ESS 411/511 Geophysical Continuum Mechanics Class #7

Highlights from Class #6 – Madeline Mamer
Today's highlights on Friday – Abigail Thienes

Not just AGI

In recognition of Earth Science Week (11-17 October 2020), we have collected some articles centered on our ever-evolving planet, covering volcanology, geology, climate science, and additional Earth-related news and research.

SCIENCE is a leading outlet for scientific news, commentary, and cutting-edge research across the sciences, with articles that consistently rank among the most cited in the world. **SCIENCE ADVANCES** is our open access multidisciplinary journal, publishing impactful research papers and reviews in any area of science, in both disciplinary-specific and broad, interdisciplinary areas. **NEWS FROM SCIENCE** is our award-winning daily news site bringing you breaking news from the world of scientific research and science policy. The **INSIGHTS** section of **SCIENCE** consists of Perspectives, Policy Forums, Book Reviews, and Letters that provide fresh viewpoints to drive fields forward and to facilitate interdisciplinary discussion and decision-making.

https://sci.scientific-direct.net/view_online.asp?1651331&bd53762d1b456a39&18

RESEARCH

Science

Report
Seismic ocean thermometry
Wenbo Wu *et al.*
Science 18 Sep 2020: Vol. 369,
Issue 6510, pp. 1510-1515
DOI: 10.1126/science.abb9519

Research Article
Strengthening of the Kuroshio current by intensifying tropical cyclones
Yu Zhang *et al.*
Science 09 May 2020: Vol. 368,
Issue 6494, pp. 988-993
DOI: 10.1126/science.aax5758

Report
Large contribution from anthropogenic warming to an emerging North American megadrought
A. Park Williams *et al.*
Science 17 Apr 2020: Vol. 368,
Issue 6488, pp. 314-318
DOI: 10.1126/science.aaz9600

Report
Colorado River flow dwindles as warming-driven loss of refelctive snow energizes evaporation
P.C.S. Milly *et al.*
Science 13 Mar 2020: Vol. 367,
Issue 6483, pp. 1252-1255
DOI: 10.1126/science.aay9187

INSIGHTS

PERSPECTIVES

Volcanology
Mount St. Helens at 40
Jon J. Major
Science 15 May 2020: Vol. 368,
Issue 6492, pp. 704-705
DOI: 10.1126/science.aaz7126

Volcanology
Calderas collapse as magma flows into rifts
Freysteinn Sigmundsson
Science 06 Dec 2019: Vol. 366,
Issue 6470, pp. 1300-1201
DOI: 10.1126/science.aaz7126

BOOKS ET AL.

Earth Sciences
The Ice at the End of the World: An Epic Journey into Greenland's Buried Past and Our Perilous Future
Jon Gertner Random House, 2019. 445 pp
A remote region, revealed
William E. Glassley
Science 28 Jun 2019: Vol. 364,
Issue 6447, pp. 1241
DOI: 10.1126/science.aax4901

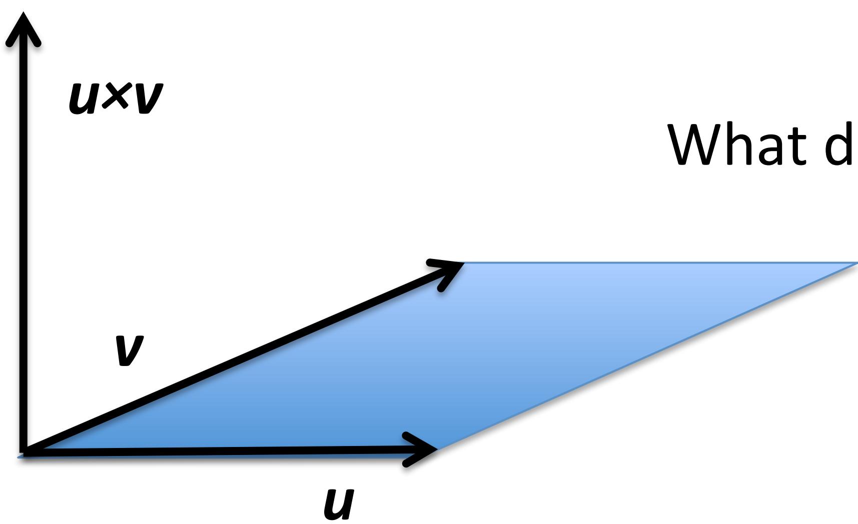
NEWS

Europe builds 'digital twin' of Earth to hone climate forecasts

Class-prep – Vector product

cross (vector) product of two vectors, Eq 2.12:

$$\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u} = (uv \sin\theta)\,\hat{\mathbf{e}} = \varepsilon_{ijk} u_i v_j \hat{\mathbf{e}}_k$$



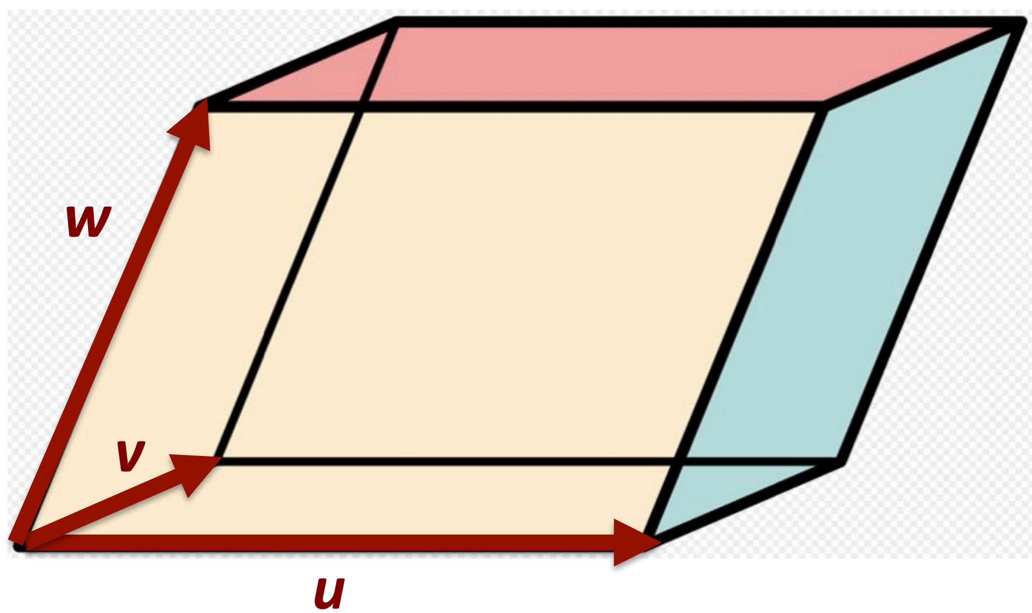
What does ***u×v*** measure?

Warm-up question – Box product

triple scalar product (box product), Eq 2.13:

$$
\begin{aligned}
[\mathbf{u},\mathbf{v},\mathbf{w}] &= u_i\hat{\mathbf{e}}_i \cdot (v_j\hat{\mathbf{e}}_j \times w_k\hat{\mathbf{e}}_k) = u_i\hat{\mathbf{e}}_i \cdot \varepsilon_{jkq} v_j w_k \hat{\mathbf{e}}_q \\
&= \varepsilon_{jkq} u_i v_j w_k \delta_{iq} = \varepsilon_{ijk} u_i v_j w_k
\end{aligned}
$$

What does [***u,v,w***] measure?



https://en.wikipedia.org/wiki/Parallelepiped

ESS 411/511 Geophysical Continuum Mechanics

For Friday class

- Please read Mase, Smelser, and Mase, CH 2 through Section 2.6

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas

- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Problem Sets

- Problem Set #1 – due today
- Problem Set #2 – work in Lab on Thursday

Kronecker delta

Since the base vectors $\hat{\mathbf{e}}_i$ ($i = 1,2,3$) are unit vectors and orthogonal

$$\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j = \begin{cases} 1 & \text{if numerical value of } i = \text{numerical value of } j \\ 0 & \text{if numerical value of } i \neq \text{numerical value of } j \end{cases}$$

Therefore, if we introduce the Kronecker delta defined by

$$\delta_{ij} = \begin{cases} 1 & \text{if numerical value of } i = \text{numerical value of } j \\ 0 & \text{if numerical value of } i \neq \text{numerical value of } j \end{cases}$$

we see that

$$\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j = \delta_{ij} \qquad (i, j = 1, 2, 3) \ .$$

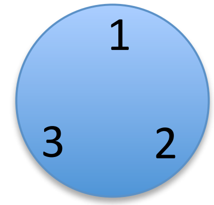
Note that δ_{ij} is like a 3x3 identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the value of δ_{jj} ?

Permutation symbol

Imagine the numbers 1, 2, 3 are written clock-wise around the circumference of a wheel



$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if the numerical values of } i,j,k \text{ are in clockwise order} \\ -1 & \text{if the numerical values of } i,j,k \text{ are anti-click-wise order} \\ 0 & \text{if the numerical values of } i,j,k \text{ are in any other order} \end{cases}$$

Cross products of basis vectors

$$\hat{e}_i \times \hat{e}_j = \varepsilon_{ijk}\hat{e}_k \qquad (i, j, k = 1, 2, 3)$$

Switching indices

$$\varepsilon_{ijk} = -\varepsilon_{kji} = \varepsilon_{kij} = -\varepsilon_{ikj}$$

Determinants

$$\det \mathcal{A} = |\mathcal{A}_{ij}| = \begin{vmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{A}_{23} \\ \mathcal{A}_{31} & \mathcal{A}_{32} & \mathcal{A}_{33} \end{vmatrix}$$

$$\begin{aligned} \det \mathcal{A} &= \mathcal{A}_{11} \begin{vmatrix} \mathcal{A}_{22} & \mathcal{A}_{23} \\ \mathcal{A}_{32} & \mathcal{A}_{33} \end{vmatrix} - \mathcal{A}_{12} \begin{vmatrix} \mathcal{A}_{21} & \mathcal{A}_{23} \\ \mathcal{A}_{31} & \mathcal{A}_{33} \end{vmatrix} + \mathcal{A}_{13} \begin{vmatrix} \mathcal{A}_{21} & \mathcal{A}_{22} \\ \mathcal{A}_{31} & \mathcal{A}_{32} \end{vmatrix} \\ &= \mathcal{A}_{11} \left(\mathcal{A}_{22}\mathcal{A}_{33} - \mathcal{A}_{23}\mathcal{A}_{32} \right) - \mathcal{A}_{12} \left(\mathcal{A}_{21}\mathcal{A}_{33} - \mathcal{A}_{23}\mathcal{A}_{31} \right) \\ &\quad + \mathcal{A}_{13} \left(\mathcal{A}_{21}\mathcal{A}_{32} - \mathcal{A}_{22}\mathcal{A}_{31} \right) \; . \end{aligned}$$

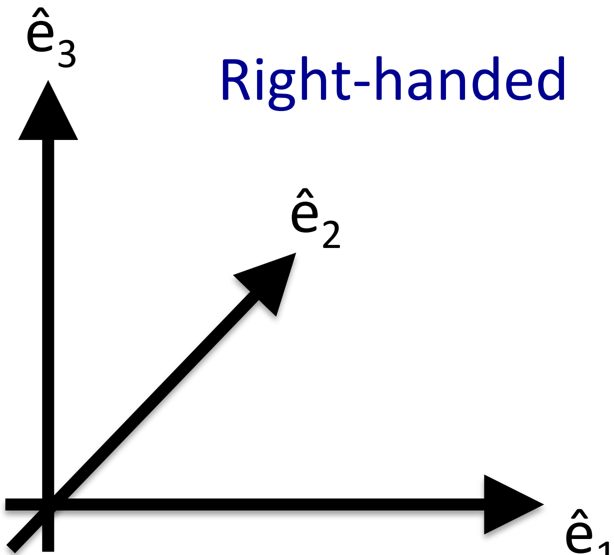
$$\varepsilon_{qmn} \det \mathcal{A} = \varepsilon_{ijk} \mathcal{A}_{iq} \mathcal{A}_{jm} \mathcal{A}_{kn}$$

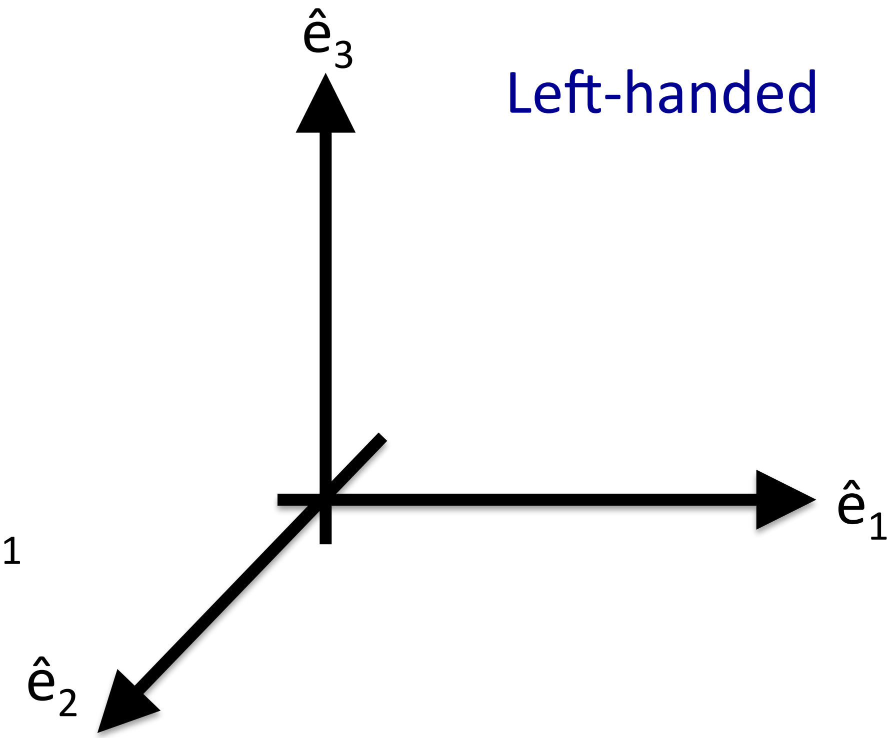
Vector algebra

Lots of details in CH 2

- Are there points that are unclear?
- Please let me know if there are things you would like us to look at specifically.

Coordinate Systems



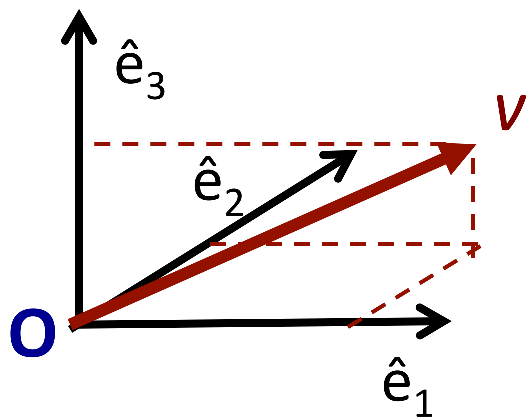


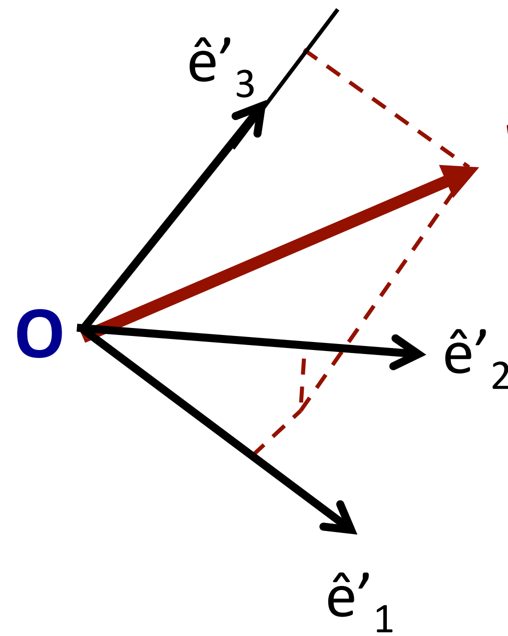
Thumb = 1
Index = 2
Middle = 3

Transformation of Cartesian Coordinates

An object such as vector ***v*** is represented as v_j in coordinate system $Ox_1x_2x_3$ with unit coordinate vectors $\hat{e}_j$

The same object (e.g. ***v***) is represented as v_j' in coordinate system $Ox_1'x_2'x_3'$ with unit coordinate vectors $\hat{e}'_j$





v is ***not*** rotated –

- its coordinates are just expressed in a different coordinate system

Transformation matrix

The new coordinate vectors $\hat{e}'_j$ can be expressed in terms of the old coordinate vectors $\hat{e}_j$

$$\hat{e}'_1 = a_{11}\hat{e}_1 + a_{12}\hat{e}_2 + a_{13}\hat{e}_3 = a_{1j}\hat{e}_j$$
$$\hat{e}'_2 = a_{21}\hat{e}_1 + a_{22}\hat{e}_2 + a_{23}\hat{e}_3 = a_{2j}\hat{e}_j$$
$$\hat{e}'_3 = a_{31}\hat{e}_1 + a_{32}\hat{e}_2 + a_{33}\hat{e}_3 = a_{3j}\hat{e}_j$$

or,

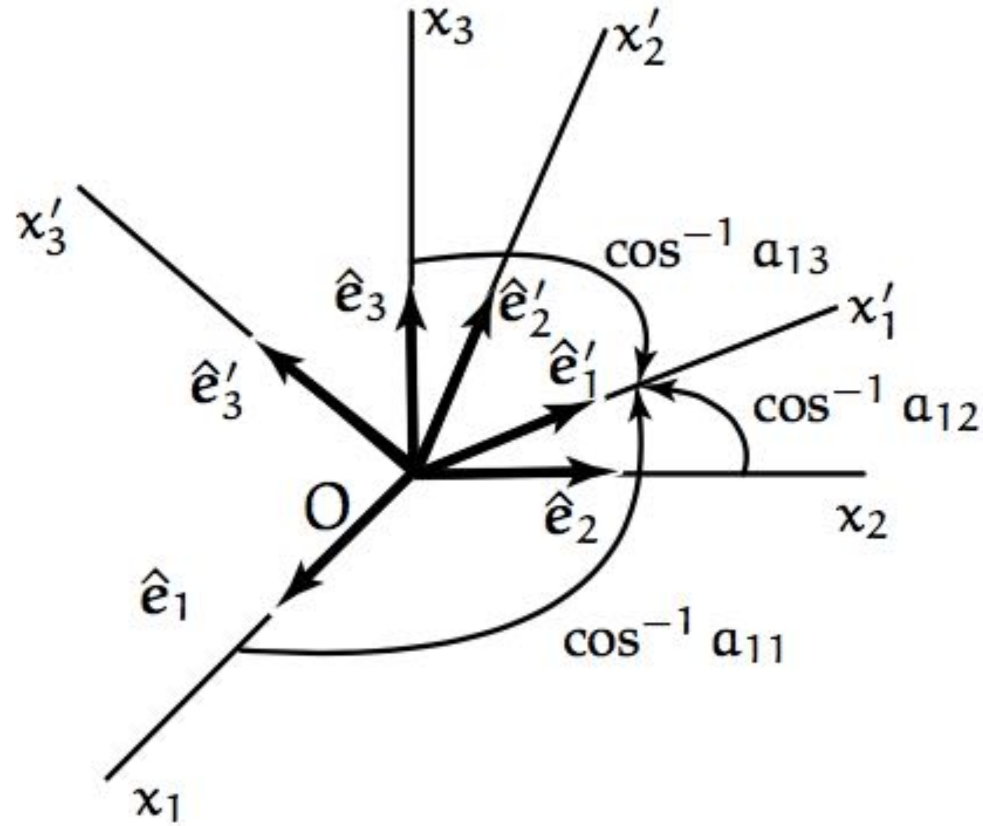
$$\hat{e}'_i = a_{ij}\hat{e}_j$$

$$\begin{bmatrix} \hat{e}'_1 \\ \hat{e}'_2 \\ \hat{e}'_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

A is the transformation matrix

Meaning of a_{ij}

The matrix elements a_{ij} are the direction cosines between $\hat{e}'_i$ and $\hat{e}_j$.



Transformation matrix

Orthogonality

$$\hat{e}'_i \cdot \hat{e}'_j = a_{iq}\hat{e}_q \cdot a_{jm}\hat{e}_m = a_{iq}a_{jm}\delta_{qm} = a_{iq}a_{jq} = \delta_{ij}$$

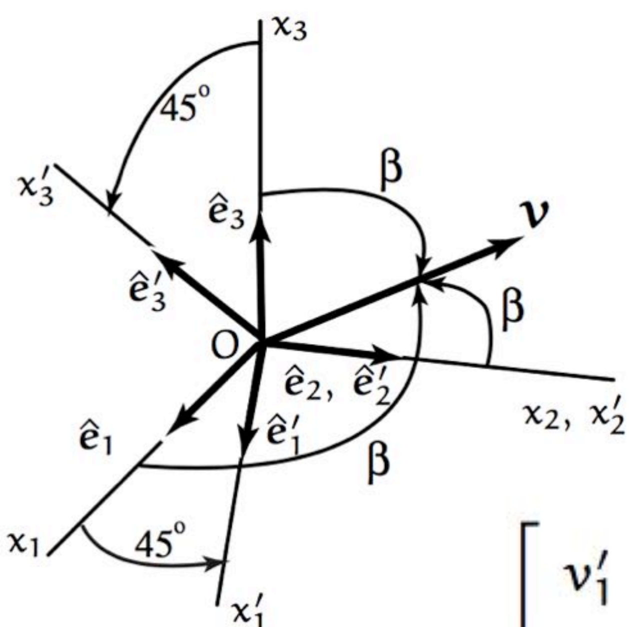
$a_{iq}a_{jq} = \delta_{ij}$ or $\mathbf{A}\,\mathbf{A}^{\mathbf{T}} = \mathbf{I}$

$a_{ij}a_{ik} = \delta_{jk}$ or $\mathbf{A}^{\mathbf{T}}\,\mathbf{A} = \mathbf{I}$

$$\hat{e}_i = a_{ji}\hat{e}'_j$$

Example 2.12

Let the primed axes $Ox'_1x'_2x'_3$ be given with respect to the unprimed axes by a 45°(counterclockwise) rotation about the x_2 axis as shown. Determine the primed components of the vector given by $\boldsymbol{v} = \hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3$.



$$[a_{ij}] = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} v'_1 \\ v'_2 \\ v'_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \sqrt{2} \end{bmatrix}$$

Change of coordinates for a scalar

$$\theta' = \theta$$

Change of coordinates for a vector

$$\boldsymbol{v} = v'_j \hat{\mathbf{e}}'_j = v_i \hat{\mathbf{e}}_i = v_i a_{ji} \hat{\mathbf{e}}'_j$$

Or without the unit vectors

$$v'_j = a_{ji} v_i$$

Change of coordinates for a 2nd order tensor

An example of second order tensor w_{ij} is

$$
\begin{aligned}
w_{ij} &= u_i\, v_j \\
&= a_{qi}\, u'_q\, a_{mj}\, v'_m \\
&= a_{qi} a_{mj}\, u'_q v'_m \\
&= a_{qi} a_{mj} w'_{ij}
\end{aligned}
$$

Or symbolically

$W = A^T W' A$ and $W' = A W A^T$

Change of coordinates for any order tensor $R_{qm\ldots n}$

$$R'_{ij\ldots k} = a_{iq} a_{jm} \cdots a_{kn} R_{qm\ldots n}$$

Multiply by transformation matrix A once
for each order in the tensor $R_{qm\ldots n}$

Proper and Improper changes of coordinates

det(**A**) = 1   is a rotation (proper)

det(**A**) = -1   is a reflection (improper)
(right-handed coordinate system becomes a left-handed coordinate system (generally not good …)