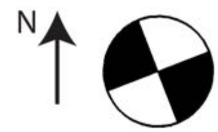
ESS 411/511 Geophysical Continuum Mechanics
Stress Homework

Use the convention that compression is negative and σ_I is the most positive principal stress.

1. Quake!

Let's assume that the accompanying beach-ball solution was released by USGS for a recent earthquake. For discussion of beach balls, see



https://www.usgs.gov/natural-hazards/earthquake-hazards/science/focal-mechanisms-or-beachballs?qt-science_center_objects=0 - qt-science_center_objects

Additional helpful information about beach balls can be found at this PNSN site, https://pnsn.org/outreach/about-earthquakes/focal-mechanisms
and at Charles Ammon's PSU class site
http://eqseis.geosc.psu.edu/cammon/HTML/Classes/IntroQuakes/Notes/notes_framed.html

(a) Fault character
- Is this a strike-slip, thrust-fault, or normal-fault earthquake?
- Is one of the principal stresses vertical? If so, why, and which one?
- Describe the 2 possible fault orientations in 3-D. Why are there 2 possible solutions?
- What can you infer about the direction of the azimuth and dip of the slip vector during the quake?

(b) Suggest how you could resolve this fault-plane ambiguity on future ruptures at this location, if you could quickly deploy a portable seismic network (e.g. approximately a dozen stations).

(c) Suppose you know that this fault has cohesion of 10^6 Pa, and coefficient of sliding friction $\mu = 0.6$.
- Using Mohr's circles, find the azimuth (degrees clockwise from North) of the most compressive principal stress for ***each of the two*** possible fault orientations.
- Also find the azimuth of the least compressive stress.
- ***Draw 2 maps*** showing the 2 solutions for fault planes and horizontal principal stresses.
- Describe your procedures and assumptions.

(d) The directions at 45° to the 2 auxiliary or nodal fault planes are often called the Pressure (P) and tension (T) axes. To get an initial estimate of principal stress directions from focal mechanisms, geophysicists sometimes assume that the principal stress directions are aligned with the P and T axes.
- Is this consistent with the assumptions in part (c)? Why or why not?

2. The rocks are frackin' crackin'!

In the vicinity of a borehole, the least-compressive horizontal stress is $\sigma_h = -10^7$ Pa, and the most-compressive horizontal stress is $\sigma_H = -2\times10^7$ Pa. Shear failure of the rock can be described with a Coulomb model, with no cohesion, and□ with $\mu=1$. The rock has zero strength in tension. The intermediate principal stress σ_{II} is vertical.

(a) If you were to apply a Coulomb failure model, at what fluid pressure p_f might you expect to first create new shear cracks in the borehole wall?

(b) Now you perform a hydrofracture experiment, and you don't see cracks *opening up* at this p_f. Why is Coulomb failure a poor model for a hydrofracturing test in a borehole?

(c) At what value of p_f might you first see cracks actually *opening up* in the borehole wall? Why?

(d) When the pumping pressure reaches the breakdown pressure, the flux of water into the hole increases dramatically, and it is difficult to raise the fluid pressure any further by further pumping.
- What is the value of the breakdown pressure?
- Explain in words what is happening in the rock surrounding the hole.

3. Icebergs and mountain ranges

A tabular iceberg with a flat top and flat bottom is floating in the Southern Ocean off Antarctica. The iceberg has thickness H, and width W on a side. (Typically, $H \approx 300$ m and $W \approx 10$ km or more, so $H/W \ll 1$.) Its density is $\rho_i = 900$ kg m^{-3}, and we assume that it has a uniform temperature throughout (an OK place to start, even if not technically true ☺). The density ρ_w of seawater is 1000 kg m^{-3} (approximately). The acceleration due to gravity is g = 9.8 m s^{-2}. The water below and beside the iceberg is stationary, i.e. there is no horizontal pressure gradient driving horizontal flow in the seawater.

(Please don't use these special-case numerical values unless you are asked to do so – a general formula gives more insight than a special numerical case. Give your answers in terms of symbolic variables H and W, g, ρ_i *etc.*).

(a) *Stress in the water*

- Pressure increases with depth in the ocean due to the increasing weight of overlying liquid. Derive an expression for how the pressure varies in the water around the iceberg.
- Write down the stress tensor in the water in a coordinate system in which z is vertical. Note that the components may be functions of position.

(b) *Flotation condition*

When the iceberg is floating, the weight of ice overlying each square meter of its base must equal the water pressure at the base.

- Find the water line on the iceberg, i.e. what fraction $(H-h)/H$ of its thickness H is below water?

(c) *Traction from seawater*

- Describe the distribution of traction applied to the edges of the iceberg by the sea water.
- What is the net horizontal force $\boldsymbol{F}_{sw}$ applied to the right-hand vertical surface of the iceberg (of area HW) by the seawater?

(d) *Vertical stress in the iceberg*

Assuming that vertical stress in the iceberg is also primarily due to overlying ice,

- How does σ_{zz} vary along a vertical plane through the iceberg (see dashed line in figure), far enough back from the free edge such that the edge does not significantly alter the stress field there?
- Find the depth-averaged value $\overline{\sigma}_{zz}$ of σ_{zz}.

(e) *Horizontal force balance in the iceberg*

Because forces and torques on the iceberg are balanced, the total horizontal force on any vertical plane through the iceberg must balance the total horizontal force exerted on the edge by the sea water. Consider the portion of the iceberg lying between the dotted plane and the right hand edge.

- Find an expression for the traction vector $\boldsymbol{t}(y,z)$ applied to the negative side of the dotted plane, in terms of the (still unknown) stress distribution $\sigma_{xx}(x,y,z)$.
- Write the horizontal component F_{int} of the total force acting on the dotted plane due to the stress σ_{xx} in terms of $\overline{\sigma}_{xx}$, the depth-averaged value of the (still unknown) normal stress σ_{xx}.
- Equate the sum of all horizontal forces to zero, and solve for $\overline{\sigma}_{xx}$.

(f) *Stress difference and its implications*

- Compare the depth-averaged values $\overline{\sigma}_{xx}$ and $\overline{\sigma}_{zz}$, and speculate how the iceberg might deform.
- Do you think that this model could also to applied to mountain ranges, which ``float" in the asthenosphere? Why or why not?