

Gravity

force (acceleration) vs potential (energy)

$$\begin{array}{ll} \text{acceleration (g)} & \Rightarrow GM/r^2 \\ \text{Potential} & \Rightarrow -GM/r \end{array}$$

G is Newton's gravitational constant 6.67×10^{-11} (S.I. units)

– you should determine what the S.I. units are

g is vector quantity (local vertical component is all that matters)

– most sensitive to nearby mass distribution

Potential is scalar field (actually mathematically easier to deal with)

– more sensitive to distant mass distributions

(because $1/r^2$ gets smaller with distance faster than $1/r$).

Total gravitational field is sum of contributions from all masses.

Surface gravity is associated with

- (1) figure of Earth,
- (2) rotation of Earth,
- (3) internal mass distribution.

(3) is point of interest. Need to understand (1) and (2) in order to determine (3).

- (1) Earth's shape can be approximated by an oblate spheroid characterized by an ellipticity or polar flattening:

$$f = (R_e - R_p)/R_e = 1/298.247 \quad R_e = 6378.14 \text{ km} \quad R_p = 6356.75 \text{ km}$$

- (2) Rotation causes axial centrifugal acceleration that reduces the effective **g**.

Gravity varies from equator to pole due to

- (1) changing distance from center of mass and (40%)
- (2) changing distance from rotation axis (60%).

The International Gravity Formula codifies these changes:

$$g(\lambda) = g_e (1 + \alpha \sin^2(\lambda) + \beta \sin^4(\lambda))$$

$$g_e = 9.7803185 \text{ m/s}^2 \quad (9.832 \text{ at poles})$$

$$\alpha = 5.278895 \times 10^{-3}$$

$$\beta = 2.3462 \times 10^{-5}$$

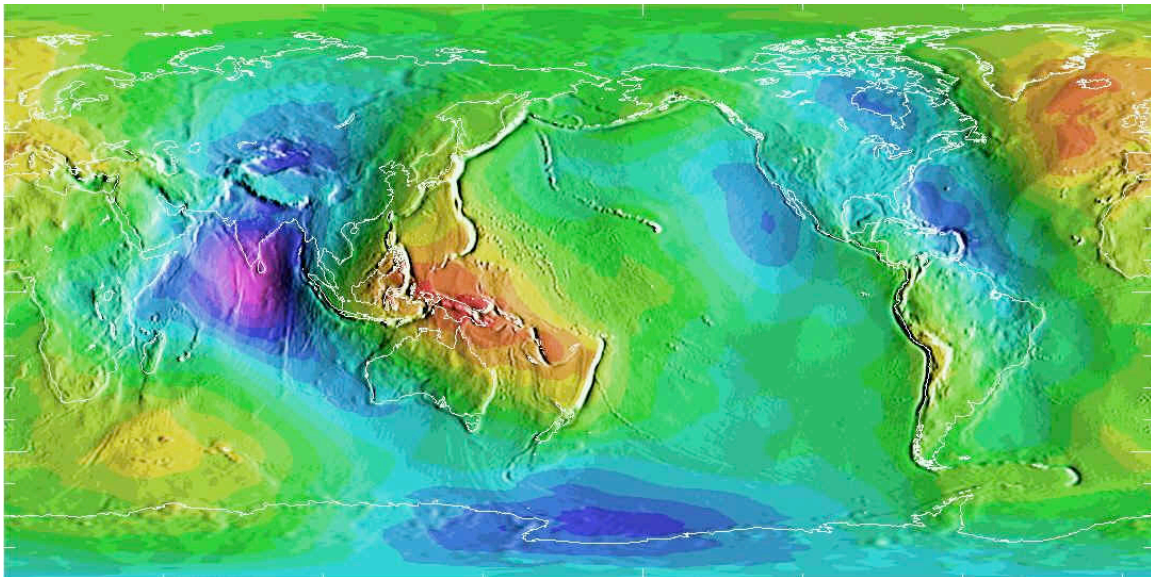
The Geoid

Actual Earth shape deviates from spheroid (by tens of meters) as a result of internal mass distribution and dynamic processes

Earth's equipotential surface is the **Geoid**

- imagine as narrow sea level channel through continents
- Measured as a distance above or below reference spheroid
- Range is +/- tens of meters (+100 m hole near India)
- More sensitive to deeper mass distribution than acceleration

Here is an image of the Geoid determined from SEASAT data (radar altimetry from a satellite). The color coding shows the large long wave-length variations (e.g. see the >100 meters “hole” near India and “highs” near Iceland and Borneo). The large areas of low Geoid relief (not much deviation from the spheroid) are colored green. Short wave length features are generally due to topographic features on the sea floor (sea mounds chains and subduction related trenches are most prominent).



Interpretation of measurements of the gravitational acceleration g

Changes in acceleration due to mass distribution are small

- measured in milligal = 10^{-5} m/s^2 or gravity unit (gu) = 10^{-6} m/s^2 .
- Field instruments are sensitive to 0.1 gu (part in 10^8)
- equator to pole variation is 52,000 gu.

Variation of gravity with height in air: 3.1 gu/m (Free air gradient)

Gravity caused by an infinite slab: $2\pi\rho Gh = 1.1 \text{ gu/m}$ for “crustal rock” (Bouguer correction)

Anomaly → a difference in gravity due to internal mass distributions

Free-air anomaly $g_{\text{FAA}} = g_{\text{obs}} - g(\lambda) + \text{FAG}$

Bouguer anomaly $g_{\text{BA}} = g_{\text{obs}} - g(\lambda) + \text{FAG} - \text{BC}$ (+ terrain correction)

Measurement of gravity

“Stable” and “Unstable” gravimeters. Draw diagram to illustrate principle.

We will use a Lacoste and Romberg “unstable” gravimeter. It has a horizontal pendulum with a test mass pulling down and a “zero-length” spring holding it up. Holding it in place requires a balance of torques that is achieved with one setting of the spring. One measures that setting. This is not an “absolute” measurement – it needs to be calibrated at locations where gravity is known precisely. The instrument is then used to measure changes in gravity from a “base station” to survey sites. The daily tidal cycle changes gravity by about 2 g.u over about 6 hours. The instrument may also suffer drift (a time change of reading associated with a change of the internal spring constant). Thus, one cycles between survey sites and the base station to determine the drift correction. Since gravity is sensitive to elevation, one needs to know the elevation of each survey point to high precision (2 cm to make use of the full precision of the instrument).

Isostasy

All large scale features on Earth are in isostatic equilibrium

- except dynamic topography – discussed below
- Interior is “weak” and flows to achieve equilibrium
- Any column extending into Earth contains the same mass
(if it did not, interior pressure would differ and cause flow)

Compensation depth

- all mass distribution differences are above compensation depth.
- same mass in columns extending to compensation depth

Two models to achieve isostasy:

Airy Compensation - same density of rock but differing thickness.

Iceberg model – mass of above water ice is compensated by below water ice. Peaks require deeper roots.
Standard idea for continental crust

Pratt Compensation – laterally varying density to a fixed depth. Higher peaks are underlain by lower density material.
This is the explanation for mid-ocean ridges.

Note:

- depth of compensation,
- possibility of “elastic” support,
- dynamic topography,
- mixed compensation in “real” examples

Note behavior of Free-air and Bouguer anomaly over isostatically compensate features.

- g_{FAA} is zero for isostatically compensated structures

- g_{BA} is zero for uncompensated structures

Note the size of features going from non-compensated to compensated.

Geoid anomalies are non-zero for compensated structures and differ depending on compensation mechanism. These details will not be considered further in this class

Examples of gravity and geoid anomalies

Profiles across structures

- Constructed in class

Mantle Dynamics and Geoid anomalies

- Static and dynamic contributions

- Consider mantle “sinker”

1. contribution from excess mass
2. surface pulled down by flow
3. internal discontinuities deflected
4. core-mantle boundary deflected by flow