ESS 411/511 Geophysical Continuum Mechanics Class #5

Highlights from Class #4 – Anna Ledeczi
Today's highlights on Monday – Peter Lindquist

Remember we are looking for just 2 or 3 *highlights*, not a transcript of the entire class. (What did you think was most important?)

Warm-up question (break-out)

- Ground displacements (and initial wave amplitudes at the epicenter of an earthquake) can be a meter or more.
- Why are amplitudes only cm or mm (or less) when the waves arrive at Seattle? (at least two reasons)

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Class-prep answers (break-out)

- In a visco-elastic material subjected to oscillatory stress at frequency ω, why is attenuation low at "high" and "low" frequencies, but higher at mid-range ($\omega\tau$ ~ 1)
- What is τ?

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For Monday class

- Please read Mase, Smelser, and Mase, CH 2 through Section 2.3

Class-prep assignment Class_06: Second-order tensors

A force can be represented by a vector, with a magnitude and a direction. However, a stress (which is the continuum analog) cannot be represented by a vector. Explain in a few qualitative sentences (maximum) why not.

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Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Problem Set #1

- How did it go yesterday?
- Questions about the actual problems?
- Suggestions to make the sessions better?

Energy and Work

Work W is force ***F*** acting through a distance *d*

Work for point particles: W = ***F*** *d*

In Continuum – work done per unit volume:

$$\frac{W}{V} = \frac{Fd}{V} = \left(\frac{F}{A}\right) \bullet \left(\frac{d}{l}\right) = \sigma e$$ *= stress × strain*

Rate of doing work per unit volume

$$\frac{d}{dt}\left(\frac{W}{V}\right) = \frac{\dot{W}}{V} = \frac{F\dot{d}}{V} = \left(\frac{F}{A}\right) \bullet \left(\frac{\dot{d}}{l}\right) = \sigma \dot{e}$$ *= stress × strain rate*

(overdots indicate time derivatives)

Energy and Work

Total energy input between from time 0 to t

$$\Delta E(t) = \int_0^t \sigma(t')\dot{e}(t')dt'$$

For elastic material, substitute: $\sigma(t) = \mu e(t)$

$$\Delta E(t) = \int_0^t \mu e(t')\dot{e}(t') \, dt' = \frac{1}{2}\mu e^2(t) = \frac{\sigma^2(t)}{2\mu}$$

$\Delta E(t)$ returns to zero whenever σ returns to zero.

- All energy is recovered

Energy and Work

Total energy input between from time 0 to *t*

$$\Delta E(t) = \int_0^t \sigma(t')\dot{e}(t')dt'$$

For viscous material:

$$\sigma(t) = \eta\dot{e}(t)$$

$$\Delta E(t) = \int_0^t \eta\dot{e}(t)^2 dt'$$

The integrand is always positive

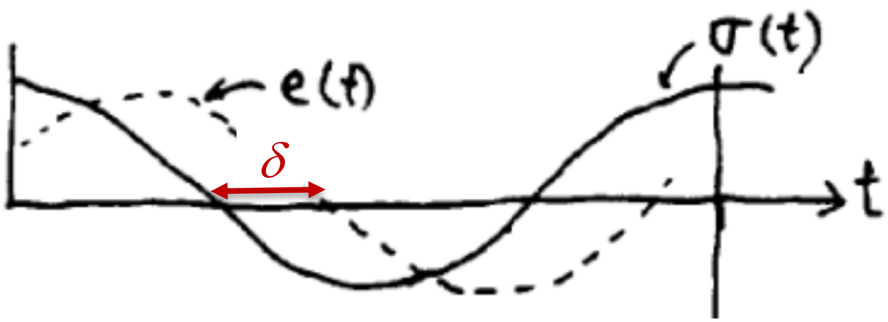
- $\Delta E(t)$ can never return to zero if strain rate is ever nonzero
- energy is always lost if any strain has occurred.

Harmonic stress loading

Stress: $\sigma(t) = \sigma_0 \, e^{\,i\omega t}$ (σ_0 is real)

Response is also harmonic (but δ is a phase lag)*

$$e(t) = e_0 \, e^{\,i\omega t} = (|e_0| e^{\,i\delta}) e^{\,i\omega t}$$



Complex modulus

$$m \equiv \frac{\sigma_o}{e_o} = |m| \, e^{i\delta}$$

$$\tan\delta = Im(m)/Re(m)$$

*Notation challenge –
be aware that *e* can be either strain or natural base

Energy in harmonically driven material

$$E(t) = E_0 + \Delta E(t)$$

Energy input from time 0 to t

$$\Delta E(t) = \int_0^t \sigma(t')\dot{e}(t')dt'$$

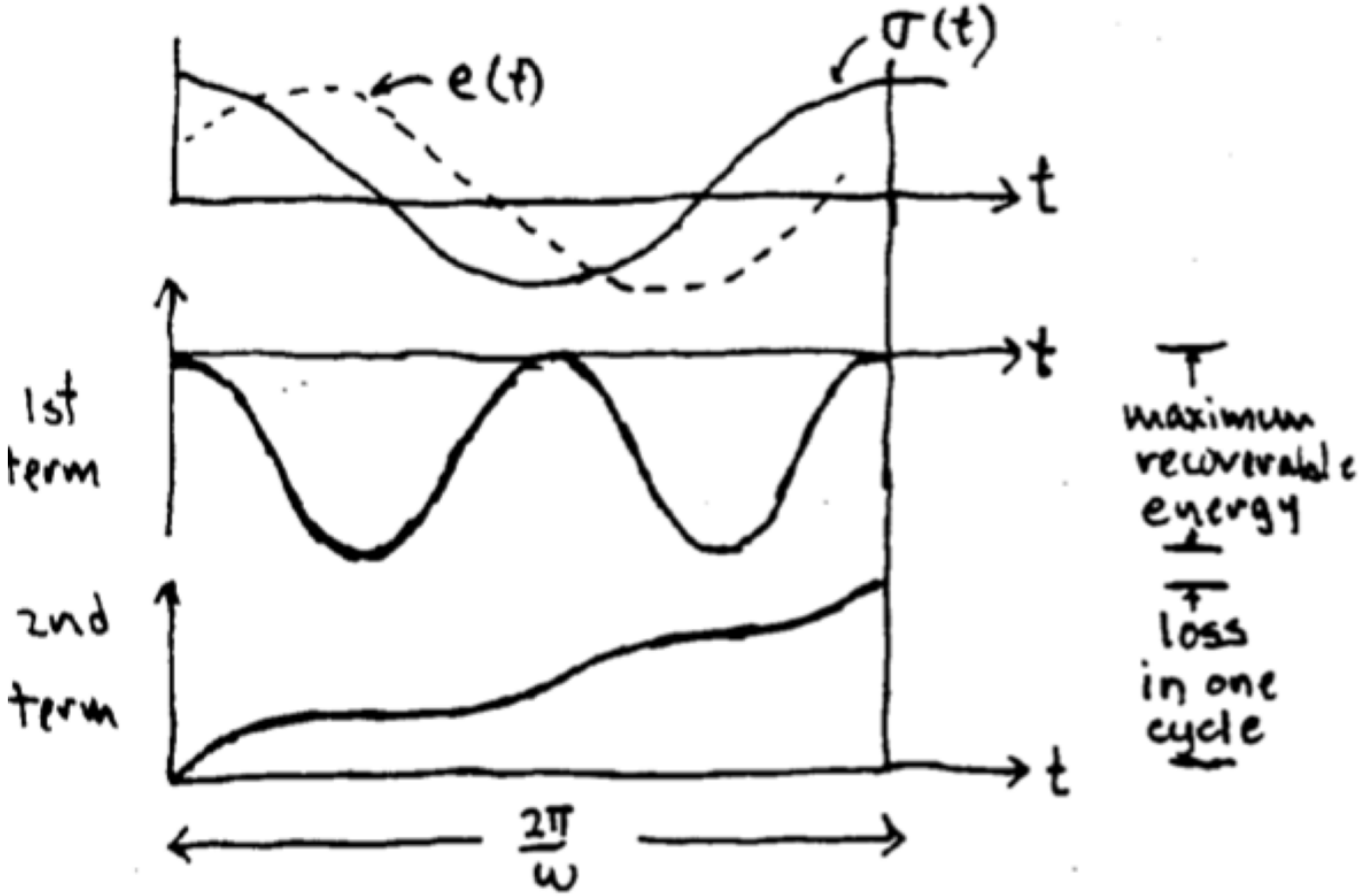
After breaking exponentials into cos and sin

$$\exp(-iy) = \cos(y) - i\sin(y)$$

$$E(t) = -\frac{w\sigma_o^2}{|m|}\int_0^t \underbrace{\cos wt'}_{\sigma} \underbrace{\sin(wt' - \delta)}_{e=|e_0|e^{i\delta}}dt' + E_o$$

$$= \frac{\sigma_0^2}{4|m|}\underbrace{(\cos 2wt - 1)\cos\delta}_{\text{Oscillatory, storage}} + \frac{w\sigma_o^2}{|m|}\underbrace{\int_0^t \cos^2 wt'dt' \sin\delta}_{\text{Monotonic, dissipation}} + E_o$$

Energy change over time

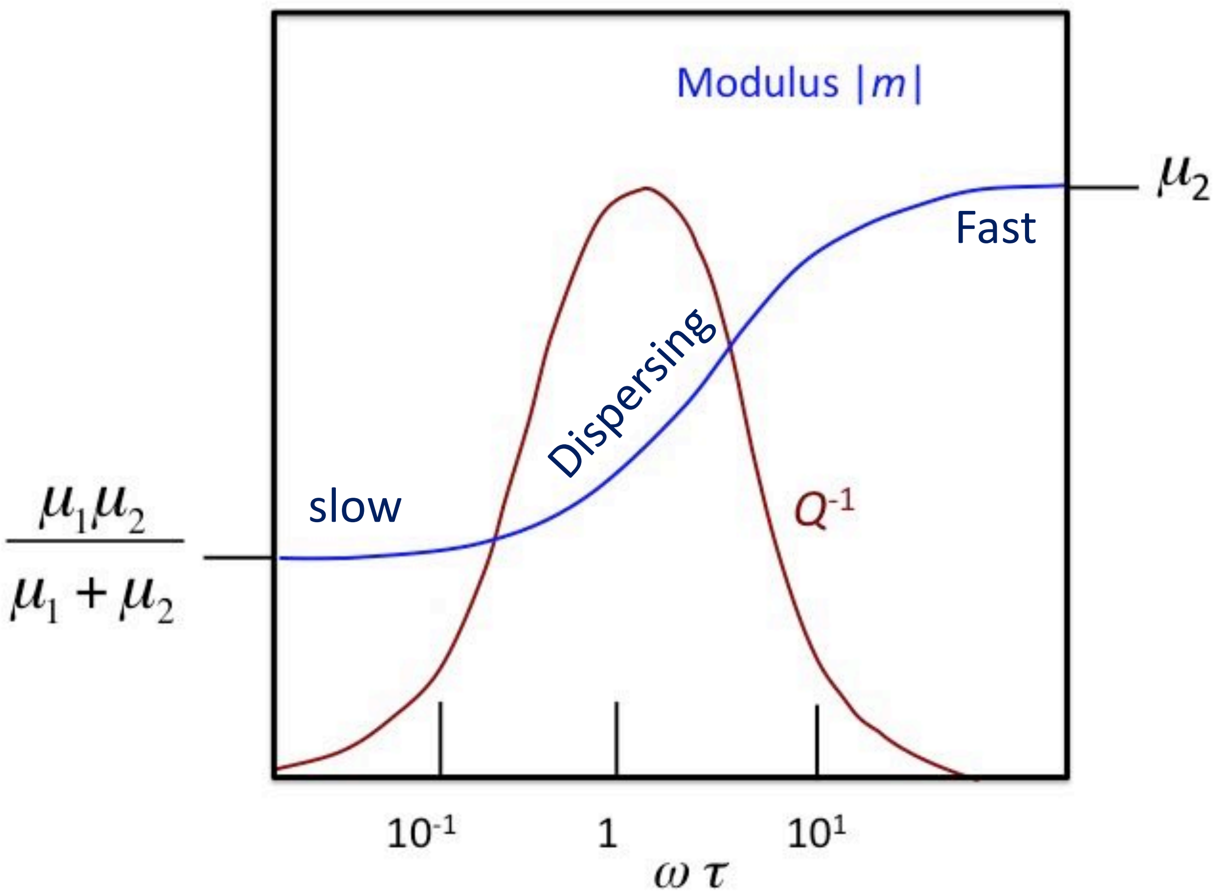


$$E = \frac{\sigma_0^2}{2|m|}$$

$$\Delta E = \frac{\sigma_0^2}{|m|} \pi \sin \delta$$

Debye Dispersion in Harmonic loading

- Elastic wave speed is proportional to elastic modulus $|m|$
- When different frequencies in a wave packet travel at different speeds, the packet breaks up or *disperses*



$$Q^{-1} = \frac{\Delta E}{2\pi E} = \tan\delta$$

What is τ?

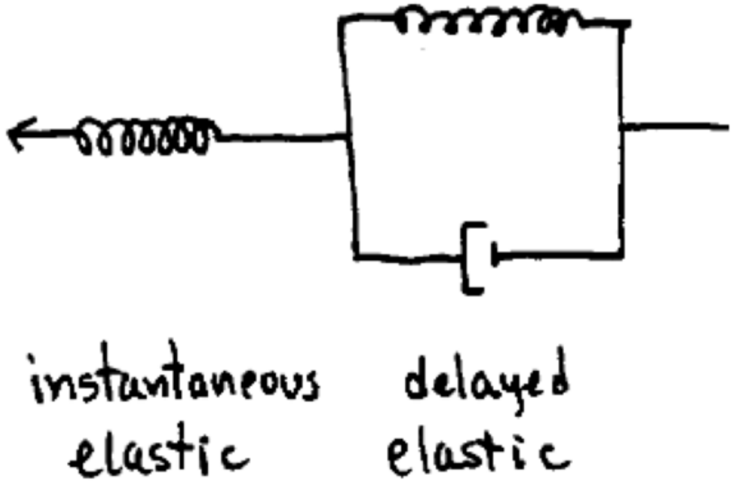
$$\tan\delta = \frac{\tau_c - \tau_r}{\tau}\frac{w\tau}{1 + w^2\tau^2}$$

where $\tau_c = \eta_1/\mu_1$ is the characteristic time in a creep experiment (Eq. 2.1), $\tau_R = \eta_1/(\mu_1 + \mu_2)$ is the characteristic time in a relaxation experiment (Eq. 2.2), and

$$\tau = \sqrt{\tau_r\tau_c}$$

which is the geometric mean of the characteristic times for the creep and relaxation experiments. Also

Why is there a maximum in dispersion?



What happens at low frequency $\omega\tau << 1$?

Strain response $$\sigma\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) = \sigma/\left(\frac{\mu_1\mu_2}{\mu_1 + \mu_2}\right)$$

What happens at high frequency $\omega\tau >> 1$?

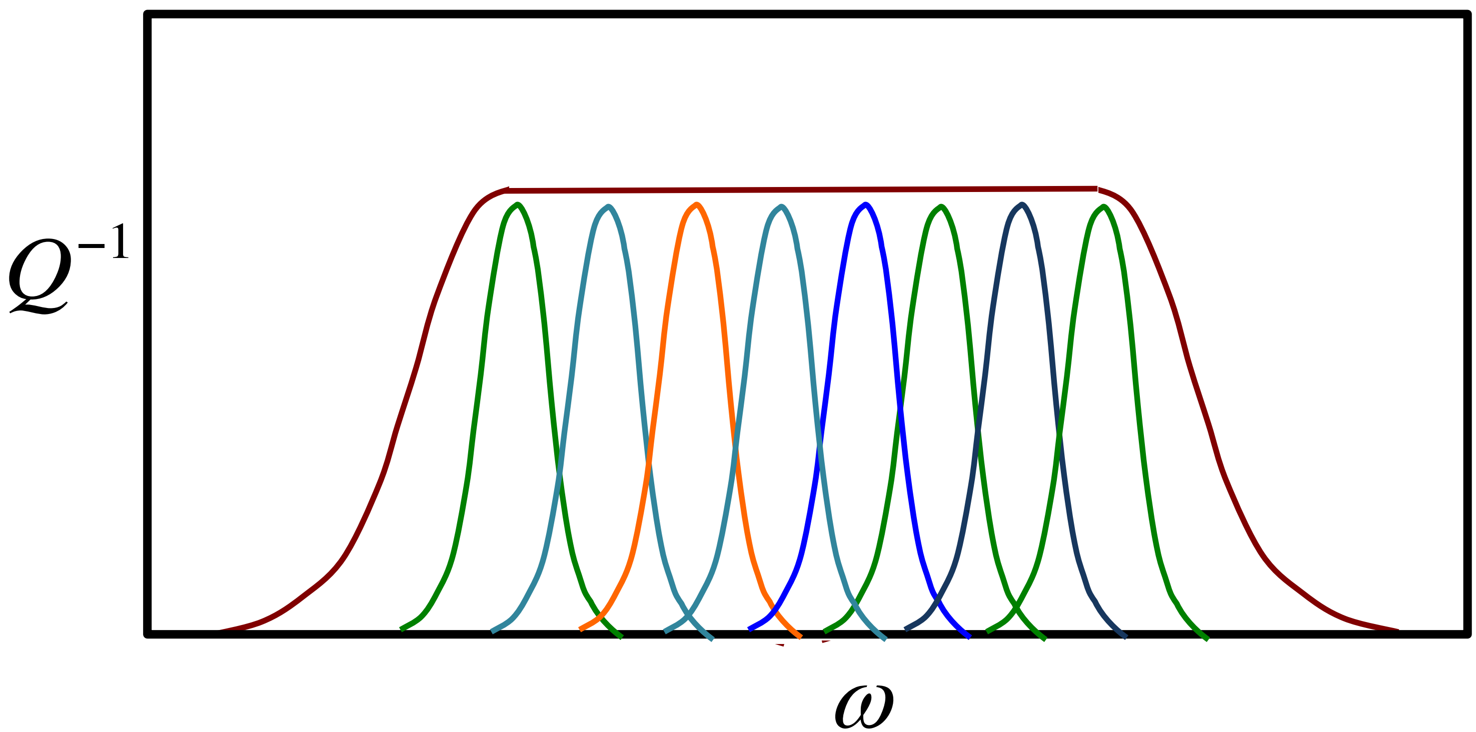
Strain response σ/μ_2

What happens in the middle range $\omega\tau \sim 1$?

Attenuation in real rocks

There is a broad plateau of low *Q* over several decades of frequency ω

- Probably a sum of various dissipative processes centered on different frequencies.
    - Dislocations inside crystals
    - Interactions among adjacent crystals
    - Pore fluids



Earthquake waves

Peak-to-peak amplitude and energy density decrease with distance from source, due to 2 processes.

(1) Geometrical spreading with distance *r* from source
- In body waves (p & s) energy is spread over expanding spherical wave front, so wave-front area increases as r^2 for
  - Energy decreases as $1/r^2$
- In surface waves (Love, Rayleigh, etc) energy is spread over expanding cylindrical wave front, so wave-front area increases as r .
  - Energy decreases as $1/r$

Earthquake waves

Peak-to-peak amplitude and energy density decrease with distance from source, due to 2 processes.

(2) Intrinsic attenuation quantified by Quality factor Q

$$Q^{-1} = \frac{\Delta E}{2\pi E}$$

- Q^{-1} is fractional energy lost per cycle of the wave.
- For most rocks, Q >> 1 (~30 - 80 is typical)

Amplitude *A(r)* of a seismic wave:

$$A(r) = A_0 \exp\left(-\frac{\omega r}{2cQ}\right)$$

- c is wave speed (e.g. 8 km/s for mantle p-waves)
- ω is angular frequency ($\omega = 2\pi f$)
- Let's use Q=75, r = 100 km, and f=1 & 10 Hz

Results:

f = 1 Hz: A = 0.59 A_0

f = 10 Hz: A = 0.0053 A_0

So a 1 Hz wave loses 40% of its amplitude for every 100 km it travels, while a 10 Hz wave loses 99.5% of its amplitude every 100 km

- **We don't see much energy at 10 Hz from teleseisms!**