Earth's Energy Budget and Heat Flow
Reading: Fowler Chapter 7 (Heat)

Note that we sorted through a collection of rocks
The green ones were "ultramafic"
- high iron and magnesium relative to silica (SiO_2)
- from the mantle

The dark rocks were "mafic"
- more silica
- basalt and gabbro
- typical of oceanic crust

The lighter "salt and pepper" rocks were higher still in silica
- more representative of continental crust
- granite and andesite for large crystal sizes
- rhyolite dacite for smaller crystal size (extrusive volcanic)

Noted the relationship associated with partial melting and chemical fractionation – both for major elements and minor (trace) radiogenic elements.

Geothermal Flux: 42×10^{12} W (80 mW/m^2)
compare to:
1. Commercial power plant ($O(10^9)$ W)
2. Sum of all earthquakes (10^{11} W)
3. Solar irradiance (2×10^{17} W - 400 W/m^2)

Sources:
Heat production from radiogenic elements
Secular cooling

Heat Production

	Mantle	"Basalt"	"Granite"		half life
U ppm	.03	.1-.8	4		U^{238} 4.5Ga
10^{-10} W/kg	.03	.1-.8	4		U^{235} 0.7Ga
Th ppm	.1	.4-2.5	15		14 Ga
10^{-10} W/kg	.03	.1-.7	4		
K^{40}-A^{40} (%)	.03	.2 -1.2	3.5		1.25 Ga
10^{-10} W/kg	.01	.1 -.4	1.3		
totals	.07	.3 – 2	9	x 10^{-10} W/kg	
	.03	.1-.5	2.5	μW/m^3	

Mass of mantle 4×10^{24} kg -> 28×10^{12} W
Mass of crust 2.8×10^{21} kg -> 5.5×10^{12} W (13% of total from 0.5% of mass)

Sum is about 80% of total -> 20% secular cooling?
Significant assumptions in this analysis – one finds a large range of estimates for this

Urey Ratio=Internal Heat Production /Surface Heat Flux ~ 0.2 at present: Korenaga, 2008

When mantle is melted to form basalt, the incompatible elements, including U, Th, and K go into the melt and ultimately remain in the crust, enriching the crust and depleting the upper mantle.

Heat Transport

Conduction
Convection/Advection
Radiative – has short mean free path in mantle so can be modeled as conduction

Conduction: heat flux Q (heat/unit time/unit area) is proportional to temperature gradient:

$$Q = -k\frac{dT}{dz}$$

k is thermal conductivity:

Metals –	10^2	W/m/°C	
Rocks –	1 to 10	W/m/°C	(2-4 most common)

For average values : $\frac{dT}{dz}$ is 30 – 40 °C/km near Earth's surface
(note absurd results for even modest extrapolation - 4000 °C in 100 km)

If the heat flux varies in the z direction, the heat entering and exiting a volume with area A and height dz will differ and the volume will heat up; The heat added to that volume per unit time is: A dz dQ/dz which will cause the temperature in that volume to change at the rate of: mass in volume * C_p * dT/dt = ρ A dz C_p dT/dt so

conduction leads to diffusion equation: $\frac{dT}{dt} = \frac{k}{\rho C_p}\frac{d^2T}{dz^2}$

C_p = specific heat: heat needed to raise 1 kg by 1 deg C

Diffusivity => $\kappa = \frac{k}{\rho C_p}$

Note units: m^2/s and typical value $O(10^{-6}\ m^2/s)$
Fundamental result of any solution is that
$\tau = L^2/\kappa$ L = sqrt(κ τ)

Example length-scale/time-scale pairs:

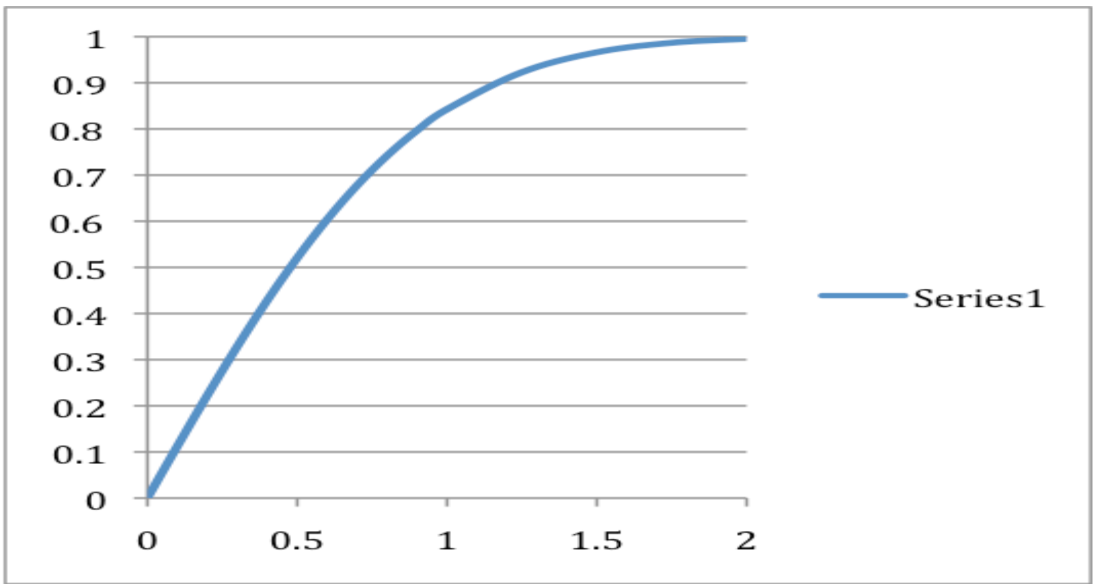
Time	1d	1y	10Kyr	1Myr	100Myr	10Gyr
Length	0.3m	5m	500m	5km	50km	500km

Length	1cm	1m	10m	1km	100km	3000km
Time	100s	12d	3.2yr	32Kyr	320Myr	300Gyr

One important case: instantaneous cooling of a half-space:
Initial Condition (t=0): $T(z,0) = T_m$ for z>0
Boundary Condition(z=0): $T(0,t) = 0$ for t>0

$$T(z,t) = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

This is a plot of erf(x):



Cooling of oceanic lithosphere:

Geotherms, topography, heat flow, hydrothermal circulation

t = distance from ridge/plate velocity

heat flow is vertical

$$T(z,t) = T_m erf\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

$$Q(0,t) = -k\frac{dT}{dz} = -\frac{kT_m}{\sqrt{\pi\kappa t}}$$

$W = const\sqrt{\kappa t}$ seafloor depth relative the depth at ridge comes from isostacy; integral of rock or ocean density from fixed depth in asthenosphere to the sea surface does not depend on x or t above, while the perturbation in density is proportional to T.

Continental Geotherm

Linear relationship between surface heat production and flux is conventional idea

Note -> oceanic (80 mW/m^2) and continental (60 mW/m^2) heat flow
different contributions- radiogenics in relatively old
continental lithosphere vs cooling in younger oceanic lithosphere

Advection and the Adiabatic gradient:

Homogeneous compression causes self-heating the adiabatic gradient is:

Note first that: $\frac{d\ln Y}{dX} = \frac{dY}{Y\,dX}$ and $\frac{d\ln Y}{d\ln X} = \frac{X\,dY}{Y\,dX}$

$$\frac{dT}{dz} = T_o \frac{d\ln T}{d\ln V}\frac{d\ln V}{dP}\frac{dP}{dz} = T_o \frac{\gamma \rho g}{K_s} = T_o g\gamma / (V_p^2 - \tfrac{4}{3}V_s^2)$$

$K_s^{-1} = -\frac{d\ln V}{dP}$; change in volume with pressure = incompressibility

$V_p^2 = (K_s + \tfrac{4}{3}\mu)/\rho$; $V_s^2 = \mu/\rho$; $V_p \approx 8 - 13\, km/s$ in the mantle

$K_s/\rho = V_p^2 - \tfrac{4}{3}V_s^2 \approx \tfrac{5}{9}V_p^2$; $if\ V_p^2/V_s^2 \approx 3$; K_s is the bulk modulus

$\gamma = -\frac{V}{T}\frac{dT}{dV} = -\frac{d\ln T}{d\ln V}$; Gruneisen Parameter $\approx 1\ to\ 1.5$ in the mantle

$P = \rho g z$; $\frac{dP}{dz} = \rho g$; $g \approx 10 ms^{-2}$ throughout the mantle

dT/dz = 2000 K * 10m/s^2 * 1.25 / (5/9*(10000 m/s)^2) = 4.5e-4 deg/m = 0.45deg/km

Gradient is 0.3 to 0.5 °C/km – O(1000°C) temperature difference in mantle

Earth Geotherm

B.C. = $T_{surface}$, T_{core}
Assumptions: conductive boundary layers, nearly adiabatic interior
Consider ICB boundary, CMB, and lithosphere
Note: melting of iron
melting of upper mantle
barriers to convection
other phase transitions

survey from surface to center of Earth