ESS 431 Principles of Glaciology
Assignment #8
Stable Isotopes in Precipitation

Assigned November 15, 2007   Due: November 27, 2007

This assignment is designed to give you a better feel for the equations that describe the evaporation and condensation of water vapor in a simple Rayleigh fractionation approximation to atmospheric processes.

Things you need to know:

A) The fractionation between water and water vapor is approximately 1.009 at 25°C:

$$\alpha = \frac{R_{liquid}}{R_{vapor}} \sim 1.009 \qquad (1)$$

where R_{liquid} refers to the ($^{18}O/^{16}O$) ratio in the liquid, and R_{vapor} refers to the $^{18}O/^{16}O$ ratio in the vapor from which the liquid condenses.

B) "delta" notation is defined for oxygen as follows:

$$\delta^{18}O_{sample} = \frac{\left(\frac{^{18}O}{^{16}O}\right)_{sample} - \left(\frac{^{18}O}{^{16}O}\right)_{SMOW}}{\left(\frac{^{18}O}{^{16}O}\right)_{SMOW}} \times 1000‰ \qquad (2)$$

where "sample" is just some ice core, or other ice or water sample. SMOW is the international standard water, "Standard Mean Ocean Water" or SMOW.

C) The Rayleigh fractionation equation for a constant value of α is:

$$\frac{R_{vapor}(f)}{R_{vapor}(f=1)} \sim f^{\alpha-1} \qquad (3)$$

where *f* if the fraction of water remaining in the air mass at any given time.

D) If the air is always saturated, then the amount of water vapor in it is given by the *saturation vapor pressure*, which in units of Torr is:

$$P(T) = \exp(\ 21.113 - 5350.3/T\ ) \qquad (4)$$

when temperature *T* is in Kelvin. (Note that this equation just gives the equilibrium line for water and vapor on the phase diagram for H_2O.) So the "fraction of water remaining" at any given time is simply

$$f = P(T(t) / P(T_0). \qquad (5)$$

where T(t) is the temperature at time t, and T_0 is the original temperature when the air mass began to cool.

Questions. Please answer on a separate sheet.

1) Using the definition of “delta” notation from (2), rewrite the Rayleigh equation using “delta” rather than “*R*” notation. This simplifies the subsequent calculations. Note that you never will need to know the actual value of R_{SMOW}.

2) Assume that an air mass starts out saturated with water vapor at 25°C, having evaporated from an ocean with $\delta^{18}O$ = 0‰ (on the SMOW scale), and that the air mass cools adiabatically while always remaining saturated. Cool the air mass until it reaches 0° C. Calculate and plot the following:

a) The $\delta^{18}O$ of the water vapor as a function of *f*.
b) The $\delta^{18}O$ of the liquid water as it is condensed from the vapor, as a function of *f*.
c) The $\delta^{18}O$ of the liquid water as it is condensed from the vapor, as a function of T.
d) Imagine that all of the liquid water condensed from the vapor over time is collected in a single bucket. What will be its $\delta^{18}O$ value?
e) ***Bonus*:** Write (but do not solve) an integral that gives the average $\delta^{18}O$ value of the water in that bucket as a function of *f*. (However, it might be harder to run around under the cloud catching all the rain drops in the bucket than to solve the equation. ☺)

3) Assume that ice sheets during the Last Glacial Maximum had an average $\delta^{18}O$ of -30‰. If the average ocean depth is 4000 m, and sea level was lower by 150 m at the last glacial maximum, what was the average $\delta^{18}O$ of the ocean, if it is 0‰ today? Ignore the small amount of glacier ice left on land today.

Note that to solve this you can (and should) use the approximation that the amount of water in the ocean, or the ice, is equal to the number of the $H_2{}^{16}O$ molecules alone. That is, you can write:

$$\text{Mass}_{ocean} = (\text{number of mols of } H_2{}^{16}O)_{ocean} \times (\text{molecular mass of } H_2{}^{16}O)$$
$$= N_{16O(Ocean)} * 18$$

This is convenient because now we can write the number of mols of $H_2{}^{18}O$ in the ocean in the same way, using the $^{18}O/^{16}O$ ratio, R_{ocean}. That is:

$$N_{18O(Ocean)} = N_{16O(Ocean)} * R_{ocean}$$

Now you can write too mass balance equations, one for the mass of the ocean, and one for the number of 18O molecules.

$$\text{Mass}_{ocean}(\text{now}) = \text{Mass}_{ocean}(\text{glacial}) + \text{Mass}_{ice}(\text{glacial})$$

$$N_{18O(Ocean)}(\text{now}) = N_{18O(Ocean)}(\text{glacial}) + N_{18O(ice)}(\text{glacial})$$

I’ll leave the rest to you. Note that to get the answer, you’ll need to keep track of the conversion from *R* notation to $\delta^{18}O$ notation.