ESS 411/511 Geophysical Continuum Mechanics Class #3

Highlights from Class #2 – Maleen Kidiwela
Today's highlights on Wednesday? – Andrew Gregovich

Warm-up question (break-out) – Units

- What are the SI units for stress?
- For strain?
- For strain rate?

Class-prep answers (break-out)

- Creep tests
- Relaxation tests

Questions about Break-out rooms

- Introductions?
- Number of participants?
- Duration?
- Choose an identity?

ESS 411/511 Geophysical Continuum Mechanics Class #3

For Wednesday class

- Read Raymond Notes Ch 2, through 2.9 (energy and energy loss).
- (Focus on the 1-D model descriptions, not the Earth properties yet)

I am also preparing a short CR/NC writing assignment (1 point) in Canvas (Assignment Group - Pre-class prep).

- It will be due in Canvas at the start of class.
- I anticipate the whole thing will be around ~half a page.
- The goal to help us get into the topic, and the points from this and similar exercises will contribute to class participation grades.
- The writing assignment should help you to be prepared to discuss the key features about energy loss during strain.

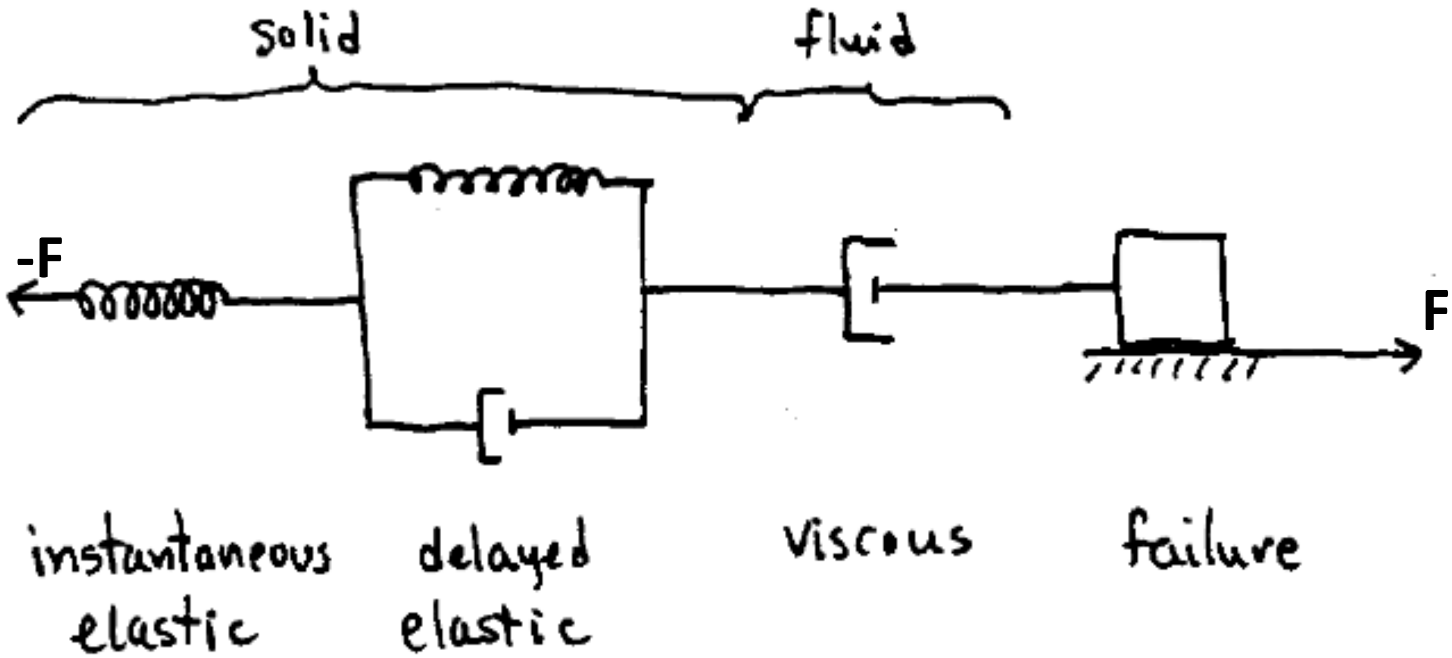
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics Class #3

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr’s circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

A model for idealized real materials



Forces are balanced

- Each element feels the same force **F**

Rheological tests

Creep tests

- Apply a constant stress σ
  e.g. put a weight on top of a sample
- Measure strain *e(t)* or strain rate $\dot{e}(t)$

Relaxation tests

- Apply an abrupt strain *e,* then hold it constant
  e.g. abrupt shortening in a vice.
- Measure stress $\sigma(t)$ as sample adjusts.

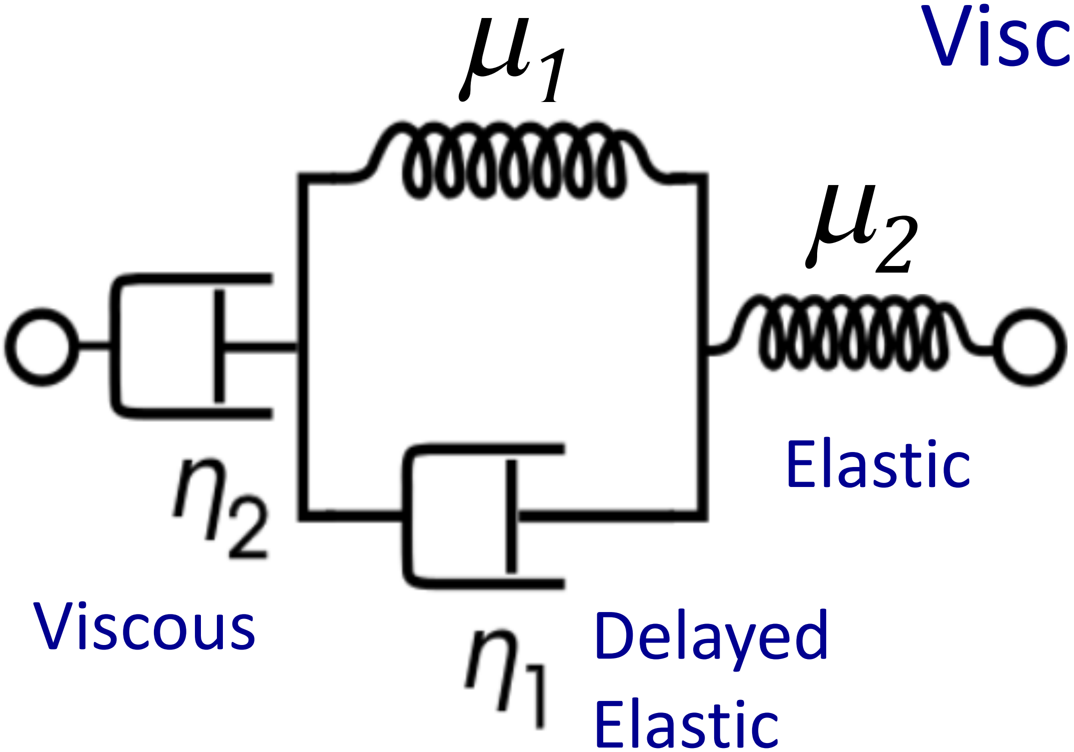
Constant strain-rate tests

- Apply a constant strain rate
  e.g. with a motor-driven vice
- Measure stress $\sigma(t)$

Models for linear solids

Those springs and dashpots …

Viscoelastic model

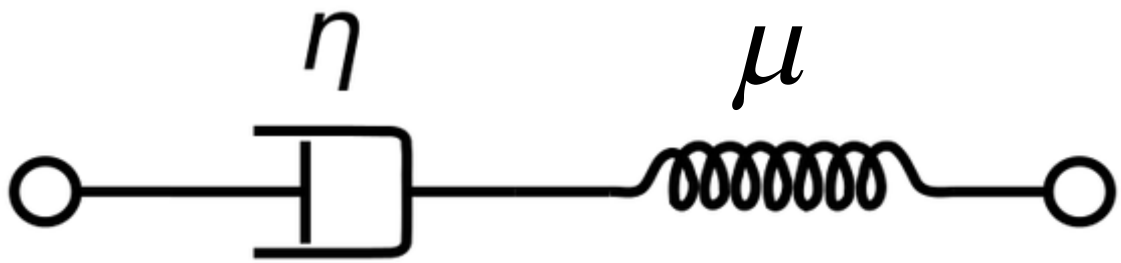


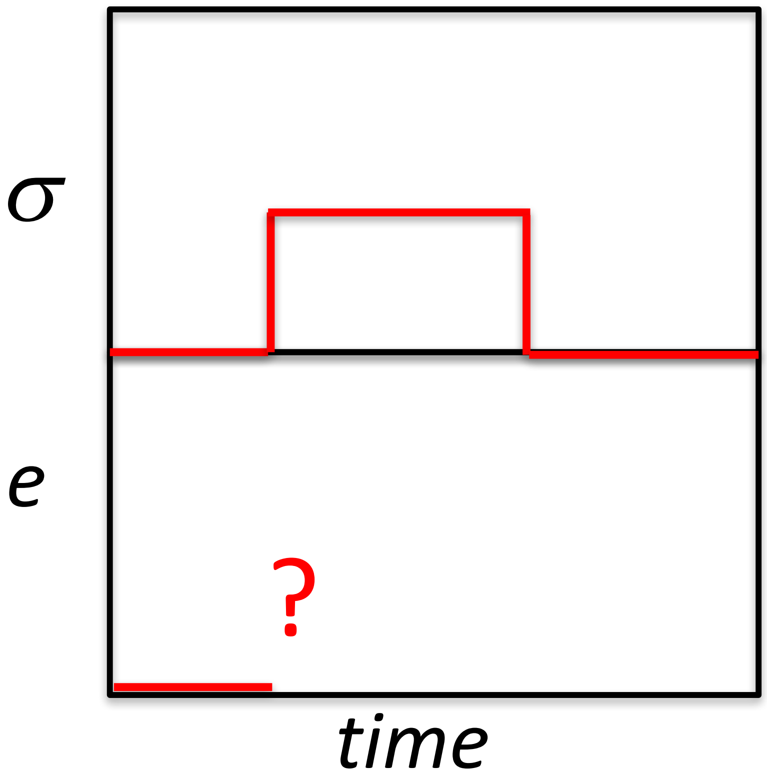
Called *Maxwell Solid*, if $\eta_1 = \infty$, $\mu_1 = \infty$

Called *Kelvin-Voigt Solid*, if $\eta_2 = \infty$, $\mu_2 = \infty$

Called *Standard Linear Solid*, if $\eta_2 = \infty$

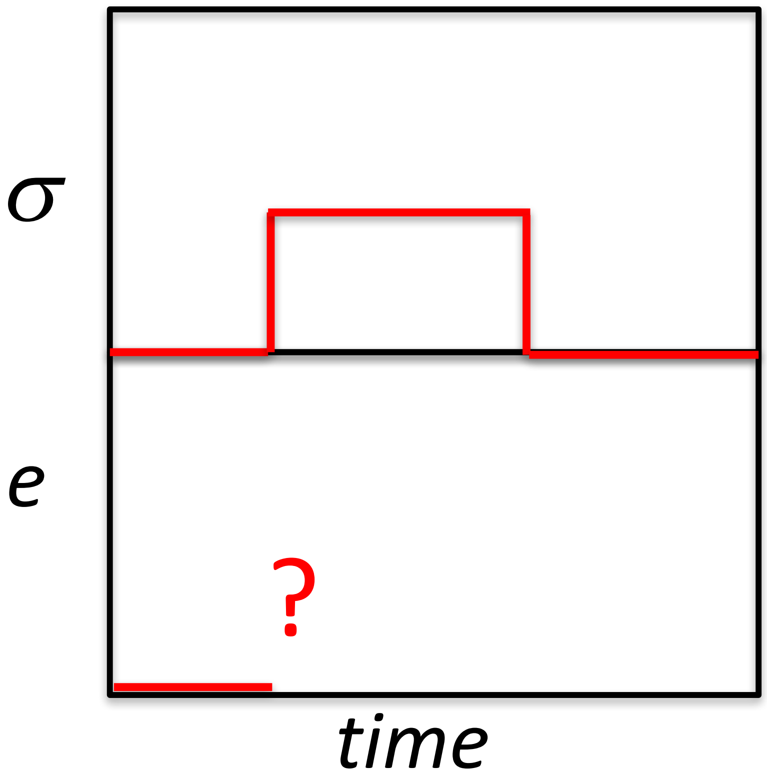
Maxwell solid

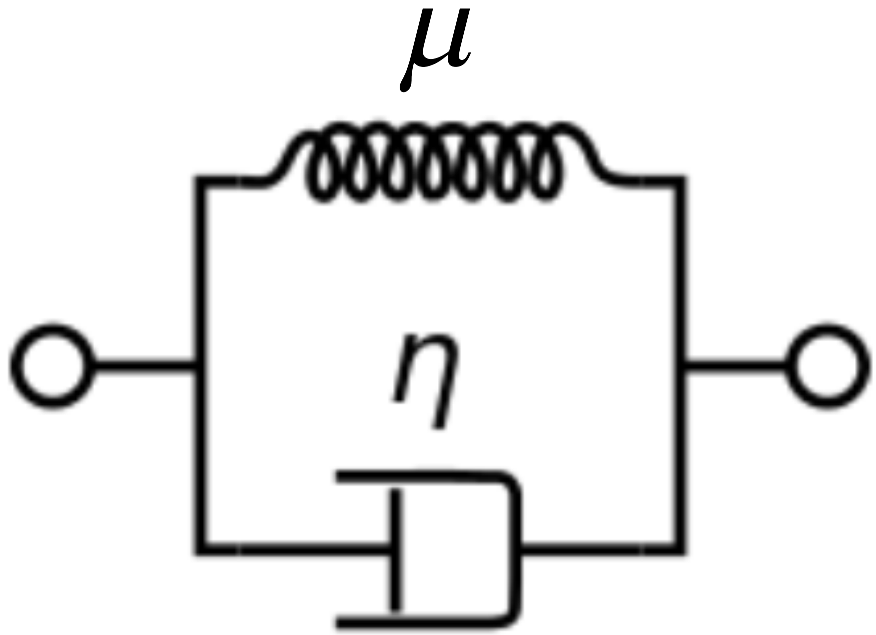


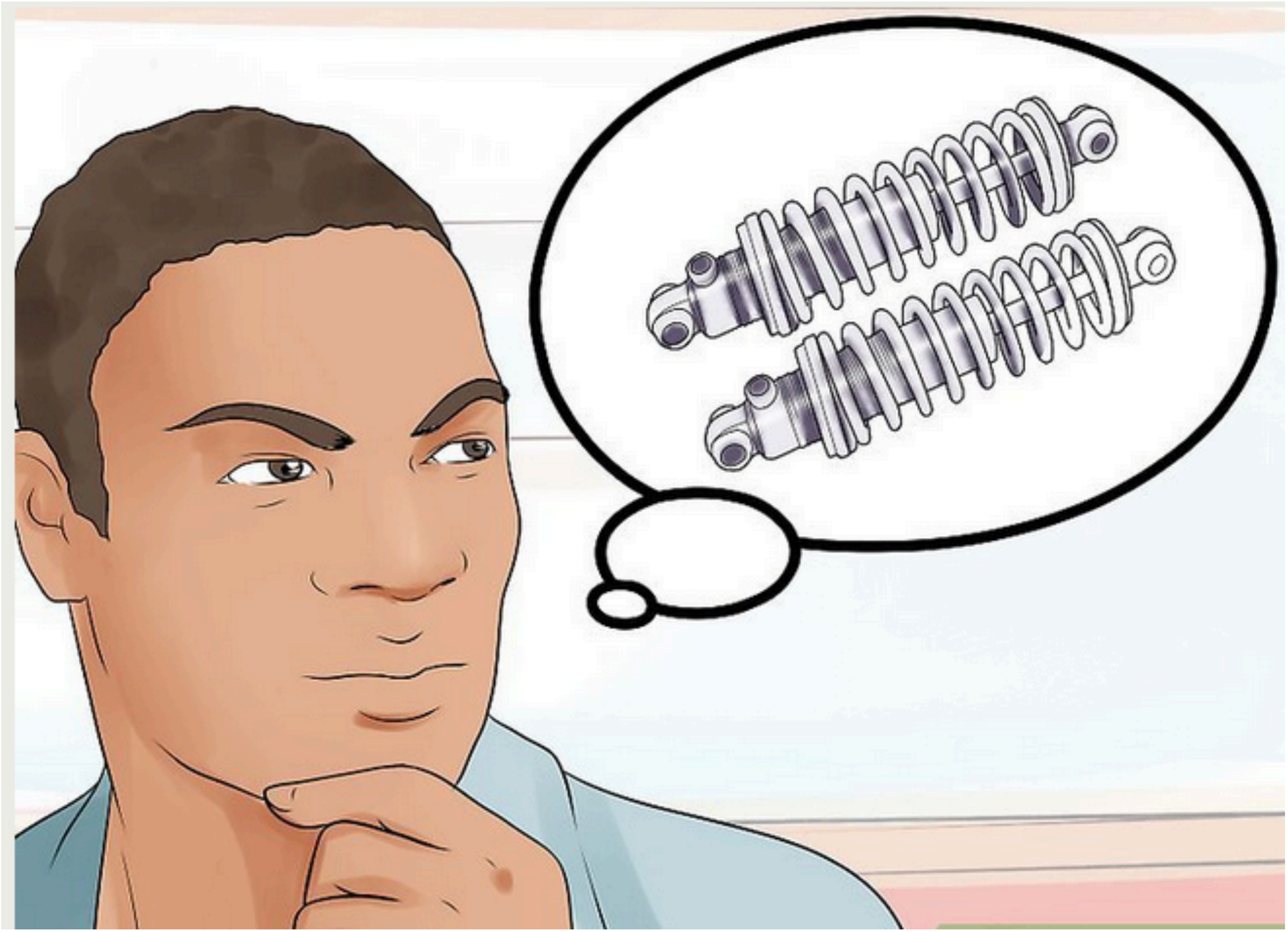


Also Homework set #1

Kelvin-Voigt solid



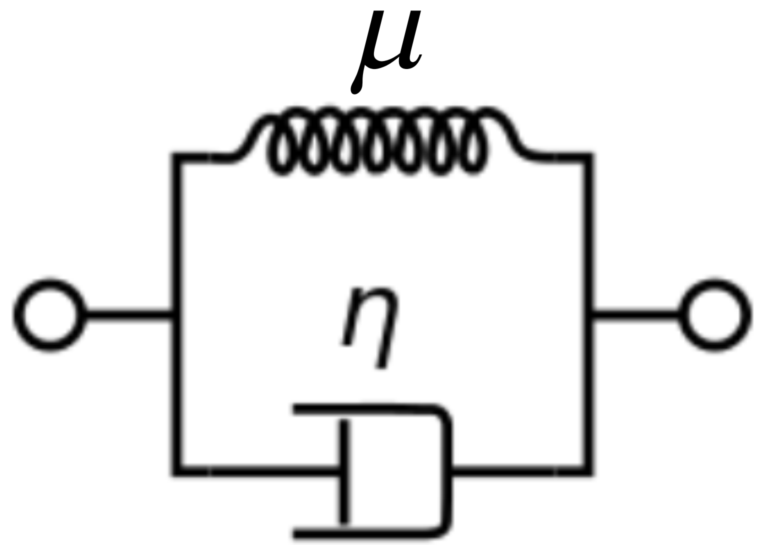


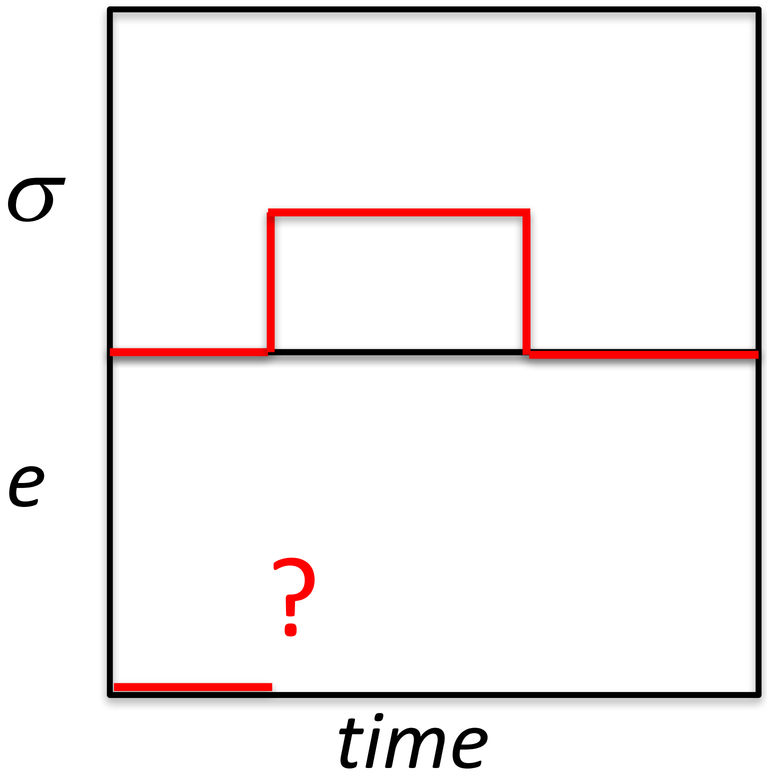


Kelvin-Voigt element - delayed elasticity

- the shock absorber in my car?!

Kelvin-Voigt solid



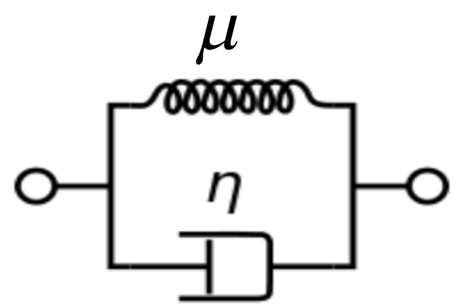


Let's figure out how this solid responds to constant load σ

…

- What are units of elasticity μ and viscosity η?
- Is there a characteristic time for the material?

Kelvin-Voigt Response



Spring and dashpot together support stress σ

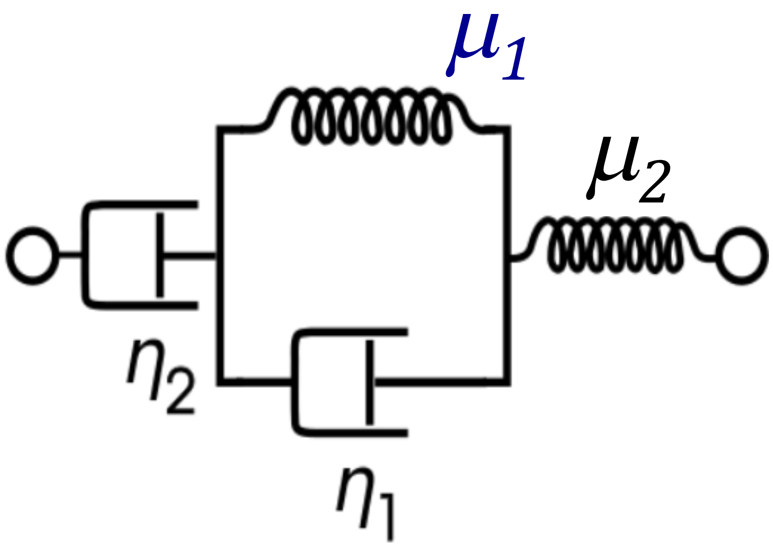
$$\sigma(t) = \mu\ e(t) + \eta\ \dot{e}(t)$$

- At $t = 0$, spring hasn't shortened; dashpot supports all the stress σ , so
  $e(0) = 0 \quad (*)$
- At $t = \infty$, dashpot has stopped; spring supports all the stress σ, so
  $e(\infty) = \sigma/\mu\ (**)$
- The transition is probably a decaying exponential.
- $\tau=\eta/\mu$ must be the time constant defining the transition.

$$e(t) = \frac{\sigma}{\mu} + \mathrm{A}\exp\left(-\frac{\eta}{\mu}t\right)$$

With the boundary conditions (*) and (**), A can be found, and solution is …

$$e(t) = \frac{\sigma}{\mu}\left[1 - \exp\left(-\frac{\mu}{\eta}t\right)\right]$$



Viscoelastic Response to constant loading σ

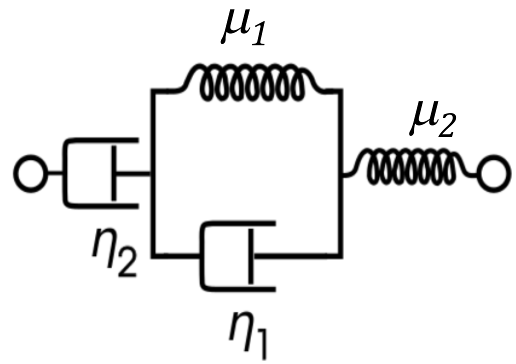
$$e(t) = \frac{\sigma}{\eta_2}t + \frac{\sigma}{\mu_2} + \frac{\sigma}{\mu_1}\left[1 - \exp\left(-\frac{\mu_1}{\eta_1}t\right)\right]$$

Viscous

Elastic

Delayed Elastic

How did we get that?!



$$e(t) = \frac{\sigma}{\eta_2}t + \frac{\sigma}{\mu_2} + \frac{\sigma}{\mu_1}\left[1 - \exp\left(-\frac{\mu_1}{\eta_1}t\right)\right]$$

Viscous Elastic Delayed Elastic

Each element feels the same stress σ,

- We just added up the strains in each element

Energy and Work

Work for point particles: W = **F** *d*

In Continuum – work per unit volume:

$$W/V = F\,d/V$$
$$= (F\,d) / (A\,l)$$
$$= (F/A)\ (d/l)$$
$$= \sigma\,e$$
$$= \text{stress x strain}$$