ESS 411/511 Geophysical Continuum Mechanics Class #15

Highlights from Class #14 – Alysa Fintel
Today's highlights on Wednesday – Jensen DeGrande

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. https://courses.washington.edu/ess511/NOTES/notes.shtml

- Stein and Wysession 5.7.2
- Stein and Wysession 5.7.3/4
- Raymond notes on failure

Also see slides about upcoming topics

- Failure and Mohr's circles – slides

ESS 411/511 Geophysical Continuum Mechanics

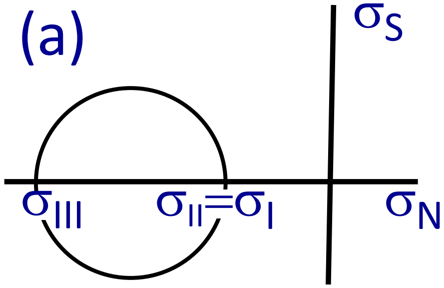
Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Warm-up questions – (break-out)

Explain what's going on in each case.

(a)



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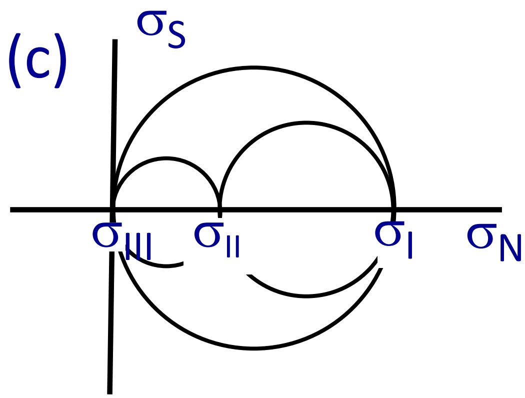

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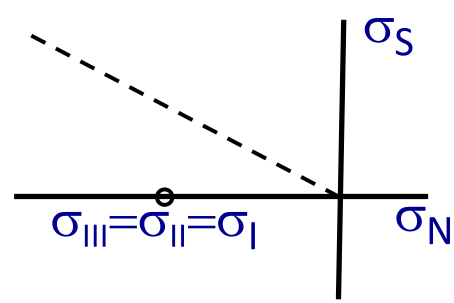

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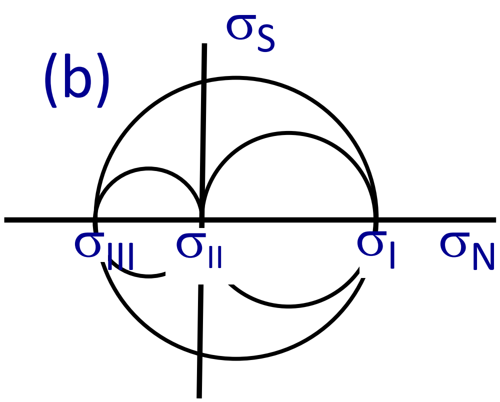

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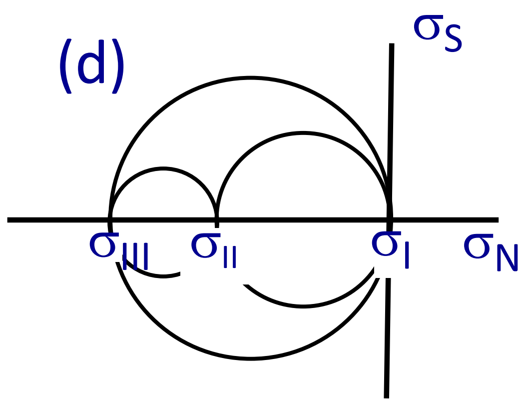


(f)

$p = -\sigma_{ii}/3$

- What is p?
- Why the minus sign?

Warm-up II

Greatest shear stress is on planes marked …….. with normal vectors n_i at 45° to σ_{III}. But failure actually happens on planes marked – – – – at an angle θ>45° between n_i and σ_{III}

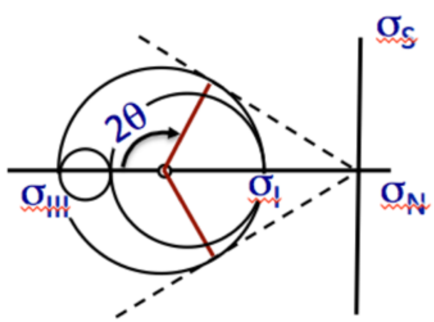
Why is failure ***not*** on the plane with maximum shear stress?

All surfaces are roughs at some scale. Relate this failure angle to how one rough surface slides over another rough surface.

Failure planes – – – are defined by their normal vectors n_i.

Why are there 2 conjugate failure planes?

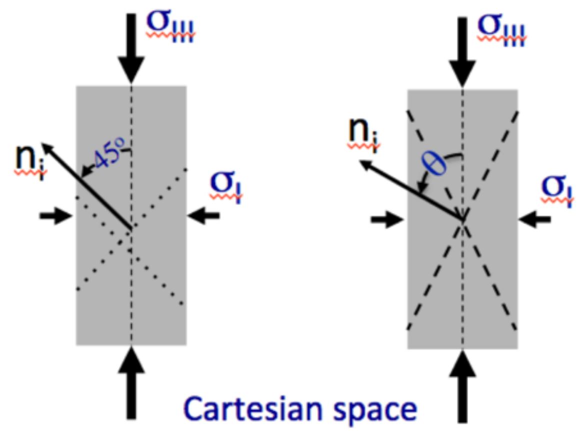
Relate this to the Mohr's circle.



Stress space

… Planes with maximum shear stress

- - - Failure Planes



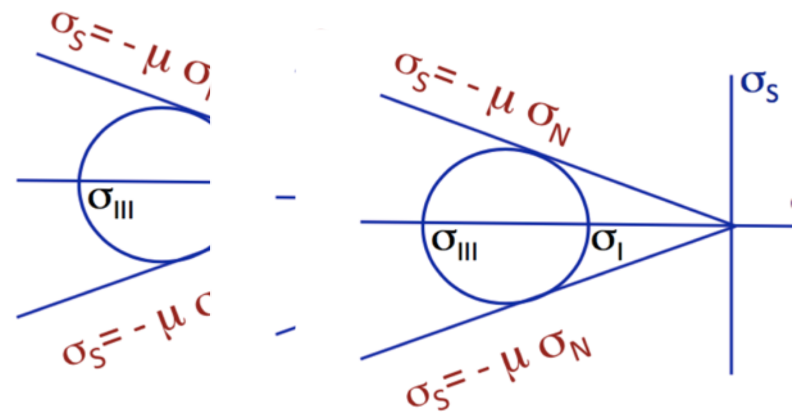
Cartesian space

Class-prep questions for today (break-out)

Failure of materials

Last class, we looked at frictional sliding on pre-existing fractures or faults with a coefficient of friction μ.

- What physical characteristics of a surface cause friction?



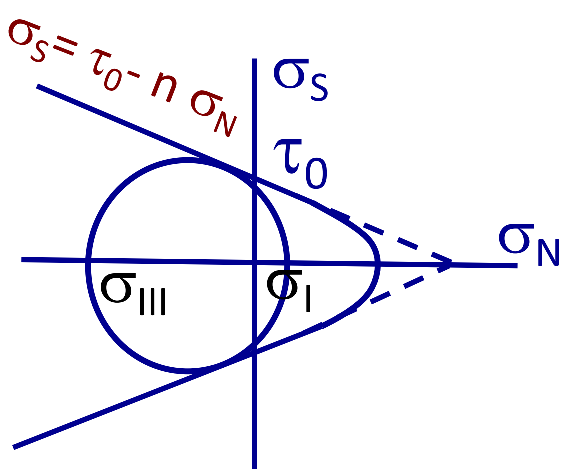
Now we are going to actually break new rocks.

Mohr–Coulomb failure

$$\sigma_S = \tau_0 - n\,\sigma_N$$

n = ***coefficient of internal friction*** for fracture on a new fault surface

τ_0 = cohesion of the material

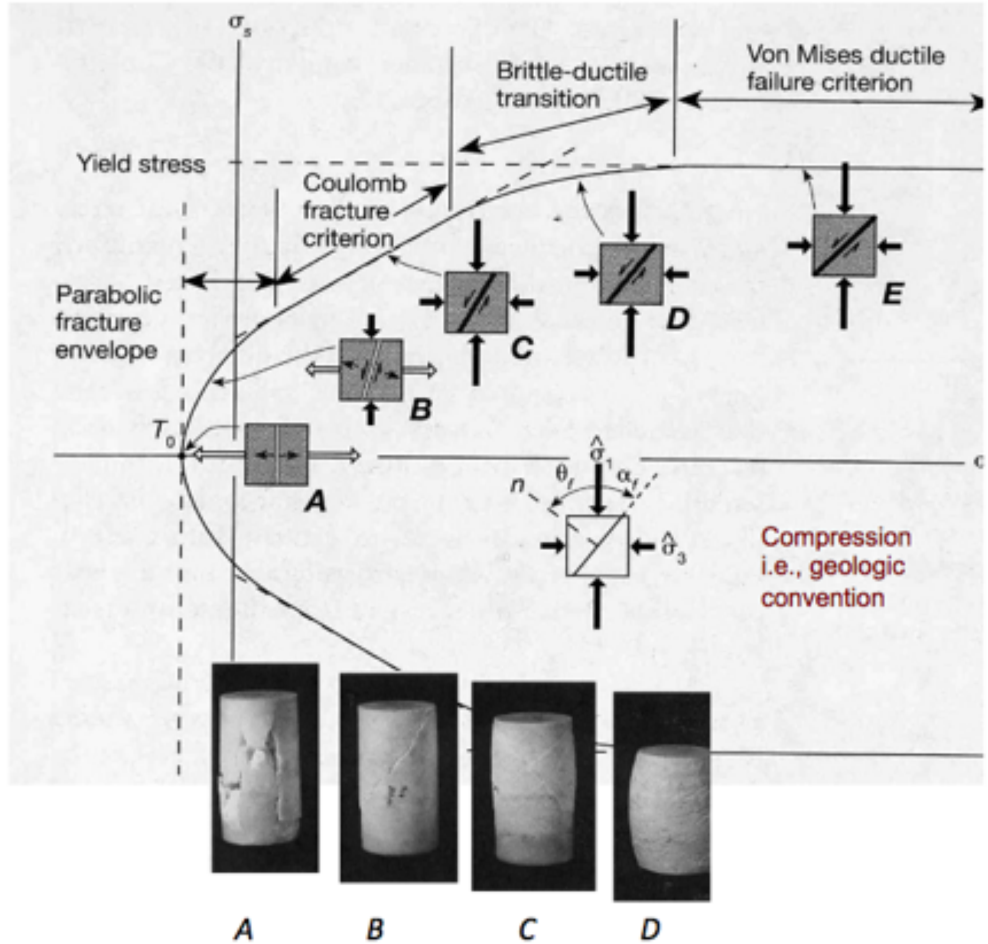


- Explain what you think n and τ_0 might mean in terms of micro-scale processes at the micro-crack, crystalline, or lattice scales.
- Why do you think the failure envelope is rounded off at the right? Think about the sign of σ_N and the processes that might contribute to internal friction.

Class-prep questions for Wednesday Class_16

Style of Failure under Various Normal Stresses σ_N

The figure shows the failure envelope and failure modes in stress space, based on experiments on rocks subjected to a range of normal stresses σ_N. Note that these authors used the convention that compression is positive (yuck …)



- Describe in words what is happening in this generalization of the failure envelopes that we have discussed in class.
- In a sentence or two for each, describe characteristics of the failure mode in each of the 5 stress regimes *A, B, C, D*, and *E*. The regime names, the angles of the failure planes, and the visual states of the samples after the experiments ended may be helpful.

4 Conventions in Stress Polarity

Engineering/Mathematical convention:
Criterion 1: Positive σ_{ii} * signifies extension
Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Mase & Mase)

or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (Stein & Wysession)

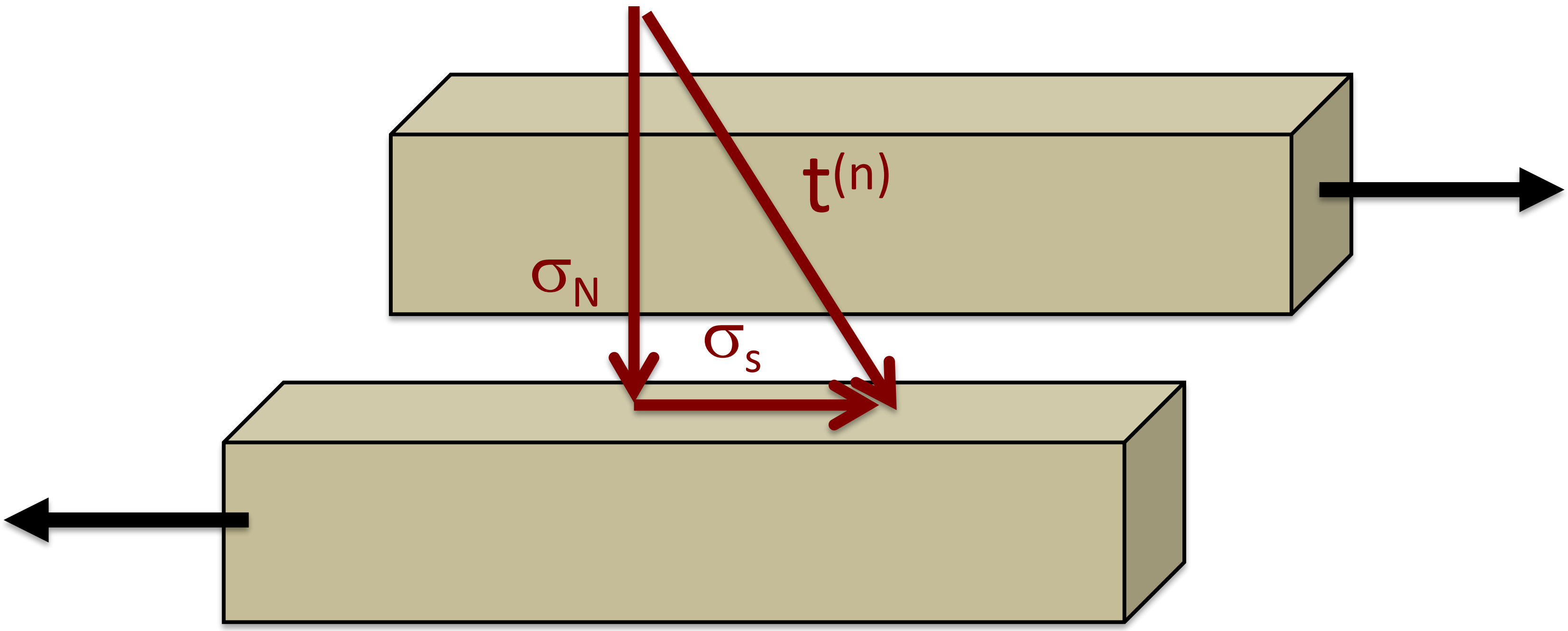
Geologic/Tectonic/Rock Mechanics convention:
Criterion 1: Positive σ_{ii} * signifies compression
(not a tensor!! Why not?)
Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Twiss & Moores)

or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (?)

* No sum implied

Sliding friction

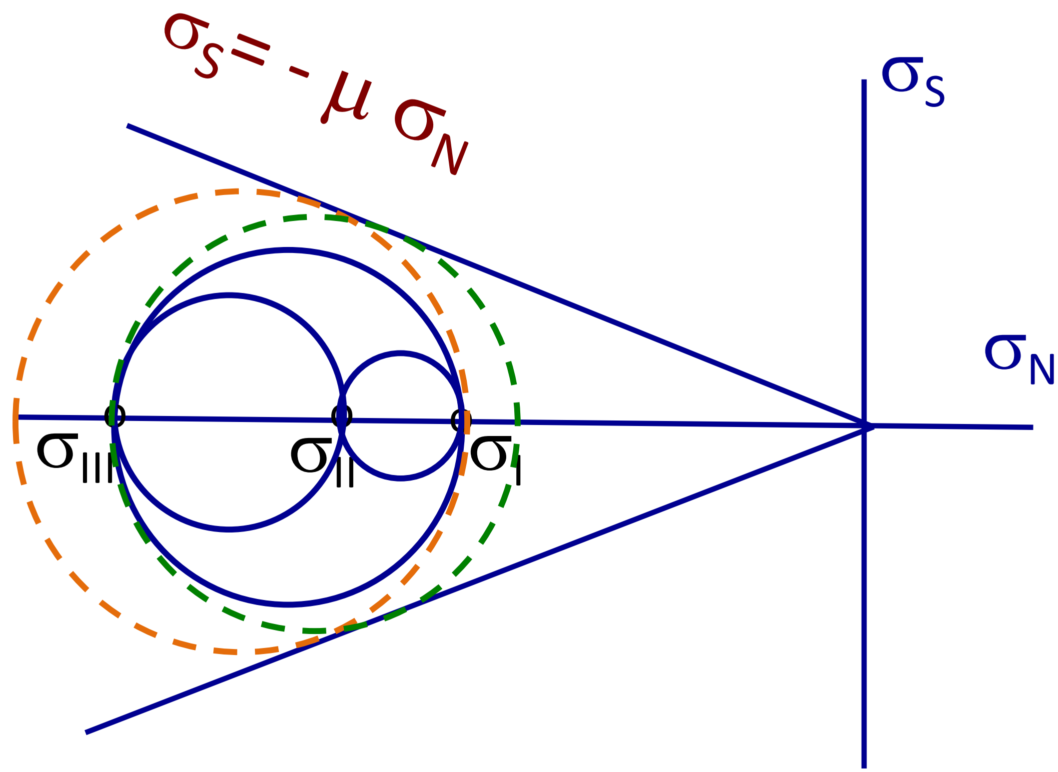


$\sigma_S = -\mu\, \sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

Differential stress

$\sigma_I - \sigma_{III}$

Differential stress is essential in order to have failure

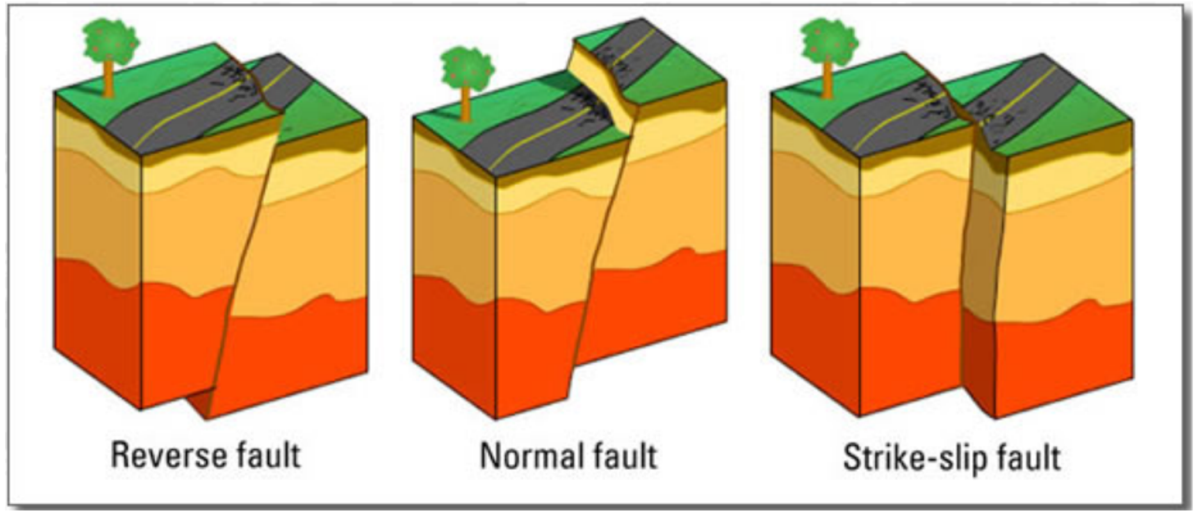


How could we change the stress state in order to cause failure?

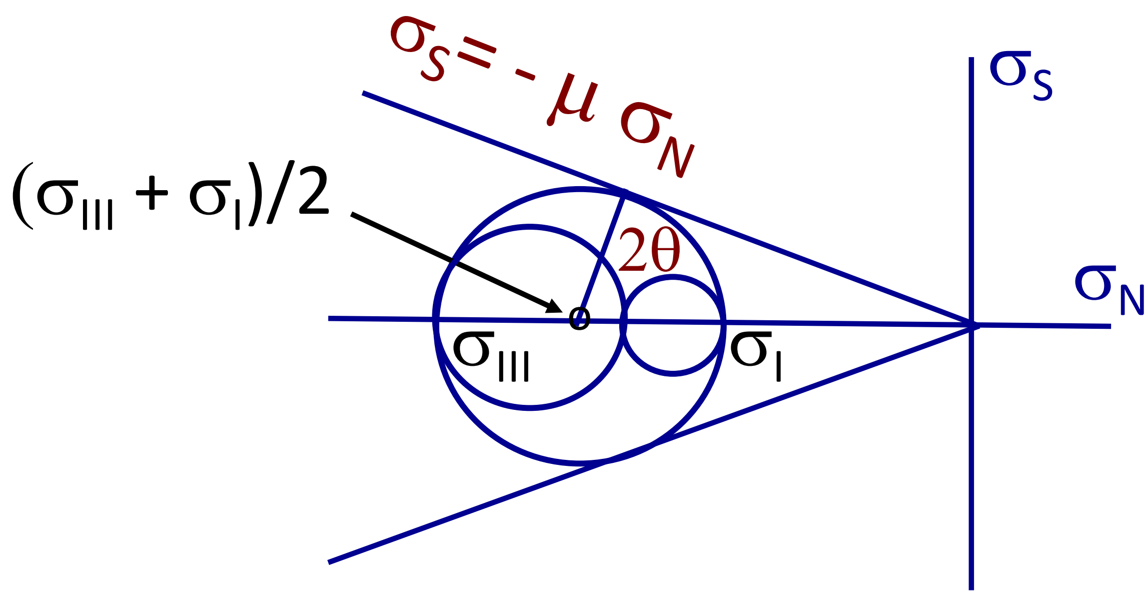
- Hold σ_I make σ_{III} more negative (squeeze harder in x_3)
- Hold σ_{III}, make σ_I less negative (don't squeeze as hard in x_1)

Types of faults

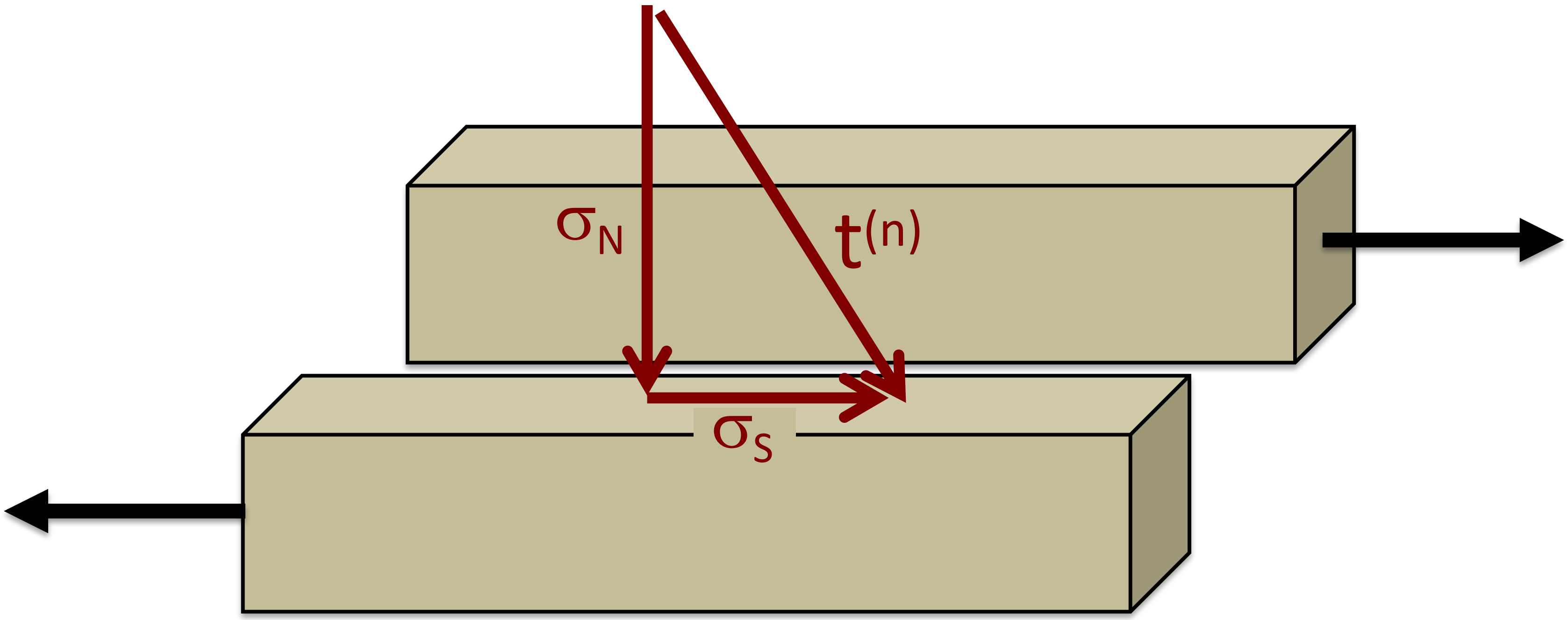
The Earth's surface is traction-free, so one of the principal directions is generally vertical



What are the orientations of the principal axes of stress $\hat{e}_1^*, \hat{e}_2^*, \hat{e}_3^*$ in each case?



Mohr-Coulomb Fracture



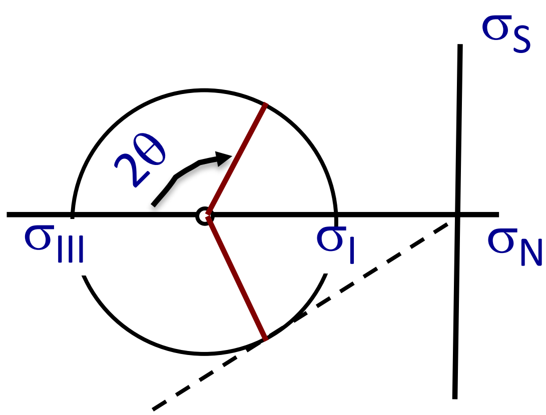
$\sigma_S = \tau_0 - n\,\sigma_N$

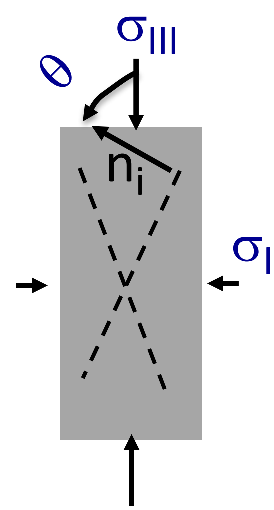
n is ***coefficient of internal friction*** for fracture on a new fault surface

τ_0 is cohesion of the material in absence of any confining stress σ_N

Failure in shear

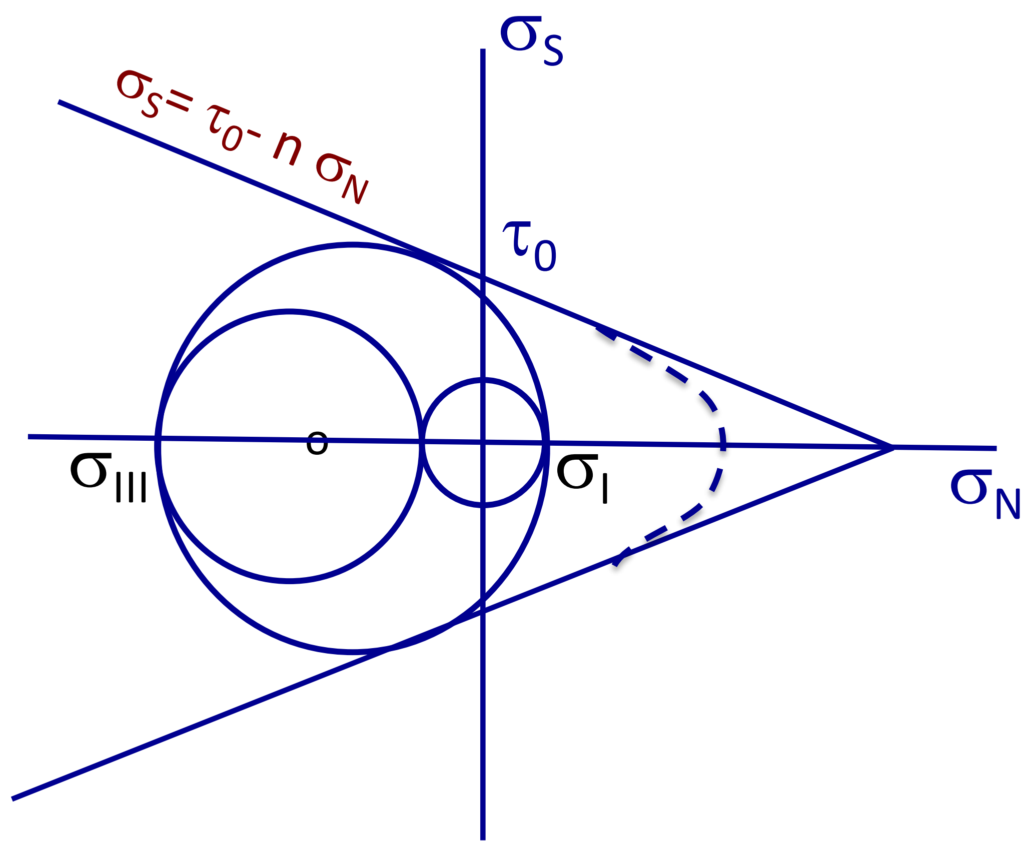
- Why is failure is not on the plane with maximum shear stress?
- Why are there 2 conjugate failure planes?





Mohr-Coulomb Fracture

Now we are actually breaking rock …



$\sigma_S = \tau_0 - n\,\sigma_N$ n is ***coefficient of internal friction*** for fracture on a new fault surface

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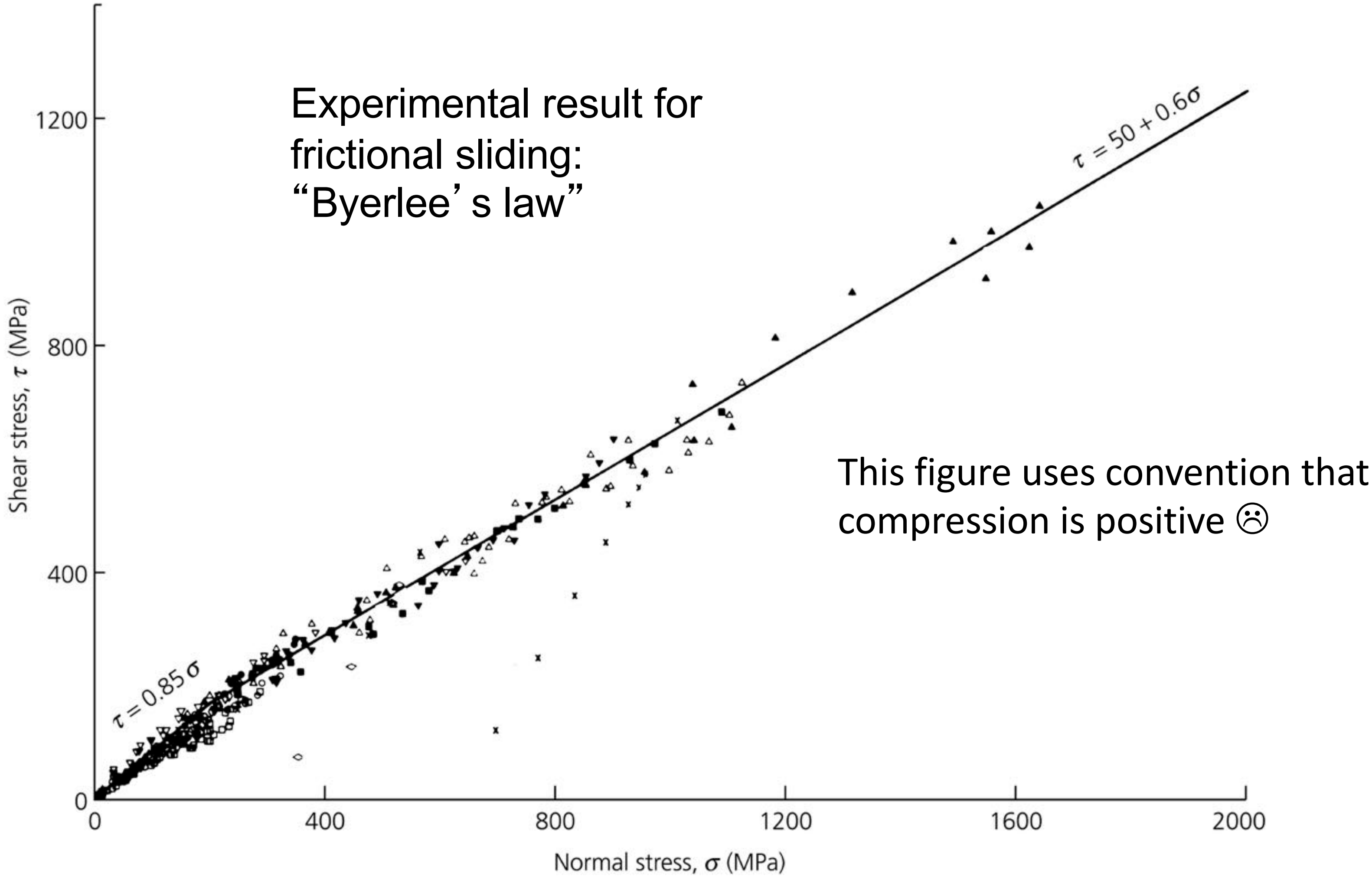
Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Lab experiments show a linear relation between the maximum shear stress that rocks can support at any given normal stress. This is called Byerlee's Law.

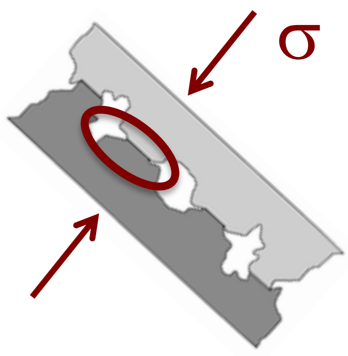
$$\tau \approx -.85\bar{\sigma} \qquad \bar{\sigma} < 200 \text{ MPa}$$

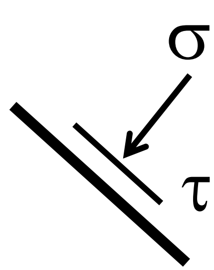
$$\tau \approx 50 - .6\bar{\sigma} \qquad \bar{\sigma} > 200 \text{ MPa.}$$

Coulomb stress

- Notion of friction:
  - More shear stress τ needed to overcome increase in normal stress σ and cause fault to slip – Byerlee's law is an example
- Coulomb stress
  - $\sigma_S = \tau - \mu (\sigma_N - p)$
  - where μ is intrinsic coefficient of friction, p is pore pressure (***not*** the mean stress $p=-\sigma_{ii}/3$, need to be careful of context)

- Basis is that real area of contact (much smaller than apparent area) is controlled by normal stress
  - deformation of asperities in response to normal stress
  - harder to over-ride asperities at higher normal stress





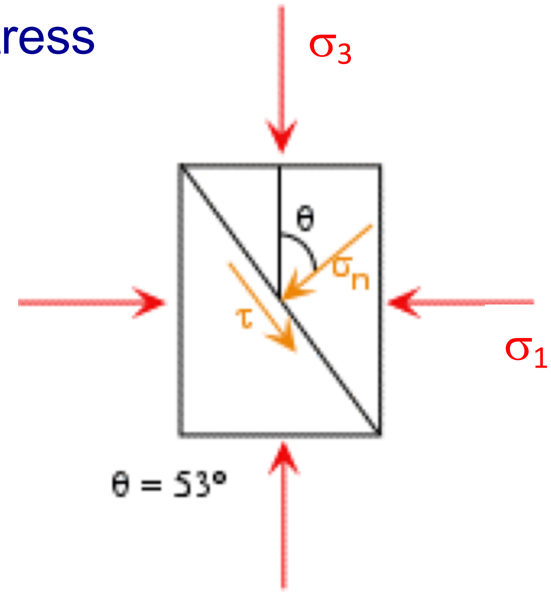
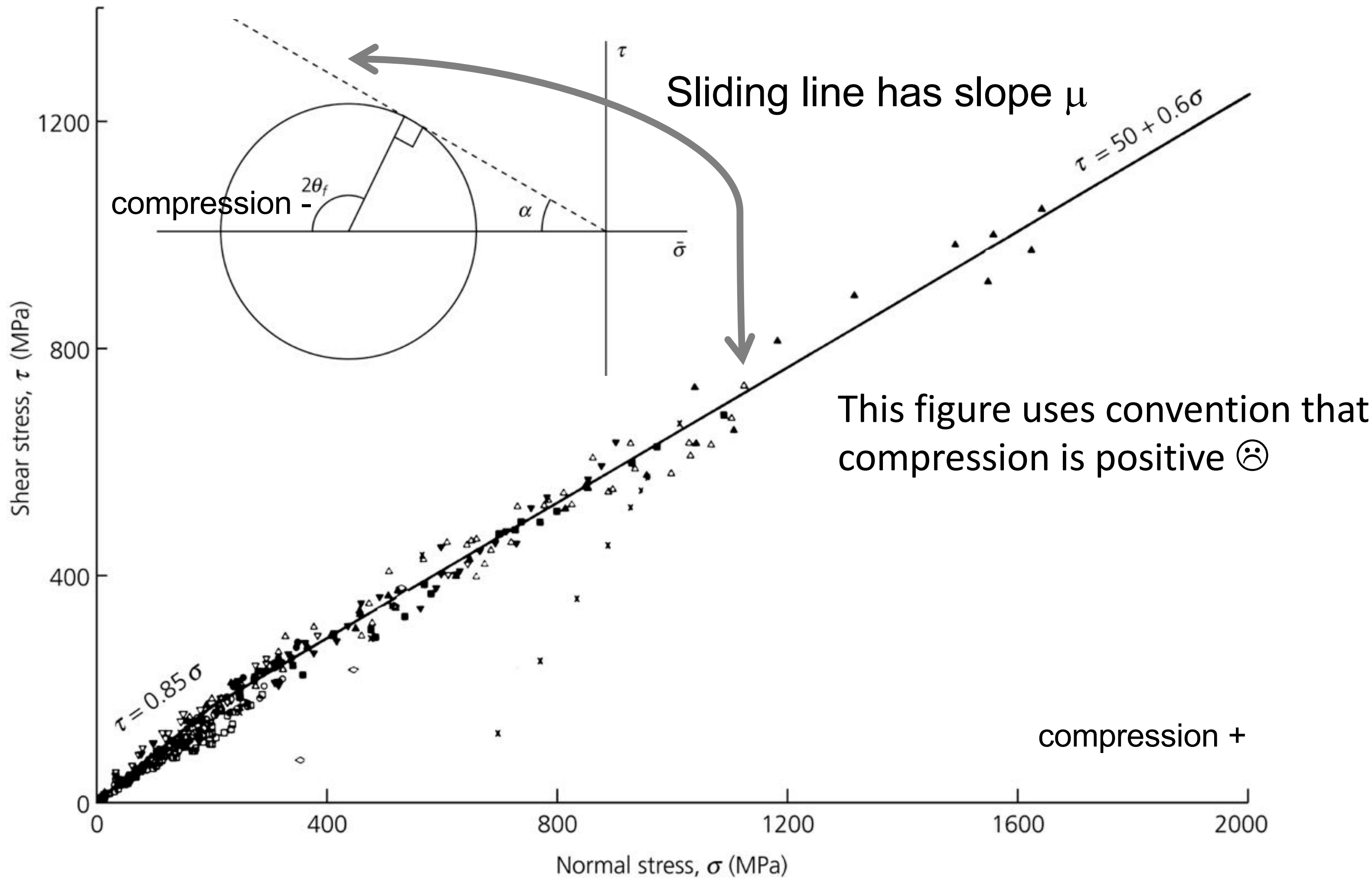


Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.

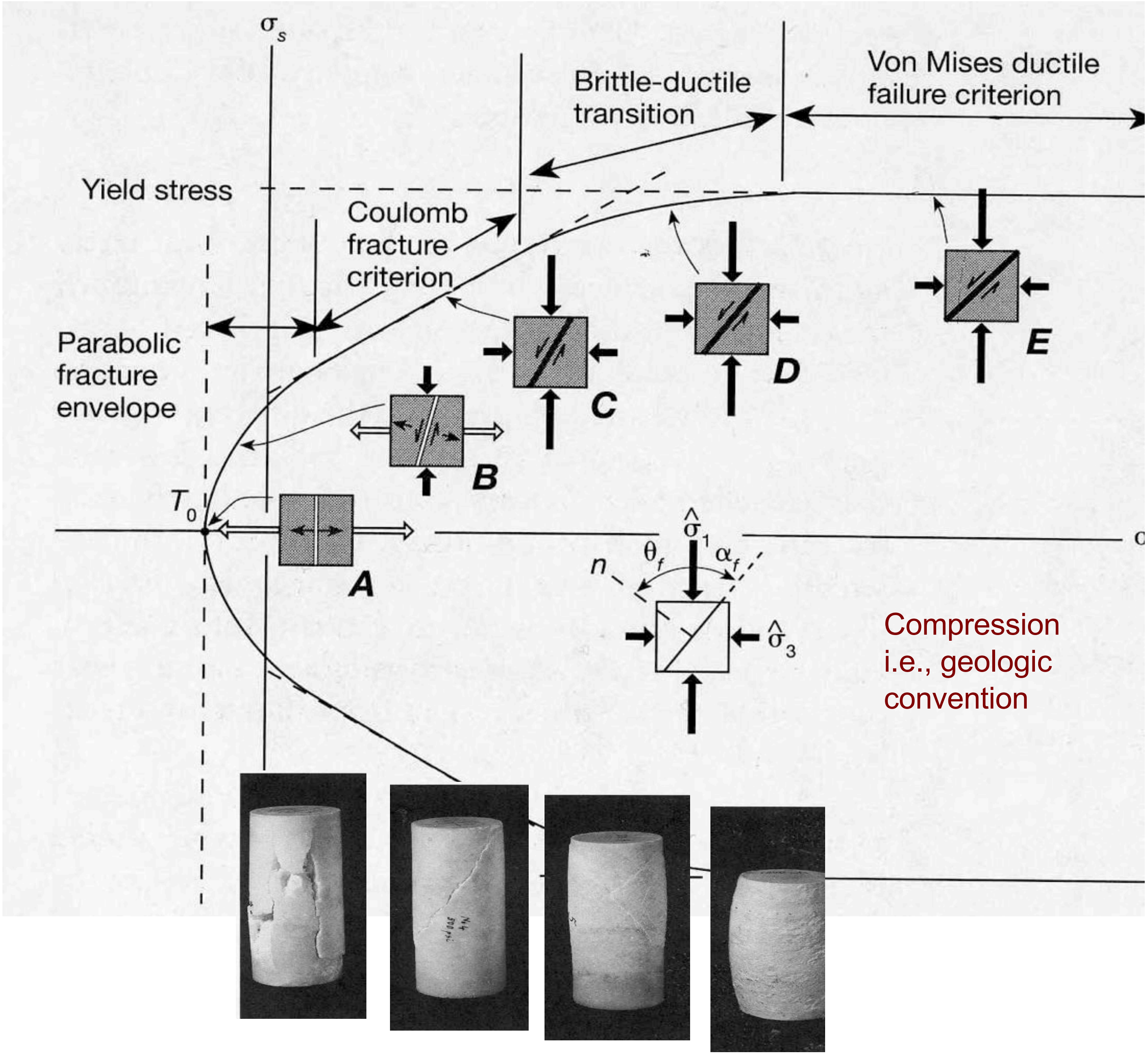


Byerlee's law

$$\tau \approx -.85\bar{\sigma} \qquad \bar{\sigma} < 200 \text{ MPa}$$

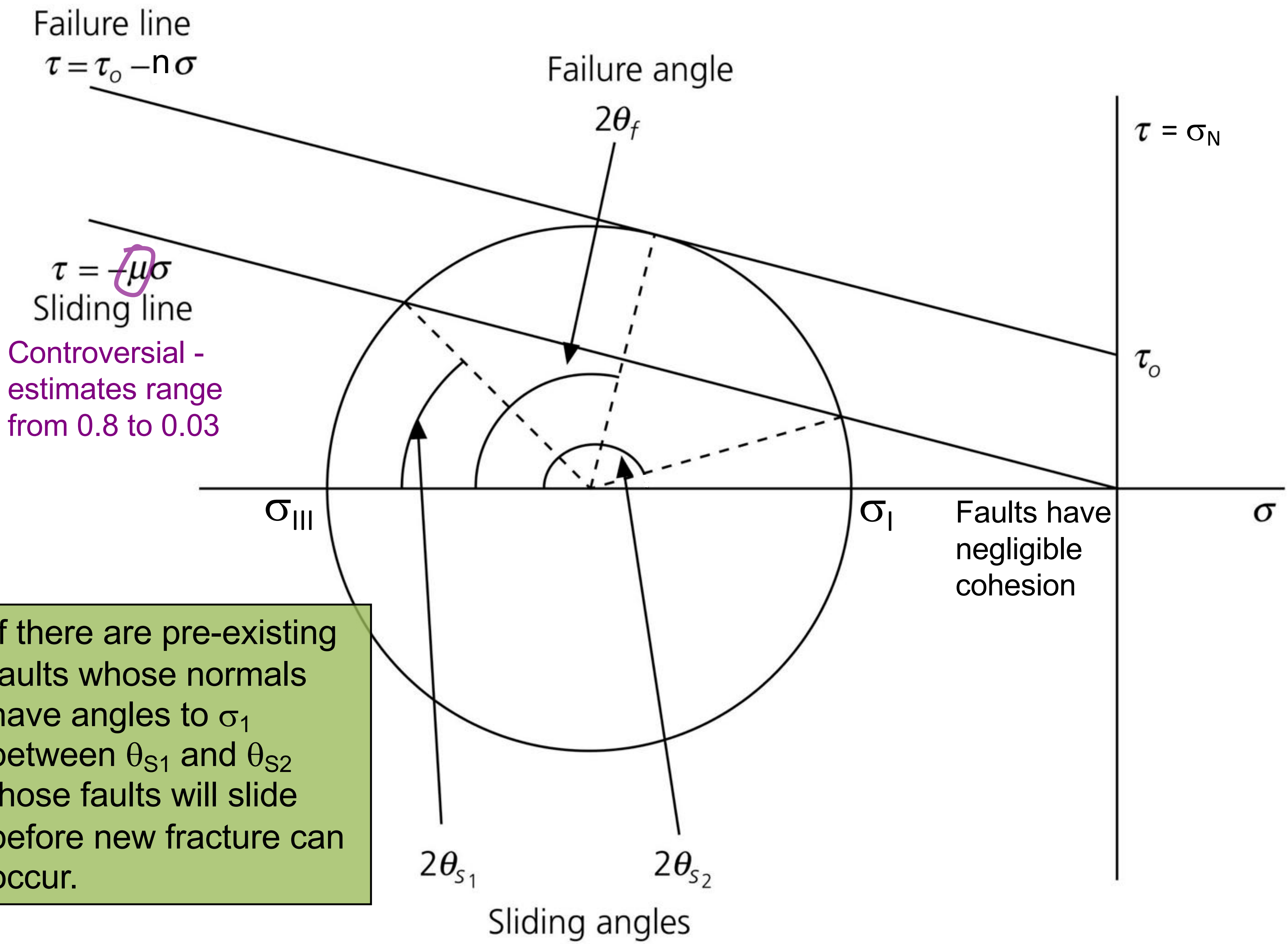
$$\tau \approx 50 - .6\bar{\sigma} \qquad \bar{\sigma} > 200 \text{ MPa.}$$

Two regions


σ_s
Brittle-ductile transition
Von Mises ductile failure criterion
Yield stress
Coulomb fracture criterion
Parabolic fracture envelope
T_0
A
B
C
D
E
$\hat{\sigma}_1$
θ_f
α_f
n
$\hat{\sigma}_3$
Compression i.e., geologic convention

A *B* *C* *D*

Figure 5.7-9: Mohr's circle for sliding on preexisting faults.



If there are pre-existing faults whose normals have angles to σ_1 between θ_{S1} and θ_{S2} those faults will slide before new fracture can occur.