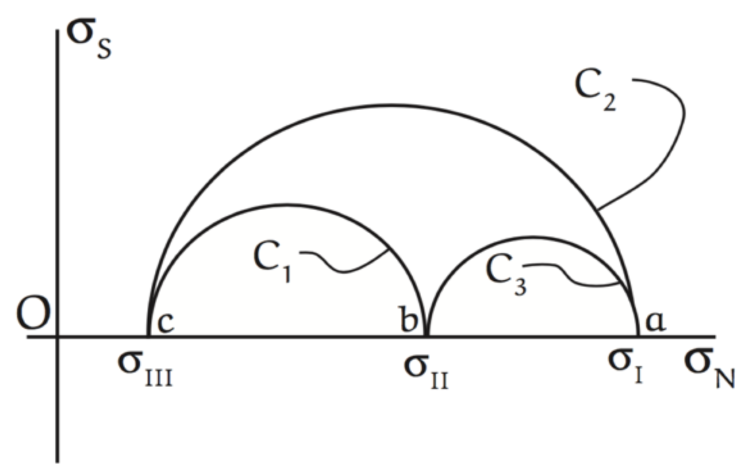
ESS 411/511 Geophysical Continuum Mechanics Class #13

Highlights from Class #12 – Yiyu Ni
Today's highlights on Friday – John-Morgan Manos

Class Prep – Mohr's circles

- Why are Mohr's circles actually *circles* in stress space, and not some other shape, such as general ellipses for example?

- For a given stress tensor t_{ij} in stress space, with principal stresses $\sigma_I > \sigma_{II} > \sigma_{III}$ the normal and shear components of t_{ij}, σ_N and σ_S for all possible planes with normal vectors n_i plot in a restricted area in stress space, and all other areas are "out of bounds".
- In which area do *all* the possible stress states (σ_N, σ_S) plot in the stress-space diagram?
- Why do Mohr's circles often stop at $\sigma_S = 0$? Is a negative shear component impossible?



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For Friday class

- Please read Mase, Smelser, and Mase, Ch 3 through Section 3.11

Frictional failure criterion

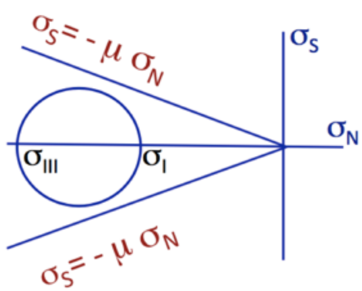
Faults can slip when shear stress σ_S is large enough to overcome frictional resistance. Frictional resistance to failure can be modeled as increasing proportional to the normal traction σ_N.

In stress space, if a stress state σ_N and σ_S exists that intersects or touches the frictional line, then the plane represented at that point can fail.

In the diagrams, all principal stresses are negative. Are they compressive or extensile?

In the first diagram, do any stress states exist outside the circle shown?

Can any faults fail in this stress field?



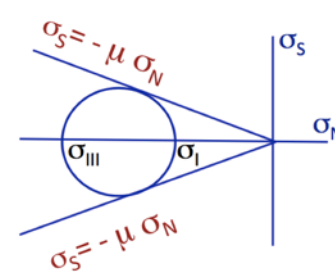
In the second diagram (below), what has changed in the stress field?

Can any faults fail in this new stress field?

If yes, how many different faults can fail?

How could you identify the orientation(s) from the Mohr's circle?

Frictional sliding

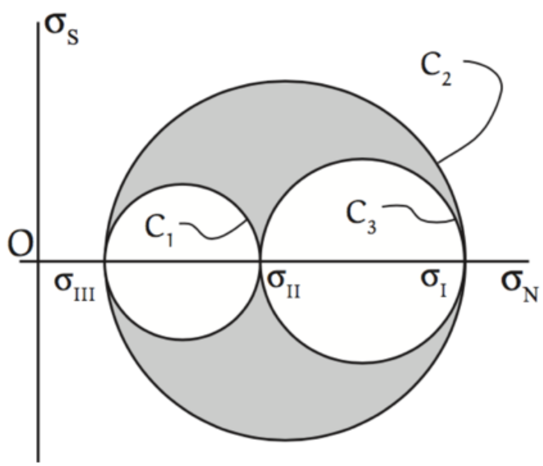


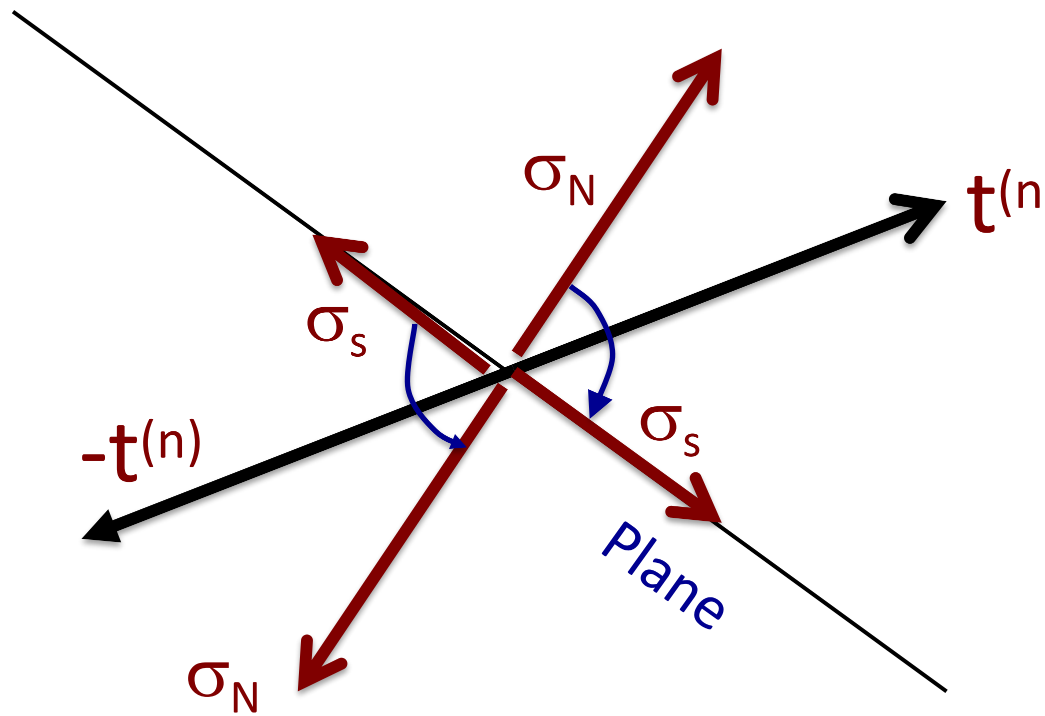
ESS 411/511 Geophysical Continuum Mechanics

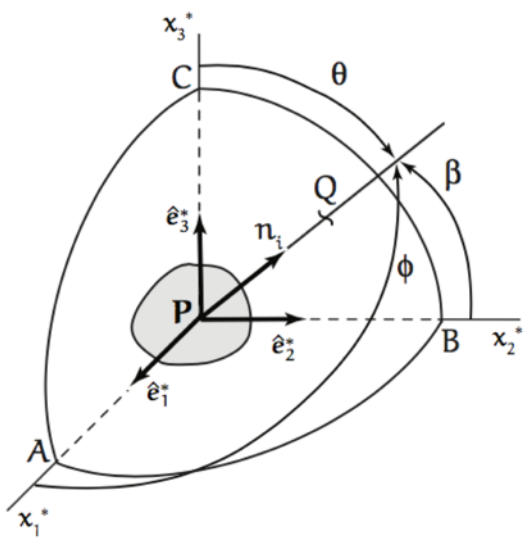
Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Mohr's circles in 4th quadrant



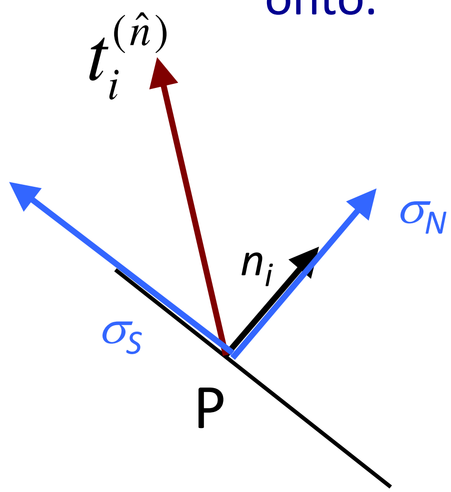


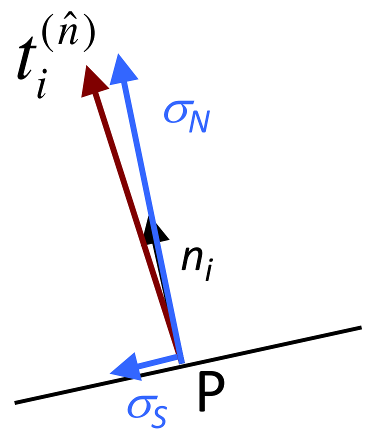


Normal vectors n_j defining a plane can point in either direction, e.g. into opposite octant in Cartesian space

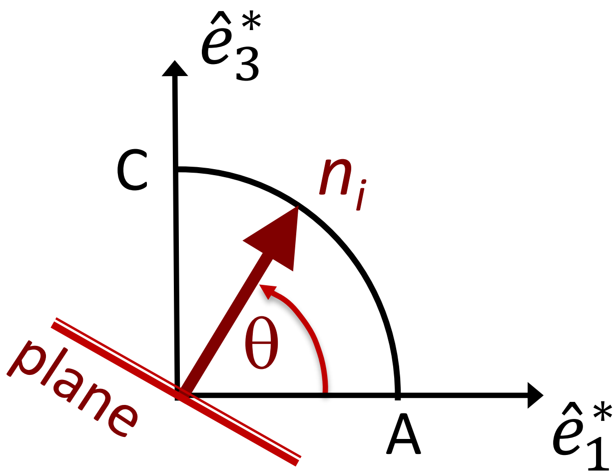
Section 3.7 – Minimum and maximum stress values

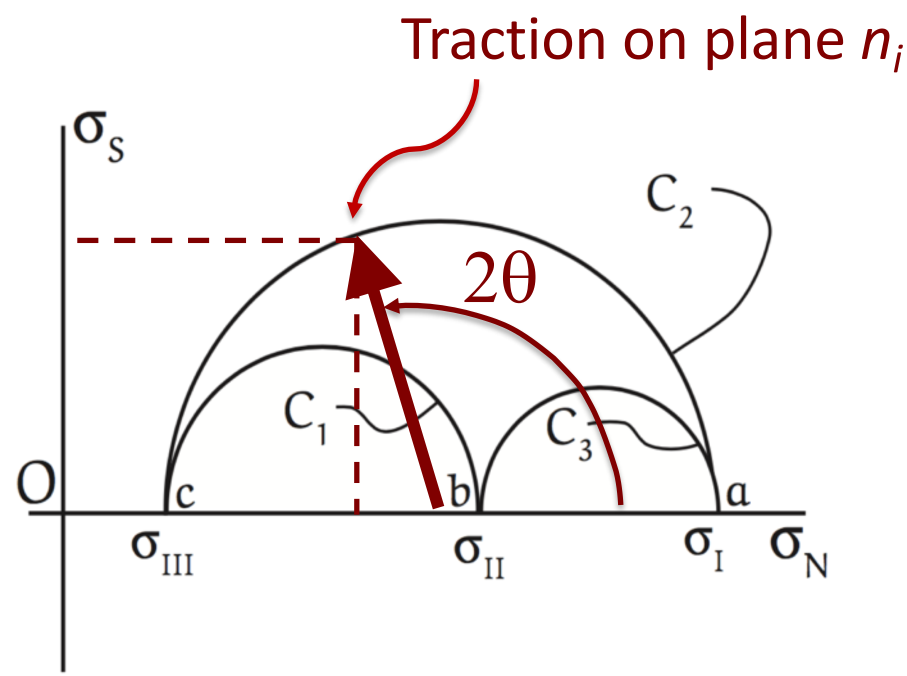
σ_N and σ_S can vary dramatically depending on which plane the traction vector $t_i^{(\hat{n})}$ is resolved onto.





A Mohr's circle in a 2-D view





- The normal vector n_i lies on the circle CA in the principal plane defined by $\hat{e}_2^*$ in Cartesian space.
- The circle C_2 (connecting *c* and *a*) is centered at $(\sigma_I + \sigma_{III})/2$ on the σ_N axis in stress space.
- Arc CA spans 90° in Cartesian space, while C_2 (*ca*) spans 180° in stress space.
- The angle 2θ between σ_I and the traction vector in stress space is double the angle θ between n_i and $\hat{e}_1^*$ in Cartesian space.

Cartesian Space vs Stress Space

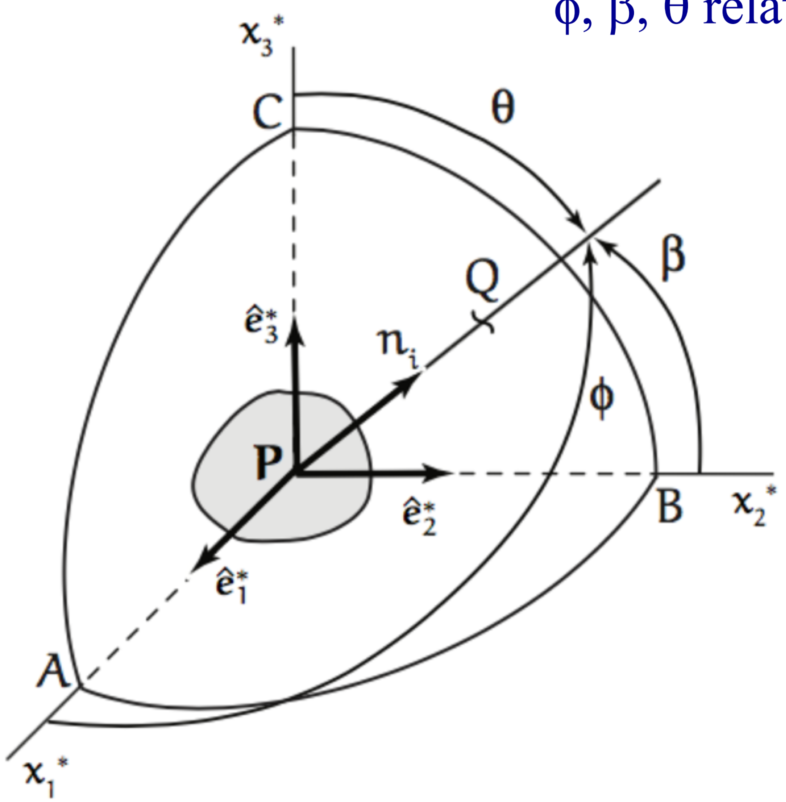
$\boldsymbol{x}_i^*$ are principal directions defining principal planes at **P**, and $\hat{e}_i^*$ are the unit vectors.

Lower-case letters in stress space correspond to upper-case letters in Cartesian space.

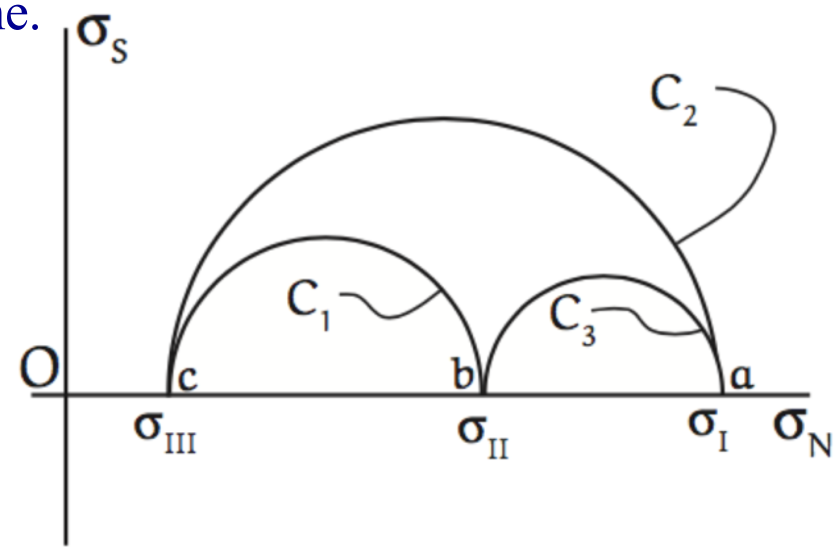
End point **Q** of unit vector n_i can fall anywhere on unit sphere centered at **P**.

ϕ, β, θ relate Q to the 3 coordinate axes $\boldsymbol{x}_i^*$.

The 3 circles in stress space correspond to Q lying somewhere in a principal plane.



(a) Octant of small spherical portion of body together with plane at **P** with normal n_i referred to principal axes $Ox_1^*x_2^*x_3^*$.



(b) Mohr's stress semicircle for octant of Fig. 3.14(a).

Cartesian Space *vs* Stress Space

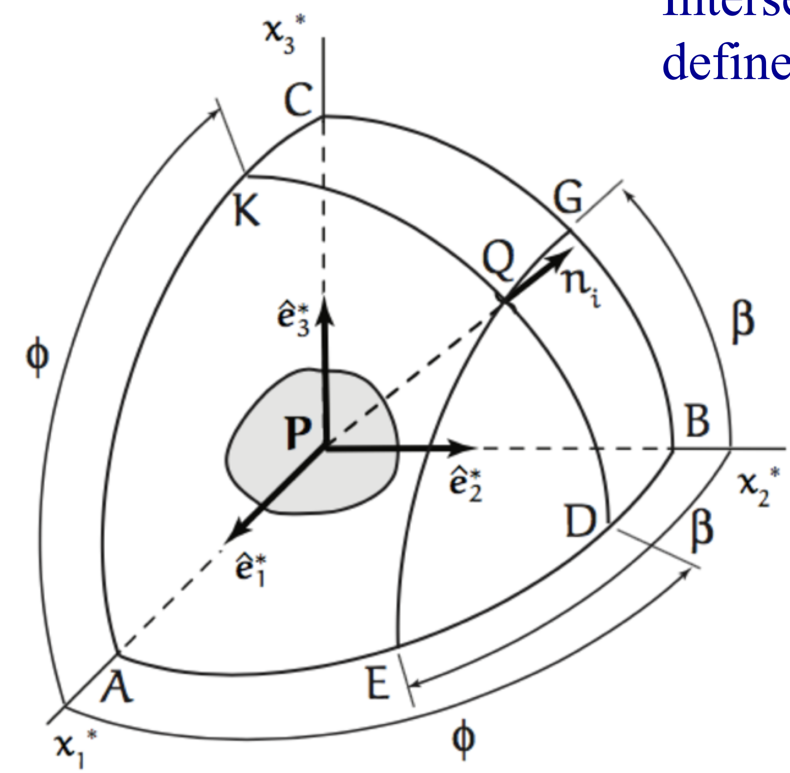
$\boldsymbol{x}_i^*$ are principal directions defining principal planes at **P**.

Small circles in Cartesian space (e.g. EQG) map onto circular arcs (e.g. eqg) concentric with primary Mohr's circles (e.g. akc).

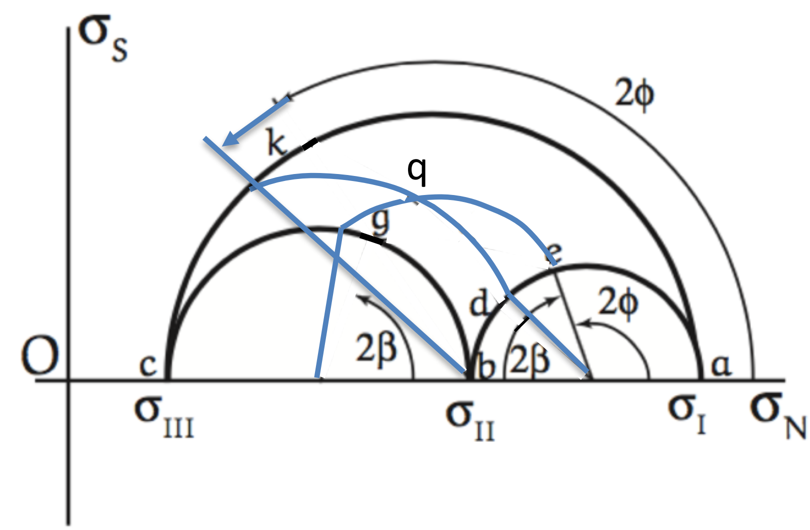
Similarly, KQD maps onto kqd.

Intersection at q shows σ_N and σ_s on plane defined by normal vector n_i at Q.

(I have attempted to (sort of) correct the stress-plane figure below. ☺)

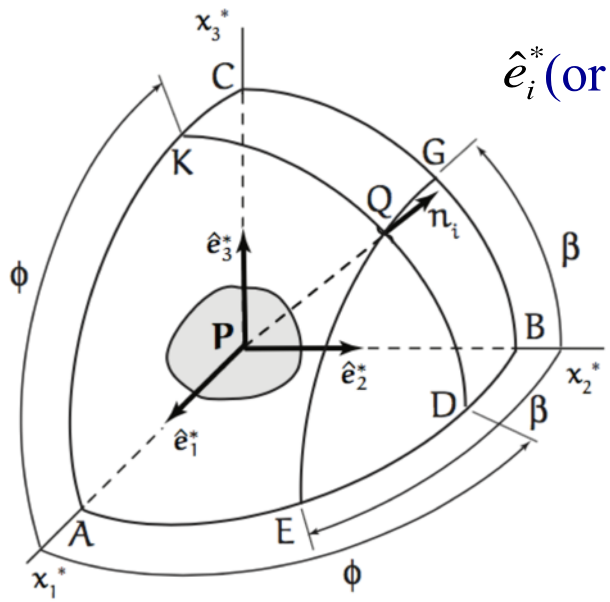


(a) Reference angles ϕ and β for intersection point Q on surface of body octant.



(b) Mohr's stress semicircle for octant of Fig. 3.15(a).

Cartesian Space *vs* Stress Space

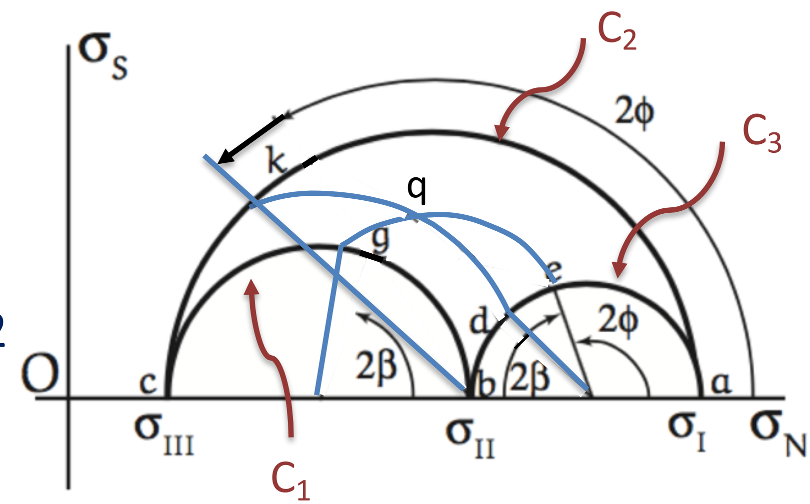


$\hat{e}_i^*$(or $\boldsymbol{x}_i^*$) are principal directions defining principal planes at **P**.

Great circles in Cartesian space (e.g. AKC) map onto circles (e.g. akc) in stress space.
Small circles in Cartesian space (e.g. EQG) that are concentric with AKC map onto arcs of circles (e.g. eqg) in stress space concentric with primary Mohr's circles (e.g. akc), but with different diameters.

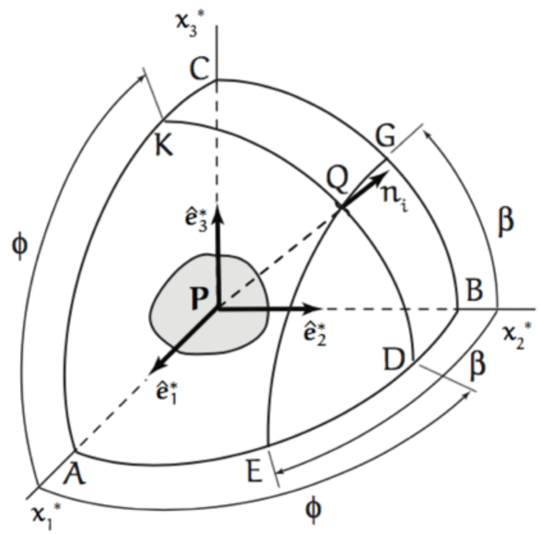
In stress space:

- The circle C_1 (cgb) is centered at $(\sigma_{II} + \sigma_{III})/2$
- The circle C_2 (cka) is centered at $(\sigma_{I} + \sigma_{III})/2$
- The circle C_3 (bdea) is centered at $(\sigma_{I} + \sigma_{II})/2$

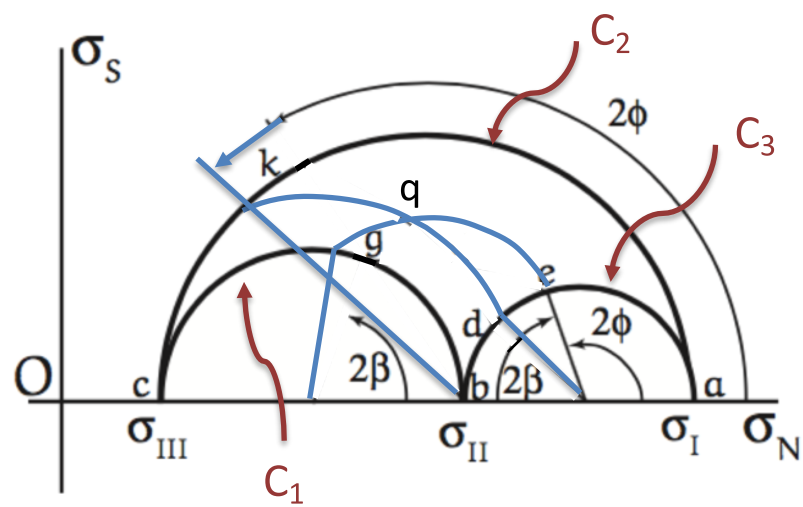


Cartesian Space *vs* Stress Space

x_i^* are principal directions defining principal planes at **P**.



Small circles in Cartesian space (e.g. EQG) map onto circles (e.g. eqg) in stress space concentric with primary Mohr's circles (e.g. akc).



In stress space:

- The circle kqd is also centered at $(\sigma_{II} + \sigma_{III})/2$, concentric with C_1
- The circle gqe is also centered at $(\sigma_I + \sigma_{III})/2$, concentric with C_2
- Their intersection locates q, the traction components on plane defined by unit vector to Q in Cartesian space
- This result can be checked with a third small circle concentric with C_3 in Cartesian space