Problem # 2: Part A

This part of the problem is designed to provide you with some insight into the uses of radiogenic isotopes as tracers in igneous petrology. Sr isotopes will be used as an example. The sequence of events is somewhat arbitrary although not totally unrealistic. The problem illustrates the point that the Sr isotopic ratios are dependent on the sum total of processes that have affected a given volume of rocks since the earth formed. Assume that the earth formed $4.6 \times 10^9$ years ago with an initial $^{87}\text{Sr}/^{86}\text{Sr} = 0.69900$. Recall that the decay constant $\lambda(\text{Rb} \rightarrow \text{Sr}) = 1.42 \times 10^{-11}$ yr$^{-1}$

$$(^{87}\text{Sr}/^{86}\text{Sr})_t = (^{87}\text{Sr}/^{86}\text{Sr})_t + (^{87}\text{Rb}/^{86}\text{Sr})_t (e^{\lambda t}-1)$$

This equation is usually called the isochron equation. Note that $t_e$ is less than $t_i$. [In applications of the isochron equation, $t_o = 0$ (i.e., present day).]

This equation can also be written in the form:

$$(^{87}\text{Sr}/^{86}\text{Sr})_t = (^{87}\text{Sr}/^{86}\text{Sr})_t + (^{87}\text{Rb}/^{86}\text{Sr})_t (1 - e^{-\lambda t - t_o})$$

This is the equation to use when, as in this hypothetical problem, you know the $^{87}\text{Rb}/^{86}\text{Sr}$ ratio at the beginning of some time interval. This equation would not be used in geochronology because we only have measurements of present day $[t_o]$ values. However, this equation is useful for calculating isotopic evolutionary trends and it is the equation you will use in this problem.

To calculate $^{87}\text{Rb}/^{86}\text{Sr}$ at successive times use the following decay equation:

$$(^{87}\text{Rb}/^{86}\text{Sr})_t = (^{87}\text{Rb}/^{86}\text{Sr})_t (e^{-\lambda t - t_o})$$

1. Assuming that the earth accreted from the solar nebula $4.6 \times 10^9$ years ago with an initial $^{87}\text{Sr}/^{86}\text{Sr} = 0.69900$ and $^{87}\text{Rb}/^{86}\text{Sr} = 0.0891$ and then evolved as a closed system until $4.3 \times 10^9$ years ago. Calculate $^{87}\text{Sr}/^{86}\text{Sr}$ and $^{87}\text{Rb}/^{86}\text{Sr}$ for the earth at $4.3 \times 10^9$ years. Do all calculations of $^{87}\text{Sr}/^{86}\text{Sr}$ to 6 decimal places and round off to 5 decimal places for your entry into the answer box.

2. Now, assume that a major differentiation episode occurred at $4.3 \times 10^9$ during which the upper mantle and crust formed. Remember that isotopes of the same element are not fractionated relative to each other but that different elements, e.g., Rb and Sr, commonly are. Assume that the crust has $^{87}\text{Rb}/^{86}\text{Sr} = 0.20$ and the mantle has $^{87}\text{Rb}/^{86}\text{Sr} = 0.076$. Now, suppose that the system evolved to $3.0 \times 10^9$ years. Calculate values of $^{87}\text{Sr}/^{86}\text{Sr}$ and $^{87}\text{Rb}/^{86}\text{Sr}$ in the mantle and crust at $3.0 \times 10^9$ yr.

3. Now, assume that at $3.0 \times 10^9$ yr another fractionation event occurred that affected the crustal rocks only, e.g., partial melting of the lower crust and transfer of the magma to the upper crust. After this event, the lower crust now has $^{87}\text{Rb}/^{86}\text{Sr} = 0.12$ and the upper crust has $^{87}\text{Rb}/^{86}\text{Sr} = 0.55$ at $3.0 \times 10^9$ yr. Let the systems evolve to $0.1 \times 10^9$ yr. Calculate the $^{87}\text{Sr}/^{86}\text{Sr}$ and the $^{87}\text{Rb}/^{86}\text{Sr}$ ratios in the mantle, the lower crust and the upper crust.

4. Now, suppose that at $0.1 \times 10^9$ yr, two basaltic magmas are formed by partial melting in the mantle. The amount of melting involved is small and the mean $^{87}\text{Rb}/^{86}\text{Sr}$ in the mantle is unchanged by this event. One of the basalts [1A] is extruded on the sea floor without interacting with any other material. At $0.1 \times 10^9$ this basalt has $^{87}\text{Rb}/^{86}\text{Sr} = 0.125$. Calculate the present day $^{87}\text{Sr}/^{86}\text{Sr}$ ratios in [a] mantle, [b] basalt 1A.
The second basalt splits into two fractions [2A and 2B]. Basalt 2A assimilates 20% of lower crustal material (at 0.1 x 10^9) before extrusion while basalt 2B assimilates 20% of upper crustal material (at 0.1 x 10^9) before extrusion. Calculate the present day \(^{87}\text{Sr}/^{86}\text{Sr}\) ratios in [a] lower crust, [b] upper crust, [c] basalt 2A after assimilation, [d] basalt 2B after assimilation. For this part of the problem, you will need to first calculate \(^{87}\text{Rb}/^{86}\text{Sr}\) after assimilation. Assume the following values for Rb and Sr in the various units and use the mixing equation we discussed in class.

<table>
<thead>
<tr>
<th>Rb [ppm]</th>
<th>Sr [ppm]</th>
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<tbody>
<tr>
<td>Basalt 2A</td>
<td>1</td>
</tr>
<tr>
<td>Lower crust</td>
<td>10</td>
</tr>
<tr>
<td>Upper crust</td>
<td>20</td>
</tr>
</tbody>
</table>

Also, you can assume that \(^{87}\text{Rb}/^{86}\text{Sr}\) = [Rb/Sr] x 2.822. The lower crust and upper crust will have \(^{87}\text{Sr}/^{86}\text{Sr}\) values at 0.1 x 10^9 identical to those in part 3. Enter your answers (rounded to 5 decimal places) in the blank boxes in the table below.

5. Plot all answers on a \(^{87}\text{Sr}/^{86}\text{Sr}\) versus time [t] diagram and compare with the uniform reservoir, i.e., whole earth, line. What you have calculated is a model Earth evolution that is a much simplified version of what actually occurred in the Earth. The real earth is, of course, much more complex but the general processes and tendencies are similar to those you calculated. Comment briefly on your answer.
Problem # 2: Part B

This part of the problem is designed to investigate the formation of alkali basalts by modeling mantle melting using trace element abundances. The problem is based on a paper by **Wendy Harrison** in *Amer. Mineral.*, 66, 242-259 (1981). Wendy did melting experiments on a natural garnet lherzolite (PHN 1611) at \( P = 35 \) kilobars in which she doped the starting material with small amounts of radioactive rare earth elements Ce, Sm, and Tm. She measured distribution coefficients of Ce, Sm, and Tm between garnet, clinopyroxene, orthopyroxene, olivine and melt at varying degrees of partial melting. **PHN 1611 has chondritic relative REE abundances at 2x chondrites.**

From 0 to \(~10\%\) melting, the melting of the lherzolite was essentially isothermal (1580°C) indicating that the melting was **pseudo-eutectic (isobaric invariant)**. What this means is that the melting reaction was essentially the same over the 0-10\% melting interval. It is also reasonable to assume that the melt was in equilibrium with all four crystalline phases over the melting interval. At \(~10\%\) melting all the garnet was used up and the melting was no longer eutectic. With increasing \( T \), melting increased until all the cpx was used up at \(~40\%\) melting leaving a harzburgite (ol + opx) residuum.

Initial assemblage: \( \text{Ol}_60\text{Opx}_{20}\text{Cpx}_{10}\text{Gnt}_{10} \)  [These modes are in wt. %]

Reaction Stoichiometry: \( 1 \text{ Gnt} + 0.67 \text{ Cpx} + 0.14 \text{ Opx} \rightleftharpoons 0.22 \text{ Ol} + 1.61 \text{ Liquid} \)

What this means is that 1 gm of Gnt + 0.67 gm Cpx + 0.14 gm Opx react to produce 0.22 gm Olivine + 1.61 gm Basaltic melt. Note that this is a peritectic reaction (which actually produces olivine) rather than a eutectic but it can still be considered to be isobaric invariant. In this case, the reaction applies as long as all 4 crystalline phases are present. If truly invariant, the major element composition of the melt will remain constant from 0 - 10\% melting.

However, as pointed out in class, the trace elements are decoupled from the major elements so the trace element abundances can vary over the same melting range. Even though the \( P \) and \( T \) are constant, the bulk distribution coefficients vary slightly over this range. The reason for this is that the melting reaction is not strictly invariant because the Fe/Mg ratios of the liquids and crystalline phases are changing and this has a small effect on the distribution coefficients.

From the reaction stoichiometry, it is clear that the various minerals are contributing to the melt in proportions markedly different from their proportions in the rock. **This is the normal case in nature** and it is commonly referred to as "**non-modal batch melting**", and it is assumed to be equilibrium melting. [Calculations using modal melting models are often seriously in error.]

For non-modal melting, the mass balance equation takes the form:

\[
C_i^{\text{liq}} / C_i^{\circ} = 1 / [D_i^{\circ} + F_{\text{liq}}(1-P_i)]
\]

where \( C_i^{\text{liq}} \) = concentration of trace element \( i \) in the liquid
\( C_i^{\circ} \) = concentration of trace element \( i \) in the total assemblage, i.e., the unmelted rock
\( D_i^{\circ} \) = Bulk distribution coefficient for the initial assemblage (before melting)
\( P_i \) = Bulk distribution coefficient of the "assemblage that is being melted which is given by the reaction stoichiometry"
\( F_{\text{liq}} \) = weight fraction of liquid produced.
DATA TABLE

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<th>D&lt;sub&gt;clpx/liq&lt;/sub&gt;</th>
<th>D&lt;sub&gt;ol/liq&lt;/sub&gt;</th>
<th>D&lt;sub&gt;ol/liq&lt;/sub&gt;</th>
<th>F&lt;sub&gt;liq&lt;/sub&gt;</th>
<th>E.F.</th>
</tr>
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<tbody>
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</tbody>
</table>

E.F. = Experimentally measured Enrichment Factor

Calculate:

1. D<sub>Ce</sub><sup>o</sup>, D<sub>Sm</sub><sup>o</sup>, D<sub>Tm</sub><sup>o</sup>

2. P<sub>Ce</sub>, P<sub>Sm</sub>, P<sub>Tm</sub>

3. C<sub>i</sub><sup>liq</sup>/C<sub>i</sub><sup>o</sup> for Ce, Sm and Tm at F<sub>liq</sub> = 0.023 and F<sub>liq</sub> = 0.08

These are the calculated enrichment factors. Compare the calculated and measured enrichment factors. How well do they agree? How might you explain any discrepancy?

4. Plot a chondrite normalized REE diagram showing your calculated data and the experimentally measured data. On the same diagram, plot the following data for an alkali basalt from Hawaii [Ce = 62 ppm, Sm = 7.8 ppm, Tm = 0.5 ppm]. Chondritic values for these elements are: Ce = .603 ppm, Sm = .147 ppm, Tm = .024 ppm.

How well does the calculated/measured values compare with those from a typical alkali basalt? Discuss your results given that most petrologists think that alkali basalt form by small degrees of melting of garnet lherzolites at P = or > 30 kilobars.