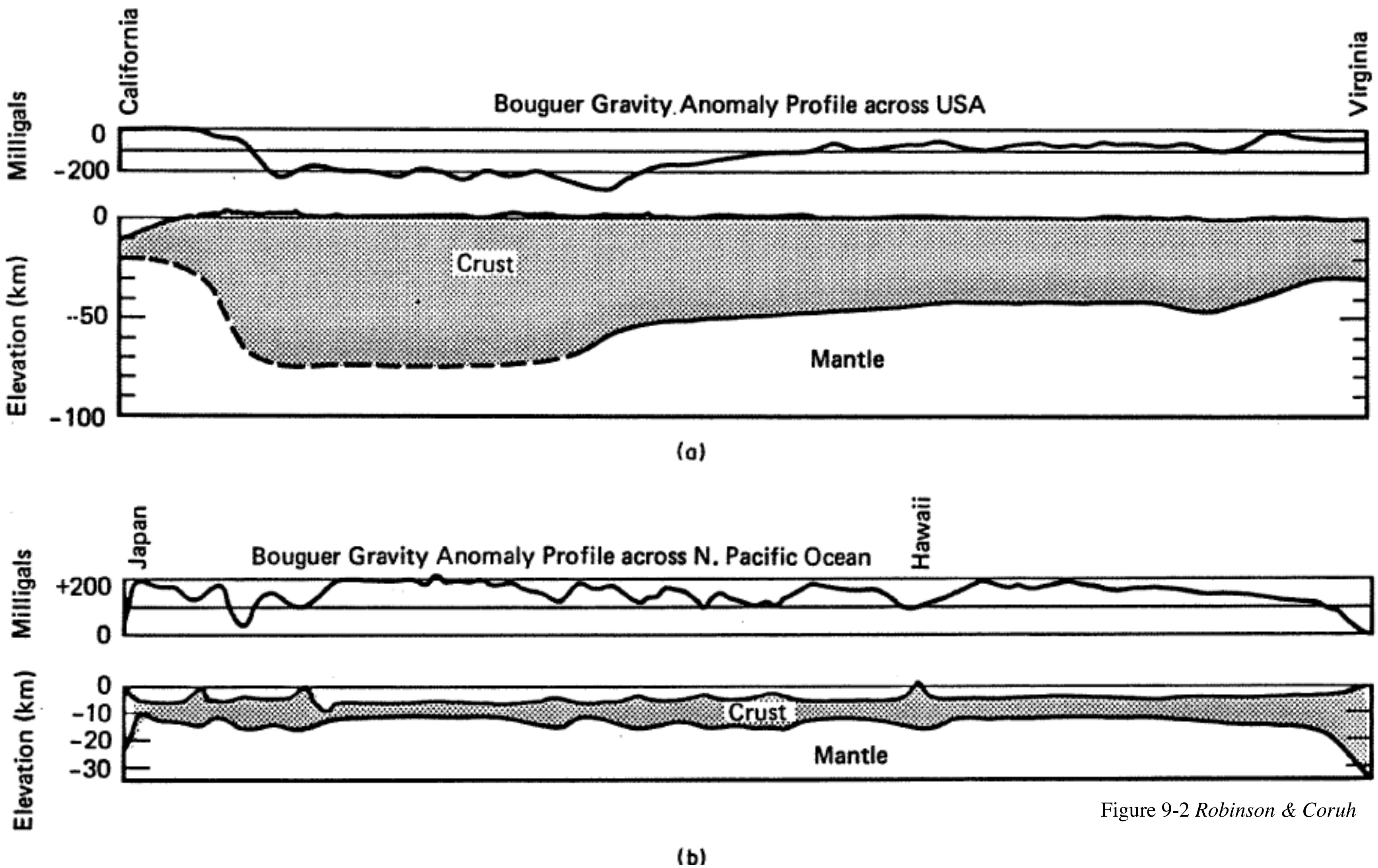
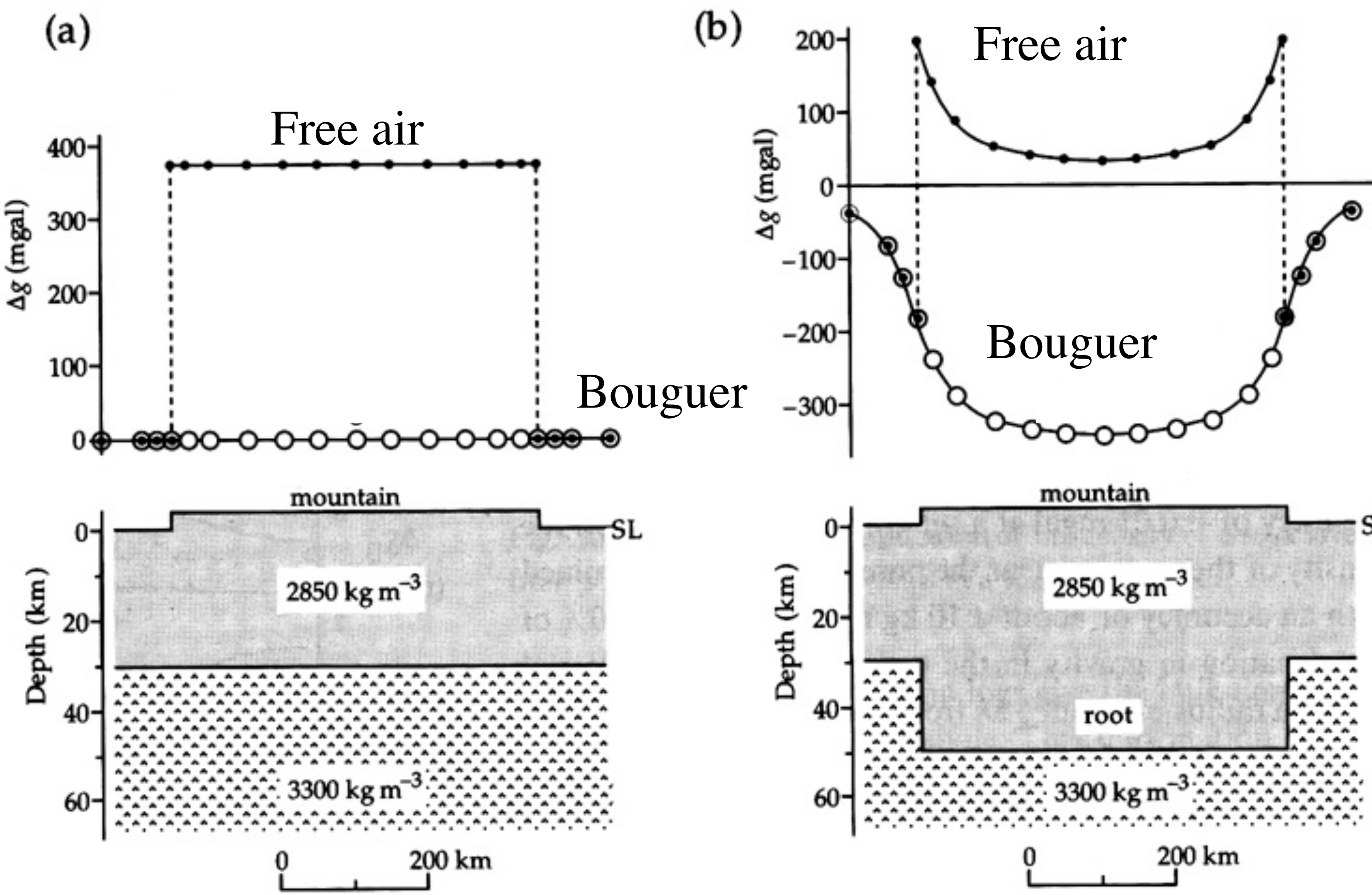


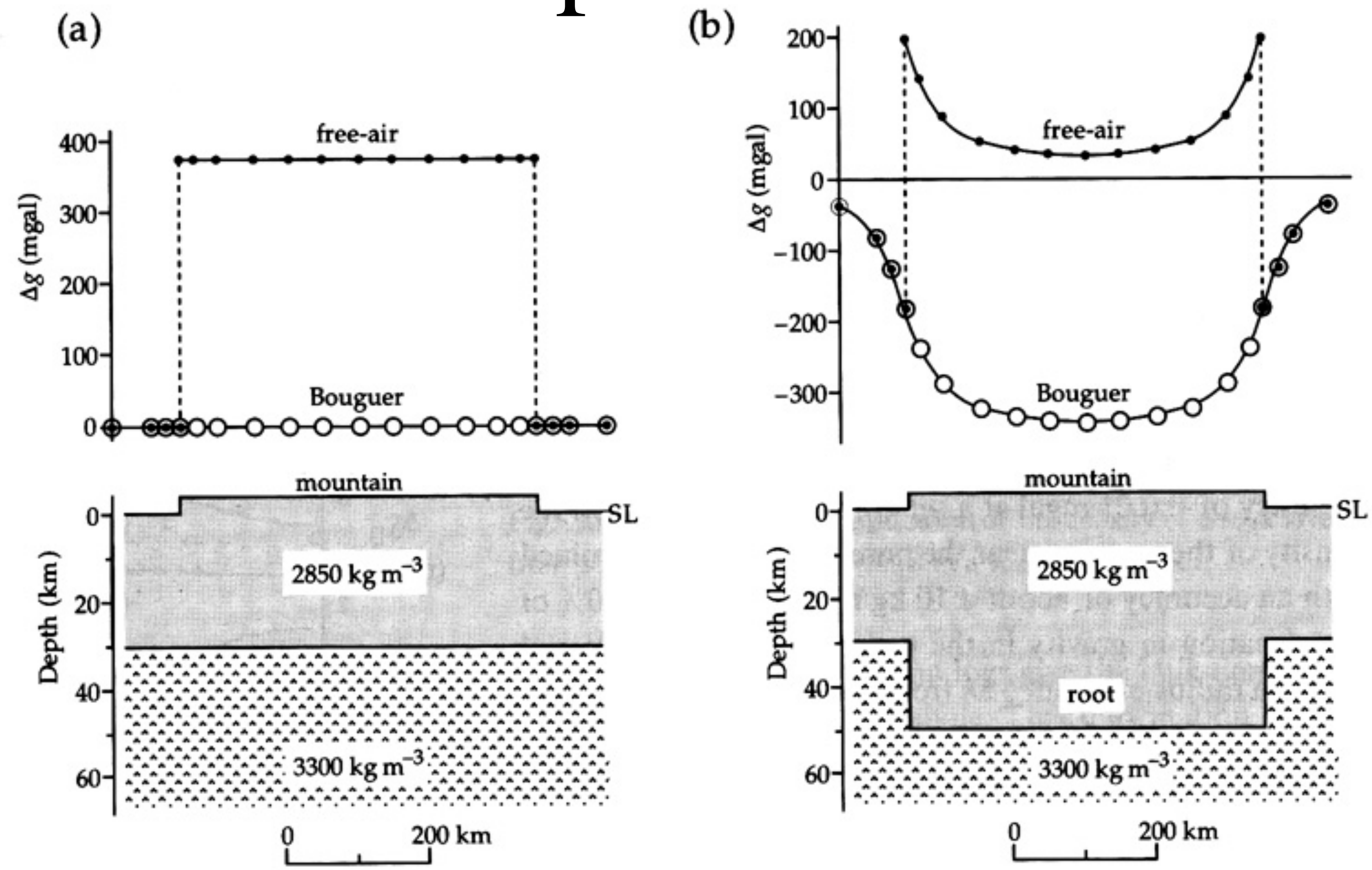
Figure 9-2 *Robinson & Coruh*

Land Bouguer is usually NEGATIVE + Ocean Bouguer is usually POSITIVE

Compensated?



Compensated?



A strong positive free-air anomaly with a weak Bouguer anomaly indicates a structure is supported by the strength of the lithosphere i.e. no compensation.

A weak free-air anomaly with a strong negative Bouguer anomaly indicates that the structure is compensated.

Isostatic Anomaly

Assume an isostasy model, e.g. Airy compensation with crustal “root”
Use that density structure as a model, calc gravity and subtract from the Bouguer gravity.

This provides a measure of the degree of compensation, e.g. zero if perfectly compensated

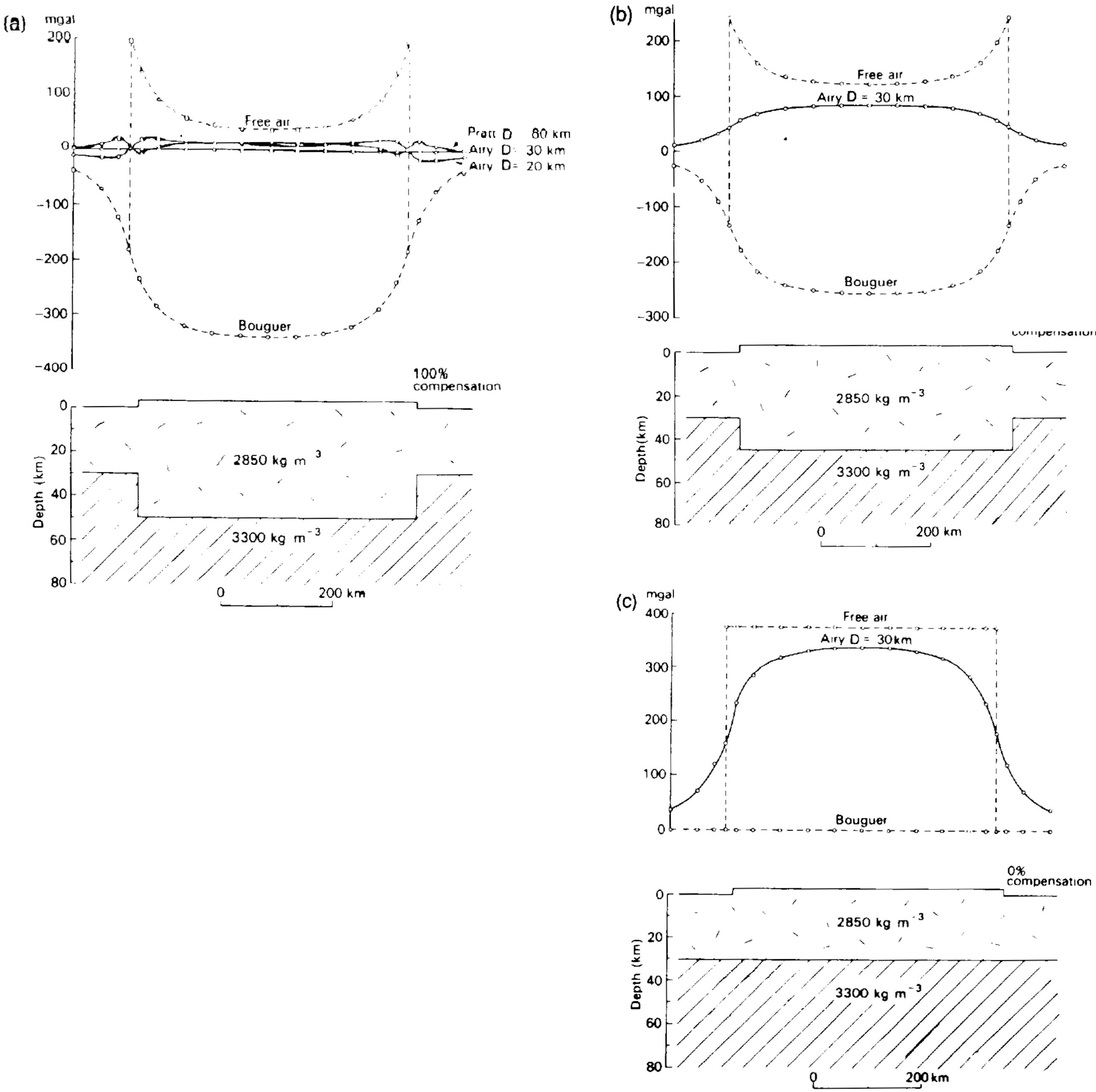


Figure 5.6. Gravity anomalies over a schematic mountain range. (a) Mountain range is 100% compensated (Airy-type). (b) Mountain range is 75% compensated (Airy-type). (c) Mountain range is uncompensated. Dashed lines, the free-air and Bouguer anomalies that would be measured over the mountain. Solid lines, isostatic anomalies calculated for the density models (Pratt compensation depth $D = 80$ km; Airy $D = 20$ km and $D = 30$ km). Densities used to calculate the isostatic anomalies are (fortuitously) those shown in the model. (From Bott 1982.)