Earth's Energy Budget and Heat Flow
Reading: Fowler Chapter 7 (Heat)

Note that we sorted through a collection of rocks
The green ones were "ultramafic"

- high iron and magnesium relative to silica (SiO_2)
- from the mantle

The dark rocks were "mafic"

- more silica
- basalt and gabbro
- typical of oceanic crust

The lighter "salt and pepper" rocks were higher still in silica

- more representative of continental crust
- granite and andesite for large crystal sizes
- rhyolite dacite for smaller crystal size (extrusive volcanic)

Noted the relationship associated with partial melting and chemical fractionation – both for major elements and minor (trace) radiogenic elements.

Geothermal Flux: 42 x 10^{12} W (80 mW/m^2)
compare to:

1. Commercial power plant (10^9 W)
2. Sum of all earthquakes (10^{11} W)
3. Solar irradiance (2 x 10^{17} W - 400 W/m^2)

Sources:
Heat production from radiogenic elements
Secular cooling

Heat Production

	Mantle	"Basalt"	"Granite"		half life
U ppm	.03	.1-.8	4		U^{238} 4.5Ga
10^{-10} W/kg	.03	.1-.8	4		U^{235} 0.7Ga
Th ppm	.1	.4-2.5	15		14 Ga
10^{-10} W/kg	.03	.1-.7	4		
K^{40}-A^{40} (%)	.03	.2 -1.2	3.5		1.25 Ga
10^{-10} W/kg	.01	.1 -.4	1.3		
totals	.07	.3 – 2	9	x 10^{-10} W/kg	
	.03	.1-.5	2.5	μW/m^3	

Mass of mantle 4x10^{24} kg -> 28x10^{12} W
Mass of crust 2.8x10^{21} kg -> 5.5x10^{12} W (13% of total from 0.5% of mass)

Sum is about 80% of total -> 20% secular cooling?
Significant assumptions in this analysis – one finds a large range of estimates for this

Urey Ratio=Internal Heat Production /Surface Heat Flux ~ 0.2 at present: Korenaga, 2008

When mantle is melted to form basalt, the incompatible elements, including U, Th, and K go into the melt and ultimately remain in the crust, enriching the crust and depleting the upper mantle.

Heat Transport

Conduction
Convection/Advection
Radiative – has short mean free path in mantle so can be modeled as conduction

Conduction: heat flux Q (heat/unit time/unit area) is proportional to temperature gradient:

$$Q = -k\frac{dT}{dz}$$

k is thermal conductivity:

Metals – 10^2 W/m/°C
Rocks – 1 to 10 W/m/°C (2-4 most common)

For average values : $\frac{dT}{dz}$ is 30 – 40 °C/km near Earth's surface
(note absurd results for even modest extrapolation - 4000 °C in 100 km)

If the heat flux varies in the z direction, the heat entering and exiting a volume with area A and height dz will differ and the volume will heat up: The heat added to that volume per unit time is: $-A\, dz\, dQ/dz = A\, dz\, k\, d^2T/dz^2$ which will cause the heat in that volume to change at the rate of: mass in volume * C_p * dT/dt = ρ A dz C_p dT/dt so conduction leads to diffusion equation (equating the terms above): $\frac{dT}{dt} = \frac{k}{\rho C_p}\frac{d^2T}{dz^2}$

C_p = specific heat: heat needed to raise 1 kg by 1 deg C

Diffusivity => $\kappa = \frac{k}{\rho C_p}$

Note units: m^2/s and typical value $O(10^{-6}\ m^2/s)$
Fundamental result of any solution is that
$\tau = L^2/\kappa$ $L = \text{sqrt}(\kappa\, \tau)$

Example length-scale/time-scale pairs:

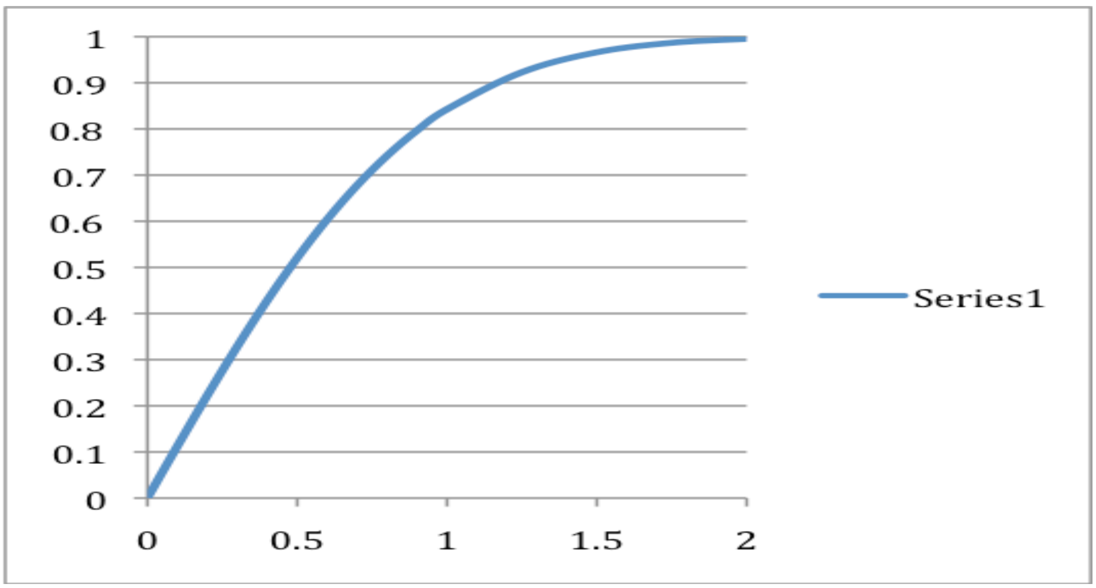
Time	1d	1y	10Kyr	1Myr	100Myr	10Gyr
Length	0.3m	5m	500m	5km	50km	500km

Length	1cm	1m	10m	1km	100km	3000km
Time	100s	12d	3.2yr	32Kyr	320Myr	300Gyr

Note: to a good approximation there are $\pi * 10^7$ seconds in a year.
One important case: instantaneous cooling of a half-space:
Initial Condition (t=0): $T(z,0) = T_m$ for z>0
Boundary Condition(z=0): $T(0,t) = 0$ for t>0

$$T(z,t) = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

This is a plot of erf(x):



Cooling of oceanic lithosphere:

Geotherms, topography, heat flow, hydrothermal circulation
t = distance from ridge/plate velocity
heat flow is vertical because thermal gradient is vertical

$$T(z,t) = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

$$Q(0,t) = -k\frac{dT}{dz} = -\frac{kT_m}{\sqrt{\pi\kappa t}}$$ heat flow at the surface (z=0)

$W = const\sqrt{\kappa t}$ seafloor depth relative the depth at ridge comes from isostacy; integral of rock or ocean density from fixed depth in asthenosphere to the sea surface does not depend on x or t above, while the perturbation in density is proportional to T. The derivation is a bit involved, but straight forward. To first order this explains the variation in world-wide seafloor depth!

Continental Geotherm

Linear relationship between surface heat production and flux is conventional idea

Note -> oceanic (80 mW/m^2) and continental (60 mW/m^2) heat flow are similar but have different contributions- radiogenics in relatively old continental lithosphere vs cooling in younger oceanic lithosphere

Advection and the Adiabatic gradient:

Homogeneous compression causes self-heating the adiabatic gradient is:

Note first that: $\frac{d\ln Y}{dX} = \frac{dY}{Y\,dX}$ and $\frac{d\ln Y}{d\ln X} = \frac{X\,dY}{Y\,dX}$

$$\frac{dT}{dz} = T_o \frac{d\ln T}{d\ln V}\frac{d\ln V}{dP}\frac{dP}{dz} = T_o \frac{\gamma \rho g}{K_s} = T_o g\gamma/(V_p^2 - \tfrac{4}{3}V_s^2)$$

$K_s^{-1} = -\frac{d\ln V}{dP}$; change in volume with pressure = incompressibility

$V_p^2 = (K_s + \frac{4}{3}\mu)/\rho$; $V_s^2 = \mu/\rho$; $V_p \approx 8-13\,km/s$ in the mantle

$K_s/\rho = V_p^2 - \frac{4}{3}V_s^2 \approx \frac{5}{9}V_p^2$; $if\ V_p^2/V_s^2 \approx 3$; K_s is the bulk modulus

$\gamma = -\frac{V}{T}\frac{dT}{dV} = -\frac{d\ln T}{d\ln V}$; Gruneisen Parameter $\approx 1\ to\ 1.5$ in the mantle

$P = \rho g z$; $\frac{dP}{dz} = \rho g$; $g \approx 10 ms^{-2}$ throughout the mantle

dT/dz = 2000 K * 10m/s^2 * 1.25 / (5/9*(10000 m/s)^2) = 4.5e-4 deg/m = 0.45deg/km

Gradient is 0.3 to 0.5 °C/km – O(1000°C) temperature difference in mantle

Earth Geotherm

B.C. = $T_{surface}$, T_{core}

Assumptions: conductive boundary layers, nearly adiabatic interior

Consider ICB boundary, CMB, and lithosphere

Note: melting of iron
- melting of upper mantle
- barriers to convection
- other phase transitions

survey from surface to center of Earth