ESS 411/511 Geophysical Continuum Mechanics Class #16

Highlights from Class #15 – Jensen DeGrande
Today's highlights on Friday – Anna Ledeczi

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. https://courses.washington.edu/ess511/NOTES/notes.shtml

- Stein and Wysession 5.7.2
- Stein and Wysession 5.7.3/4
- Raymond notes on failure

Also see slides about upcoming topics

- Failure and Mohr's circles – slides

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

ESS 511 Term projects

This Friday Nov 5:

- 60 second updates

Next Friday Nov 12:

- 1-page reports (outline, refs, …)

The following Friday Nov19:

- 60 second updates

4 Conventions in Stress Polarity

Engineering/Mathematical convention:

Criterion 1: Positive σ_{ii} * signifies extension

Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Mase & Mase)

or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (Stein & Wysession)

Geologic/Tectonic/Rock Mechanics convention:

Criterion 1: Positive σ_{ii} * signifies compression

(not a tensor!! Why not?)

Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Twiss & Moores)

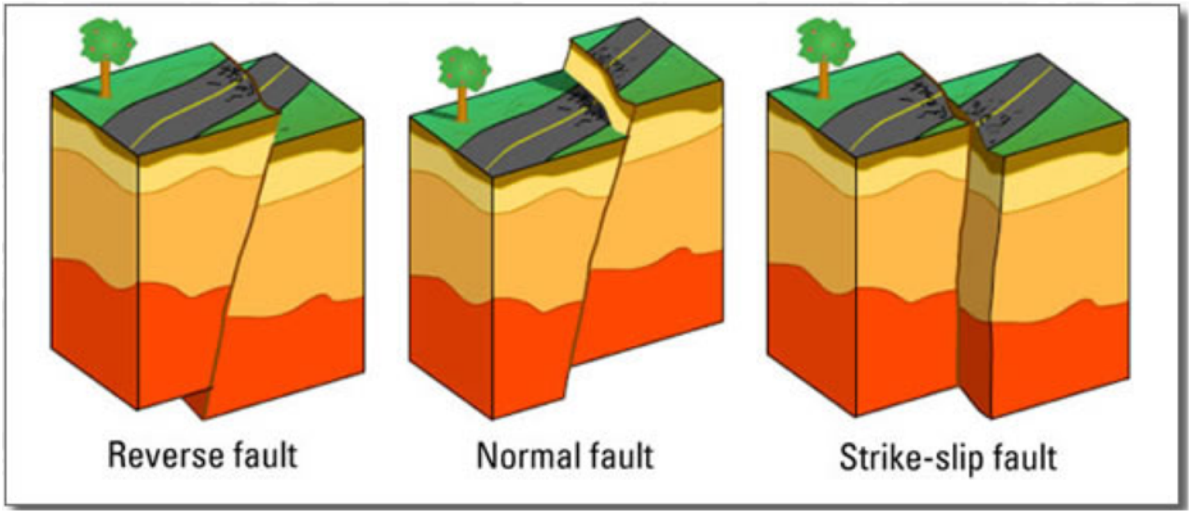
or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (?)

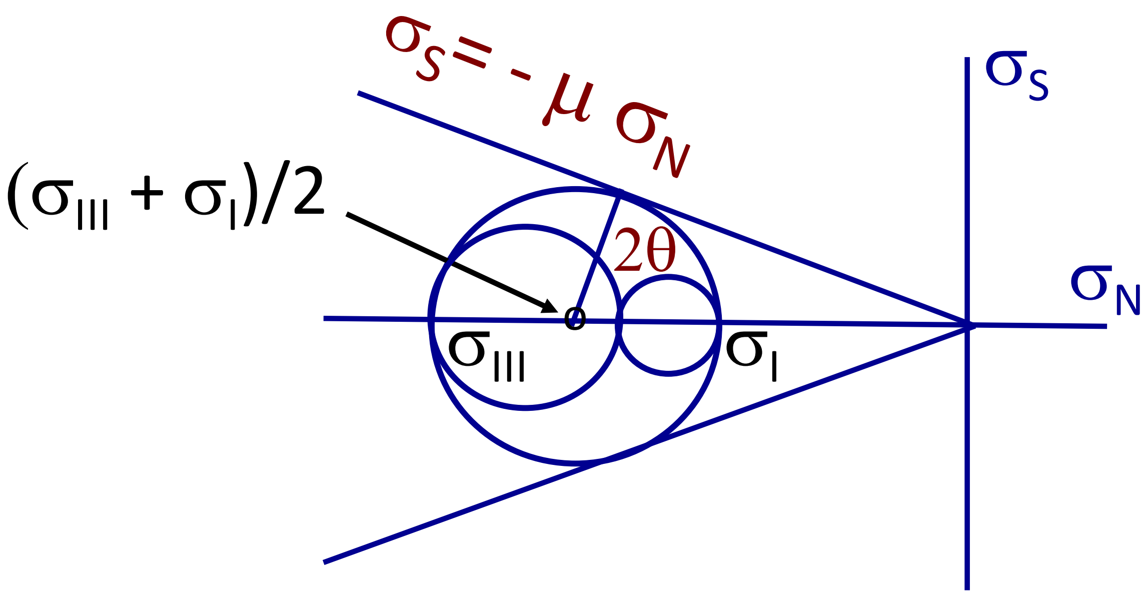
* No sum implied

Types of faults

The Earth's surface is traction-free, so one of the principal directions is generally vertical



What are the orientations of the principal axes of stress $\hat{e}_1^*, \hat{e}_2^*, \hat{e}_3^*$ in each case?



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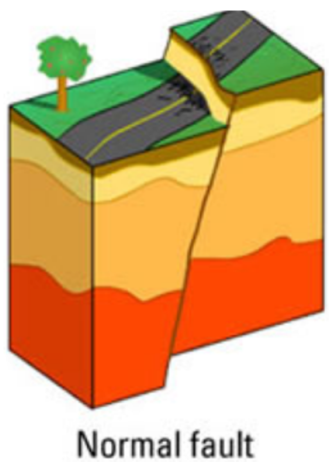
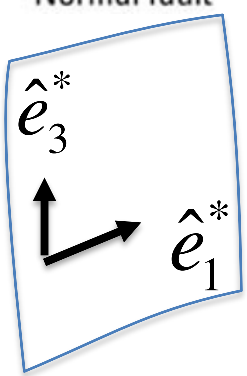
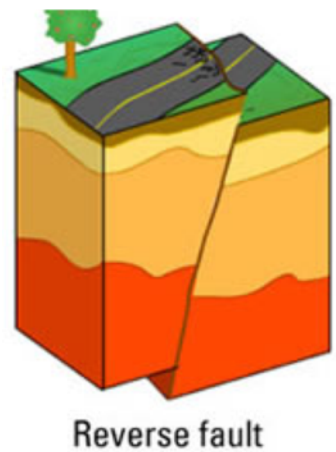
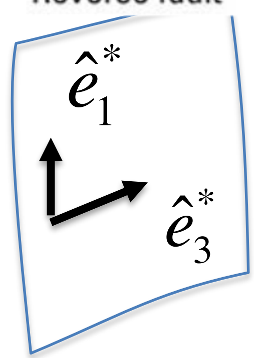


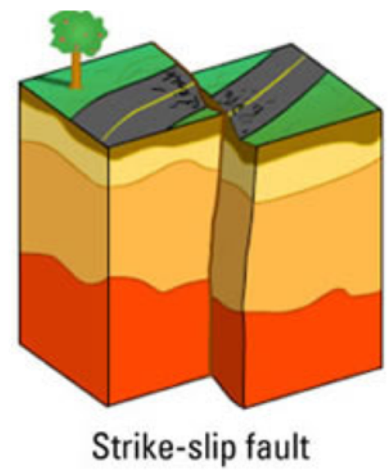


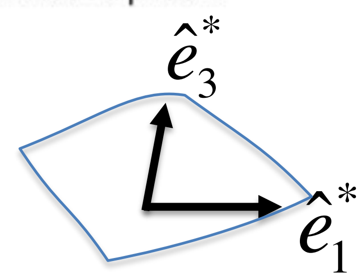
In vertical plane





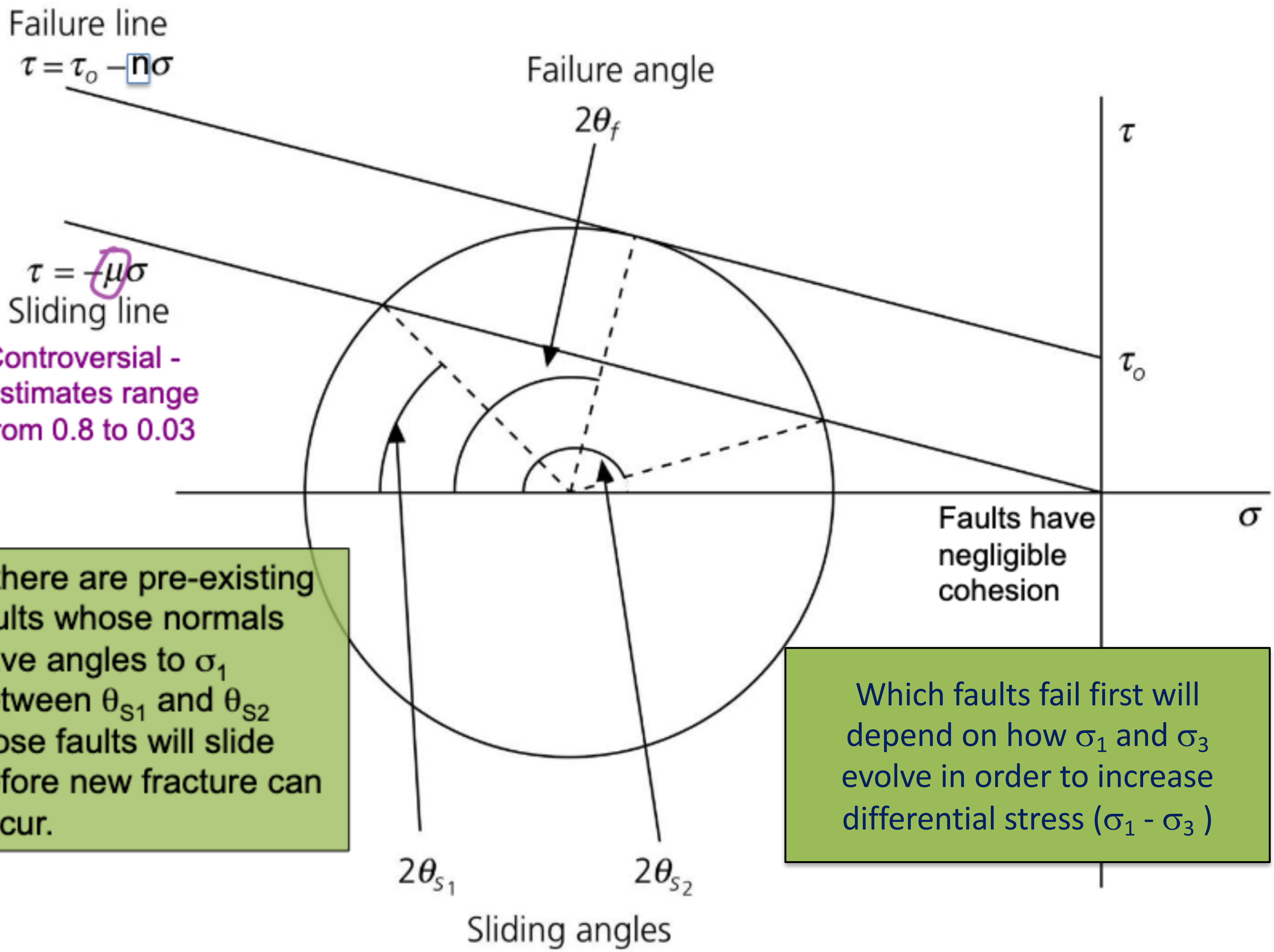
In vertical plane





In horizontal plane

Figure 5.7-9: Mohr's circle for sliding on preexisting faults.



If there are pre-existing faults whose normals have angles to σ_1 between θ_{S1} and θ_{S2} those faults will slide before new fracture can occur.

Which faults fail first will depend on how σ_1 and σ_3 evolve in order to increase differential stress (σ_1 - σ_3)

Strength of rocks

Shear stress needed to break a rock or to make an existing fault slip) increases as the normal stress on the rock increases.
This can be measured in rock-mechanics labs for a wide range of normal stresses

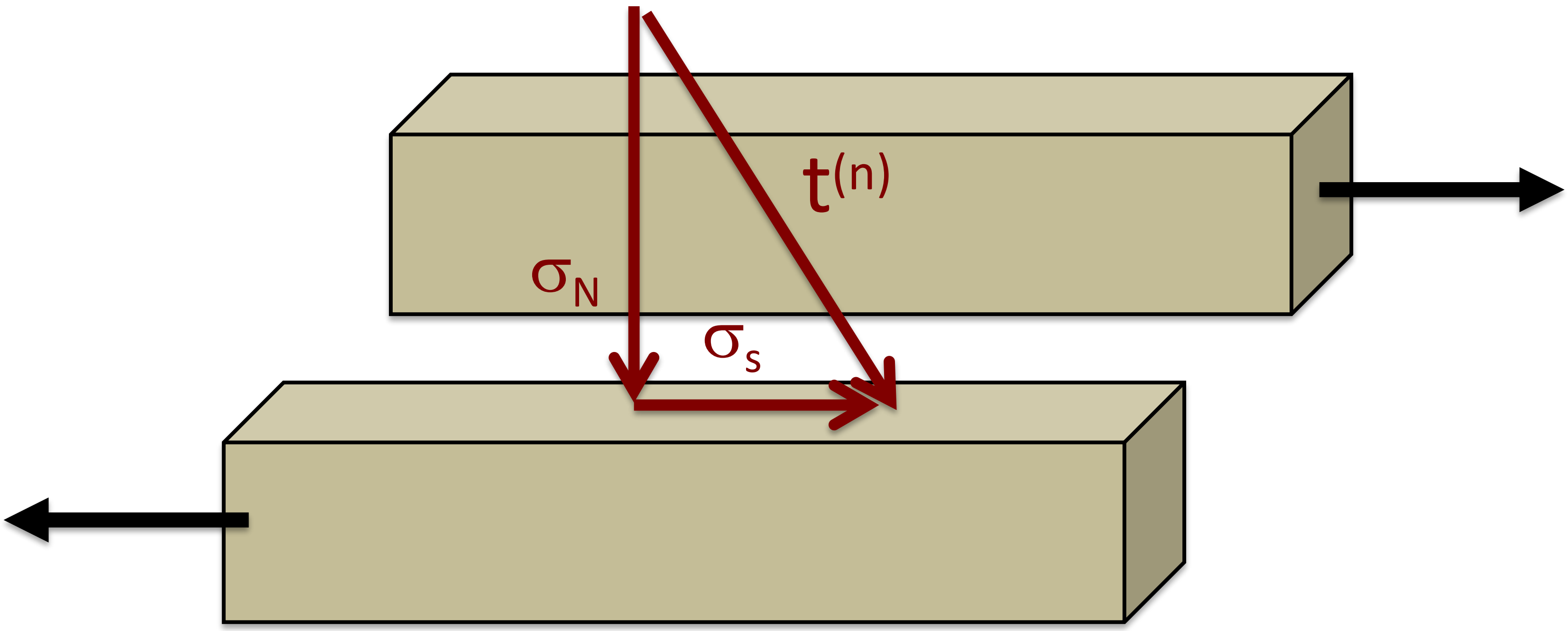
Byerlee's Friction Law (empirical)

Byerlee's law[1] *concerns the shear stress (τ, or σ_S) required to slide one rock over another.*

- *The rocks have macroscopically flat surfaces, but the surfaces have small asperities that make them "rough."*
- *For a given experiment and at normal stresses (σ_N) below about 2000 bars (200 MPa) the shear stress increases approximately linearly with the normal stress ($\sigma_S = 0.85\ \sigma_N$) and is highly dependent on rock type and the roughness of the surfaces (Mohr-Coulomb friction law).*
- *Byerlee's law states that with increased normal stress σ_N, the required shear stress σ_S continues to increase, but the rate of increase decreases ($\sigma_S = 0.5 + 0.6\ \sigma_N$), and becomes nearly independent of rock type*
- *The law describes an important property of crustal rock, and can be used to determine when slip along a geological fault takes place.*

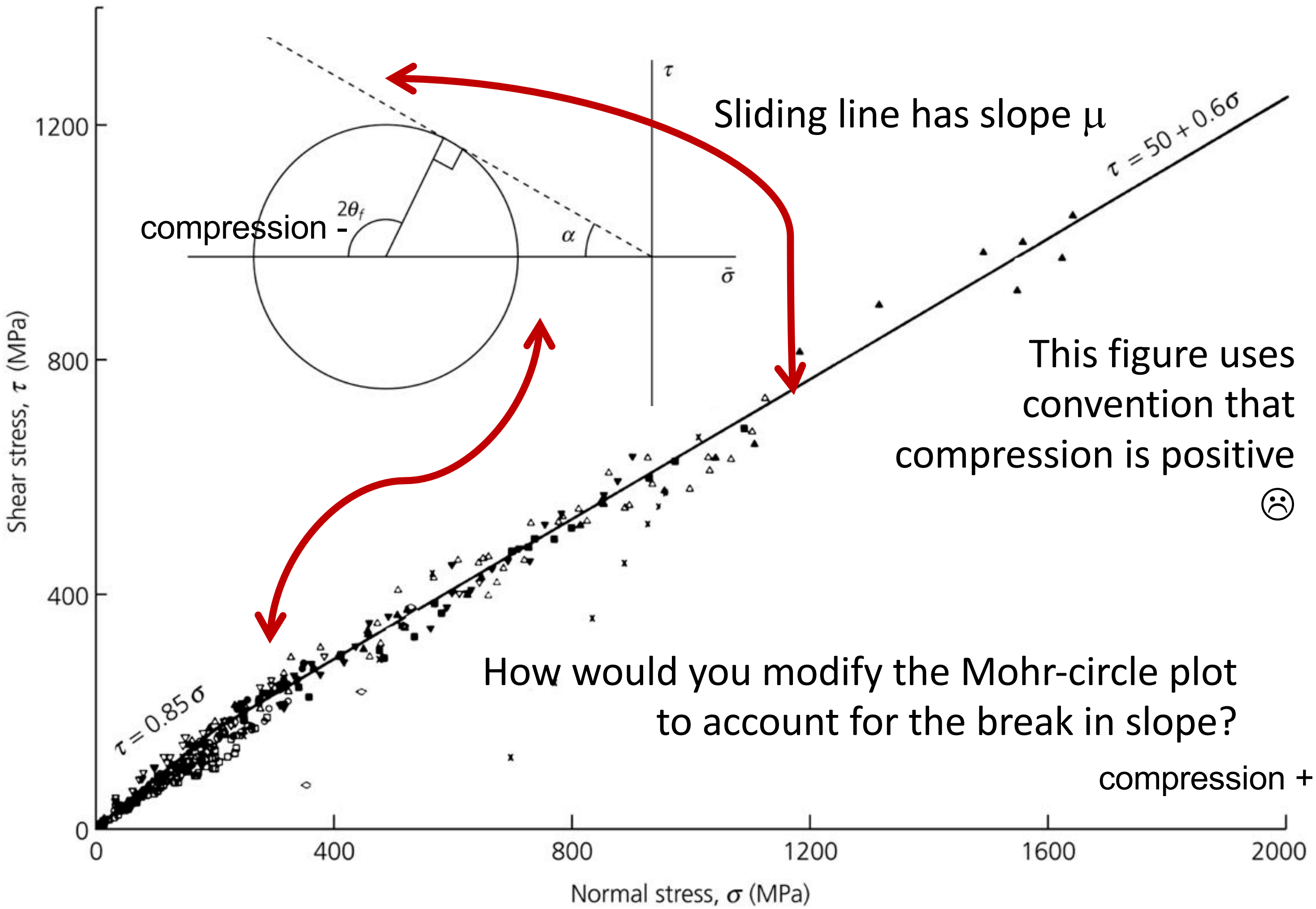
Adapted from Wikipedia

Sliding friction



$\sigma_S = -\mu\,\sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Byerlee's law

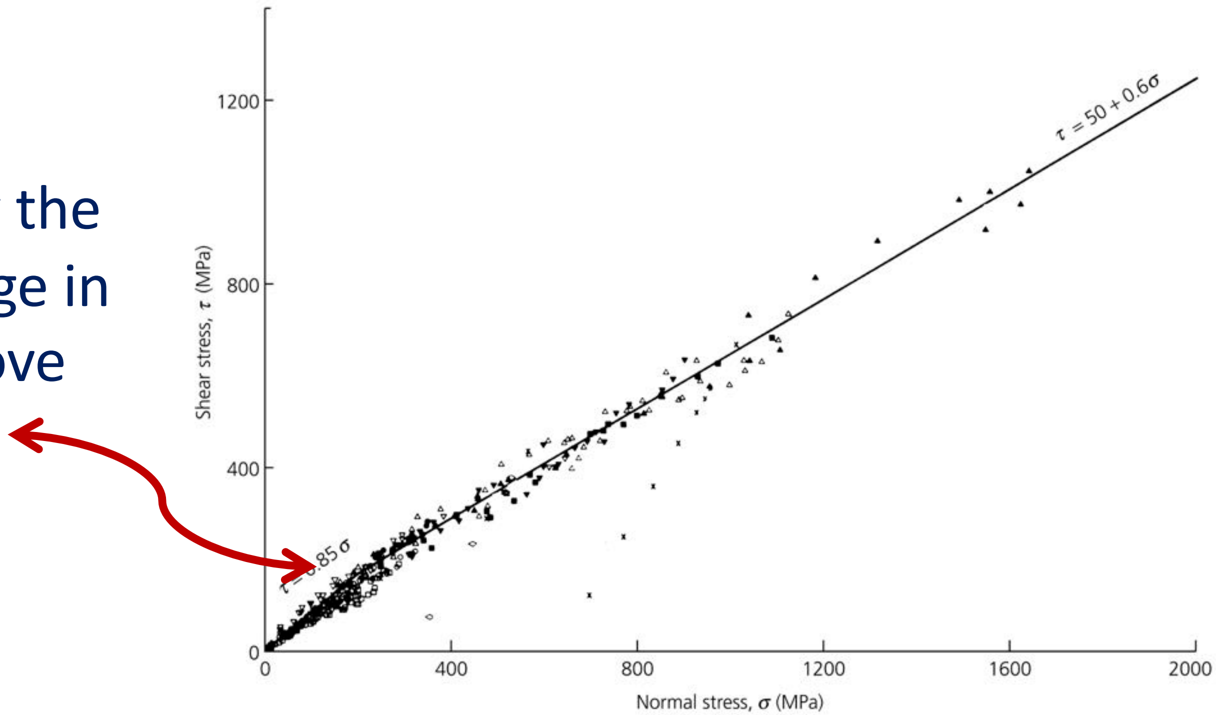
$$\tau \approx -.85\bar{\sigma} \qquad \bar{\sigma} < 200 \text{ MPa}$$

$$\tau \approx 50 - .6\bar{\sigma} \qquad \bar{\sigma} > 200 \text{ MPa.}$$

Two regions

Break-out discussions

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Speculate on why the slope might change in Byerlee's Law above $\sigma_N \approx 200$ MPa?

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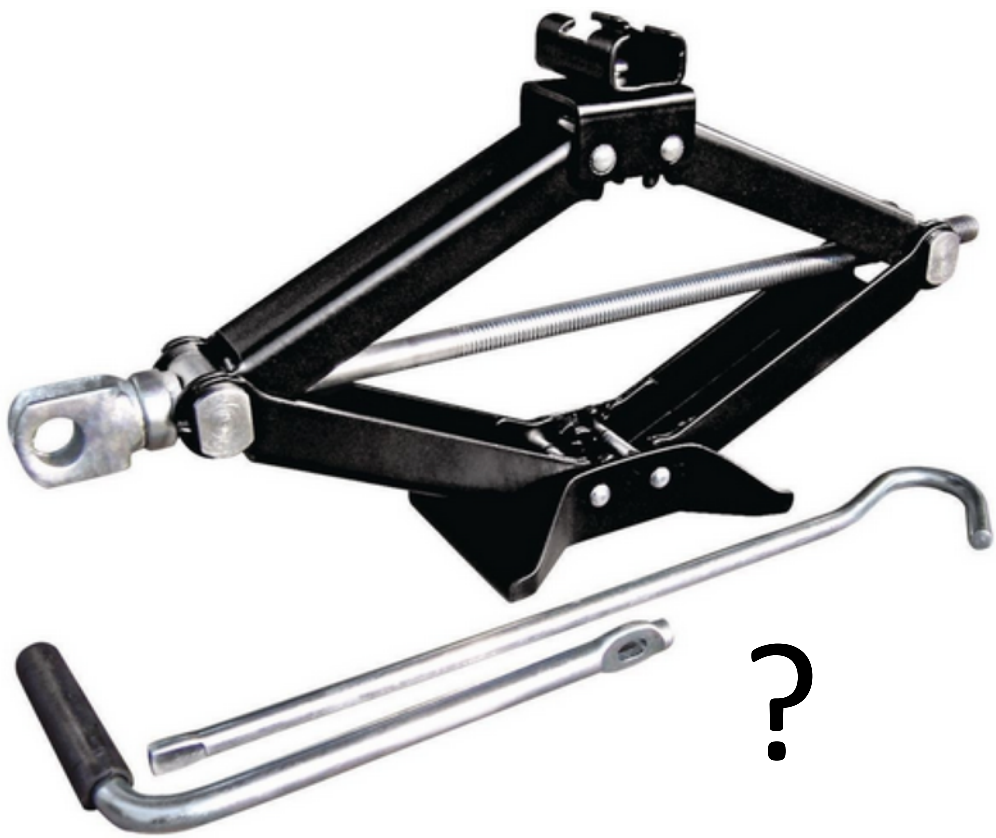
- Can high normal stress actually elastically flatten small asperities?
- Have asperities been already worn off as σ_N was rising in Lab experiments?

Speculate on why the slope might change in Byerlee's Law above $\sigma_N \approx 200$ MPa?

- Can high normal stress actually elastically flatten small asperities?
- Have asperities been already worn off as σ_N was rising in lab experiments?

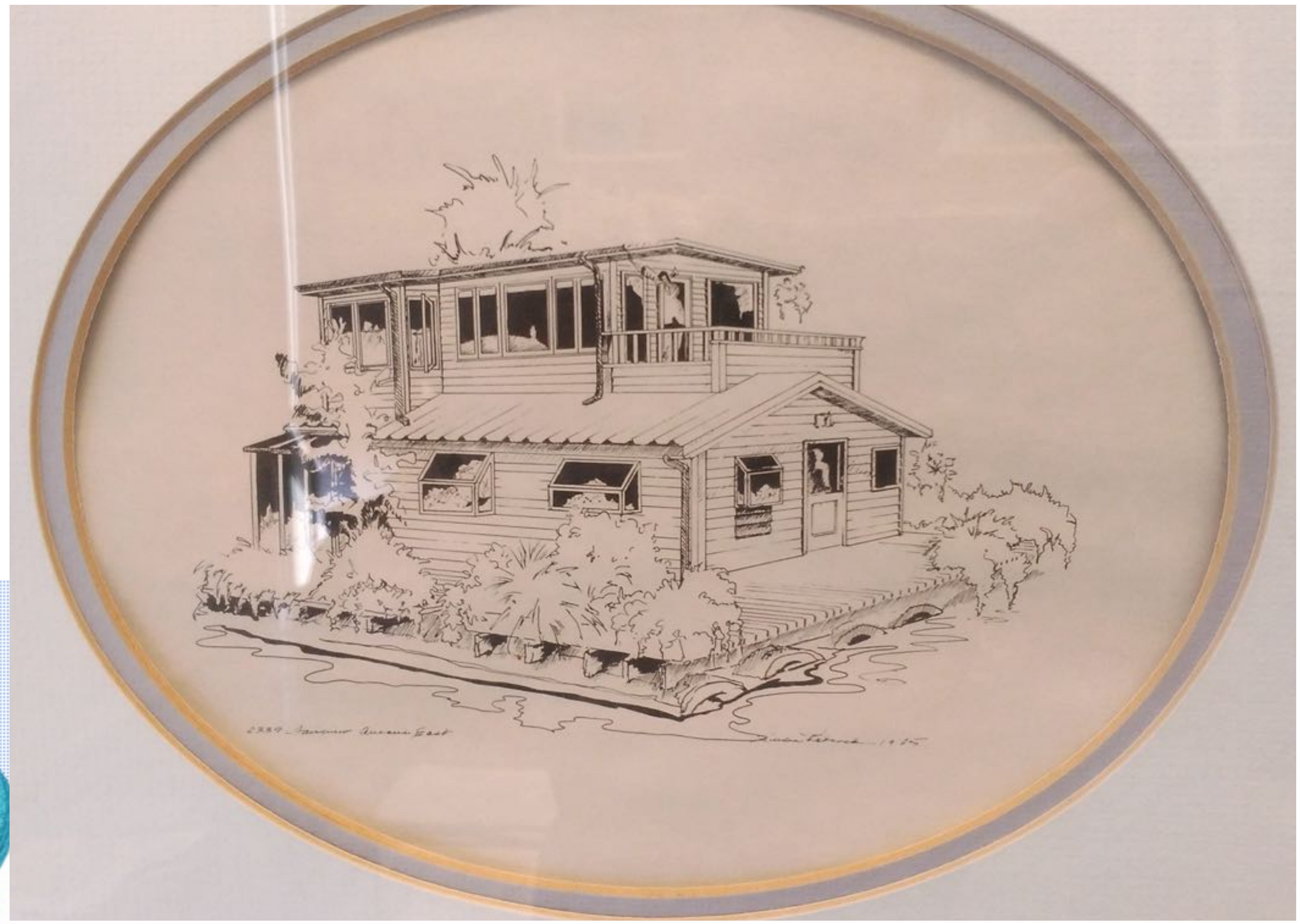
- Could you test these hypotheses experimentally?

How to jack up a structure (or a car) when there is no uniform solid ground to support the jack?

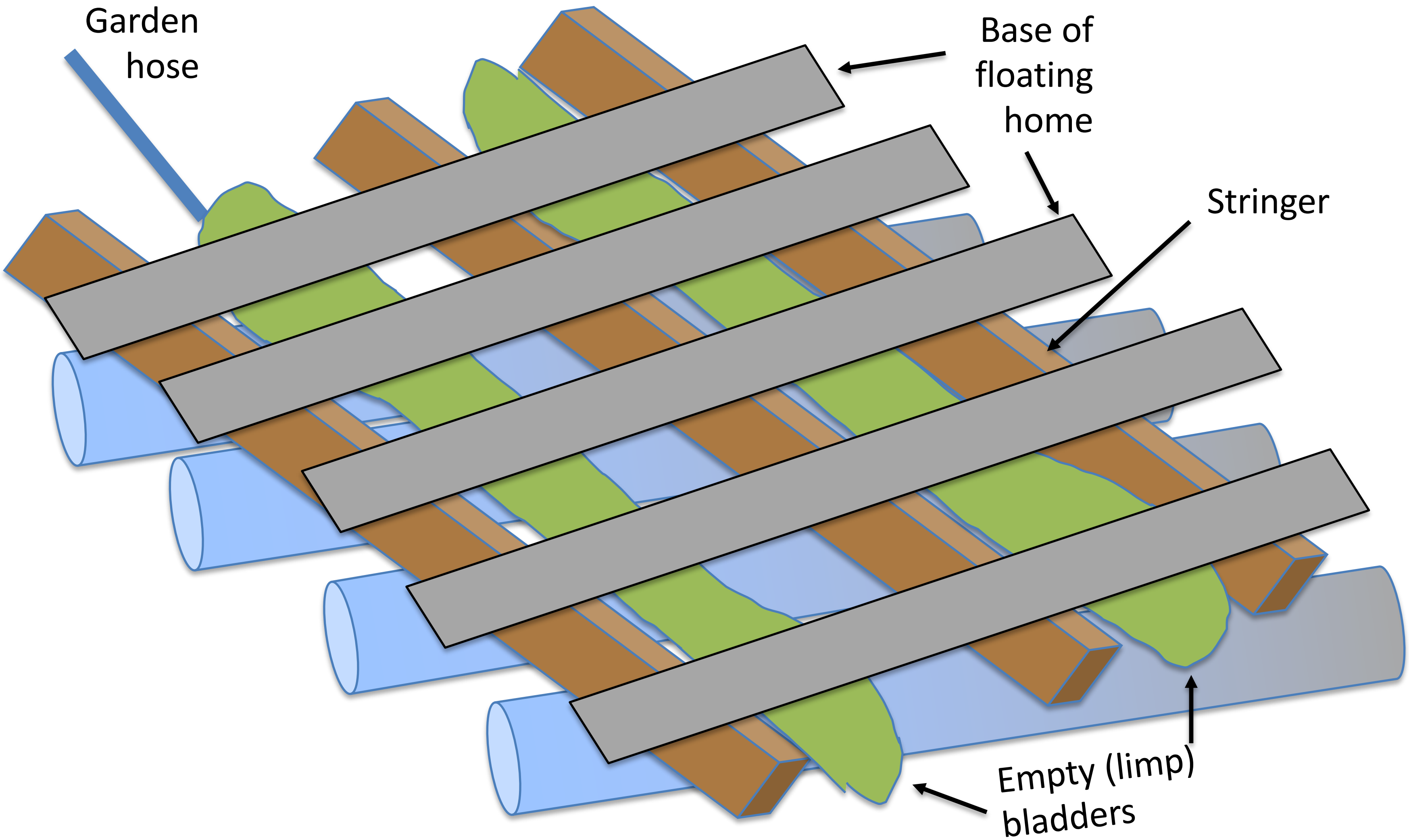


?

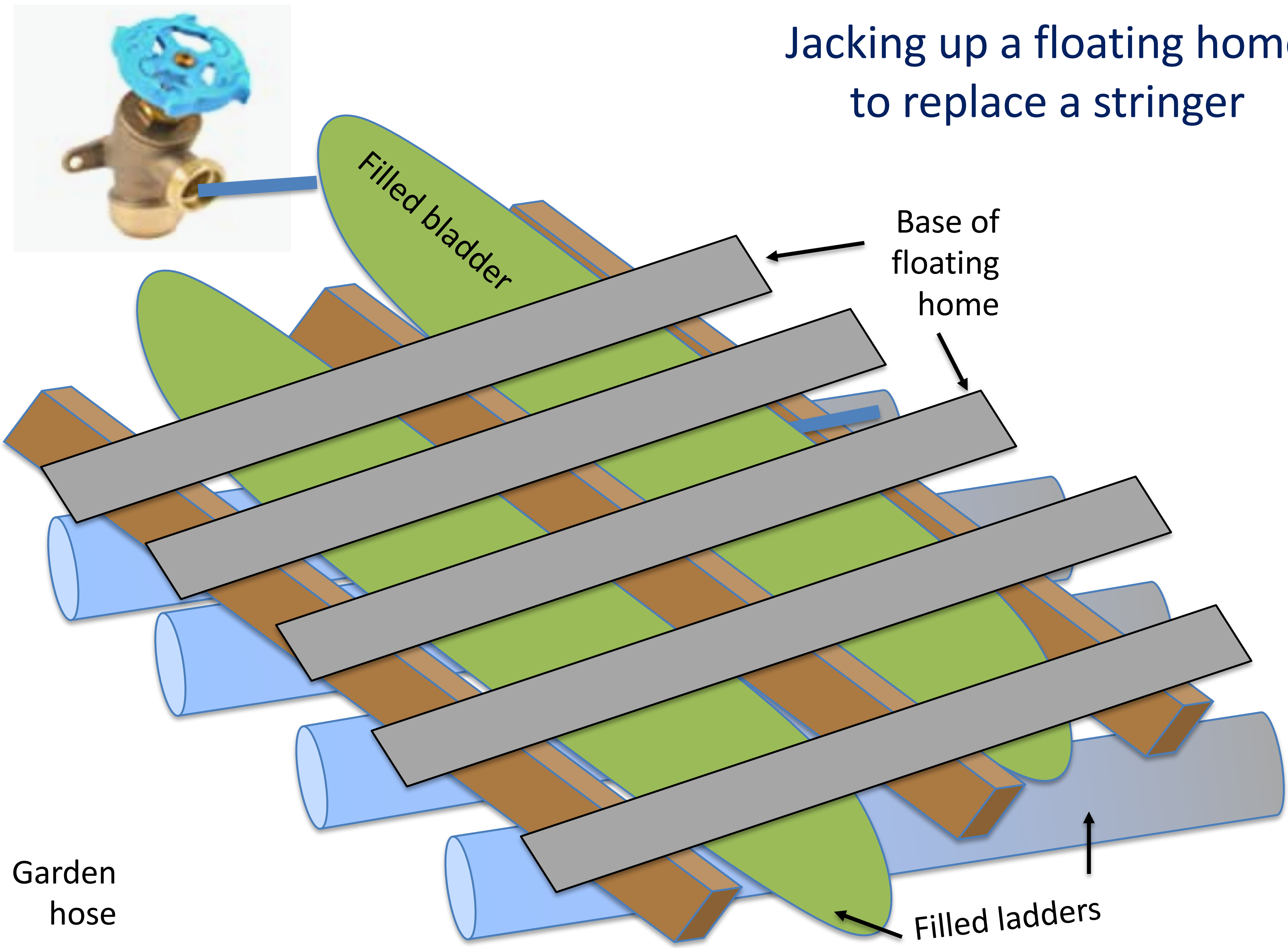
How about lifting a structure with a garden hose?



Let's jack a floating home off its float for stringer replacement



Jacking up a floating home to replace a stringer



Pore pressure

Most crustal rocks are wet

- Pores in the rock are filled with fluid
- What is the most common fluid?
- Are there other possibilities?

Pore pressure p

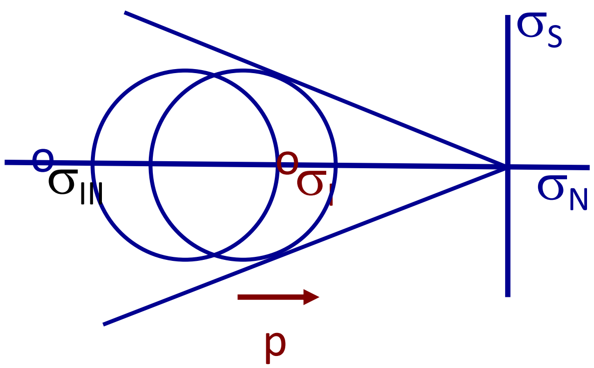
Fluids in rock pores and cracks is a lubricant

Failure when pore fluid is present with pore pressure p

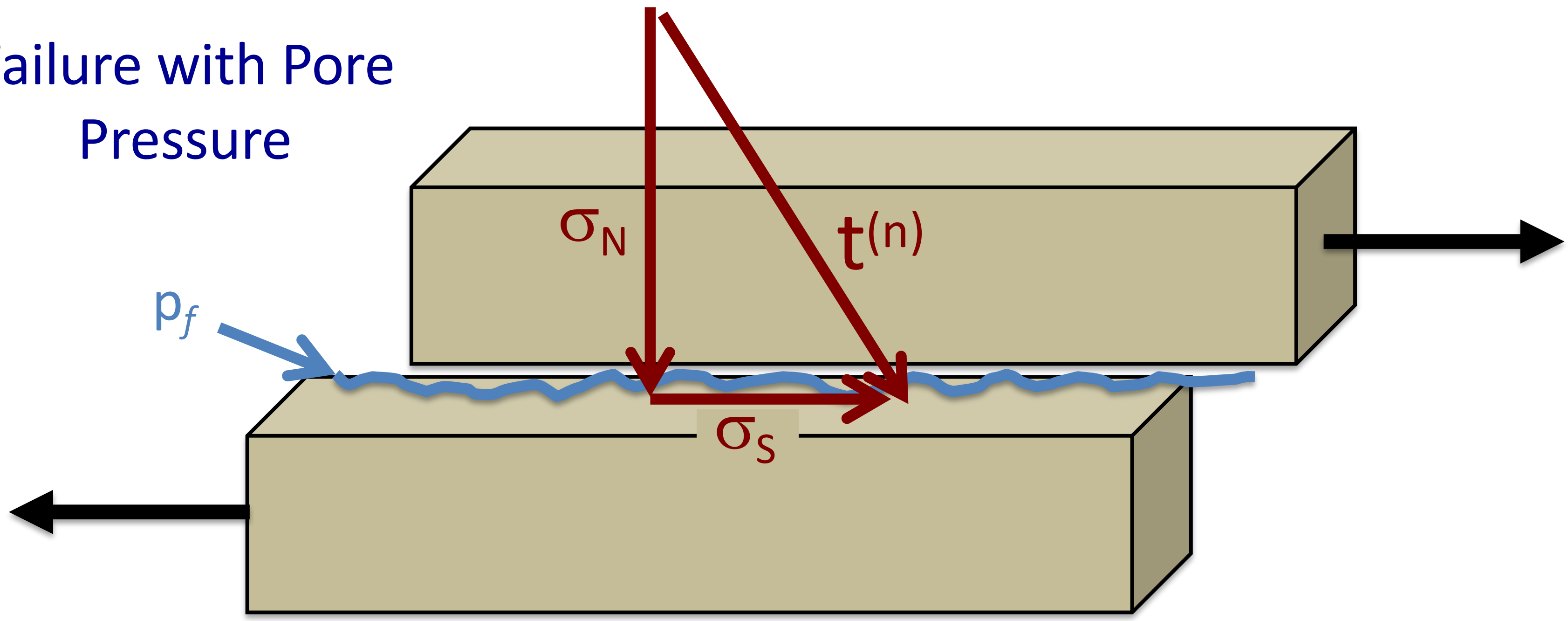
$$\sigma_S = -\mu(\sigma_N + p)$$

remember that p is positive, but compressive stress is negative

- Pore pressure reduces the clamping effect of σ_N
- Pressure jacks apart the locking asperities on faults
- Can lead to failure



Failure with Pore Pressure



Friction

$\sigma_S = \tau_0 - \mu\,(\sigma_N + p_f)$

- μ is ***coefficient of friction*** for sliding on an existing fault
- τ_0 is cohesion of the fault (generally small)
- p_f is fluid pore pressure

Fracture

$\sigma_S = \tau_0 - n\,(\sigma_N + p_f)$

- n is ***coefficient of internal friction*** for fracture on a new fault
- τ_0 is cohesion of the material in absence of any confining stress σ_N
- p_f is fluid pore pressure

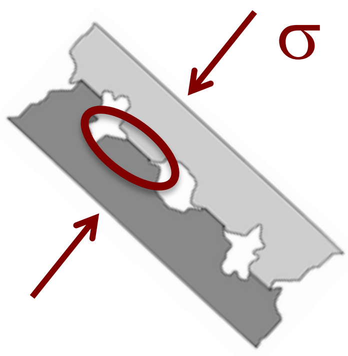
Possible states of pore pressure

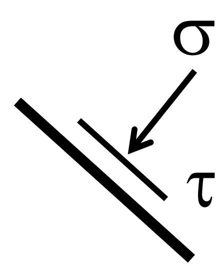
- Rocks are dry
- p_f is hydrostatic
  (what does this say about pore connectivity?)
- p_f is lithostatic
  (what does this say about pore connectivity?)

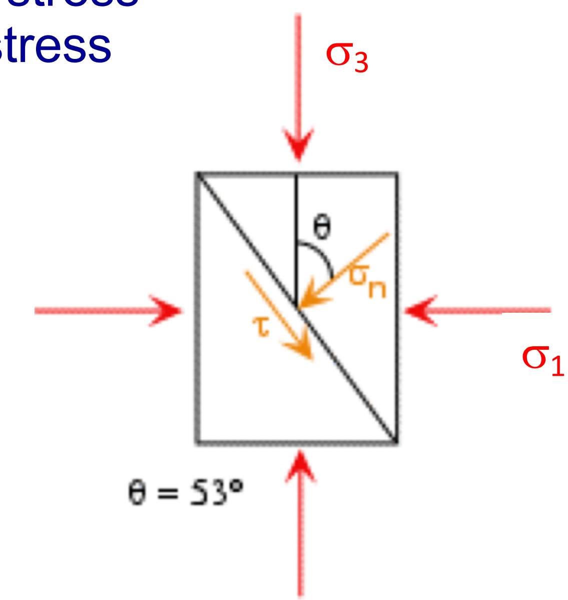
Coulomb stress

- Notion of friction:
  - More shear stress τ needed to overcome increase in normal stress σ and cause fault to slip – Byerlee's law is an example
- Coulomb stress
  - $\sigma_S = \tau - \mu (\sigma_N + p)$
  - where μ is intrinsic coefficient of friction, p is pore pressure (***not*** the mean stress $p=-\sigma_{ii}/3$, need to be careful of context)

- Basis is that real area of contact (much smaller than apparent area) is controlled by normal stress
  - deformation of asperities in response to normal stress
  - harder to over-ride asperities at higher normal stress



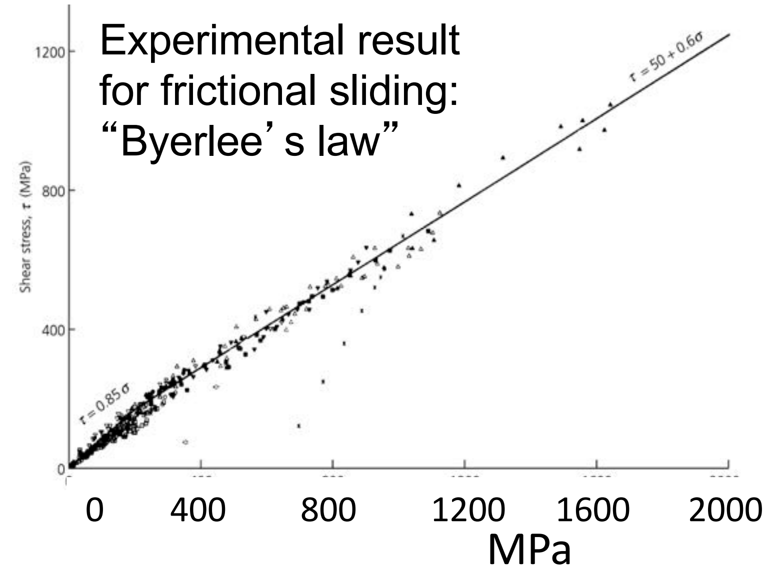




Lithostatic stress or Overburden stress

$p = \rho g z$

$\rho = 2700\ \mathrm{kg\ m^{-3}}$
$g \sim 10\ \mathrm{m\ s^{-2}}$
$z = 8\ \mathrm{km}$



Estimate overburden stress at 8 km depth

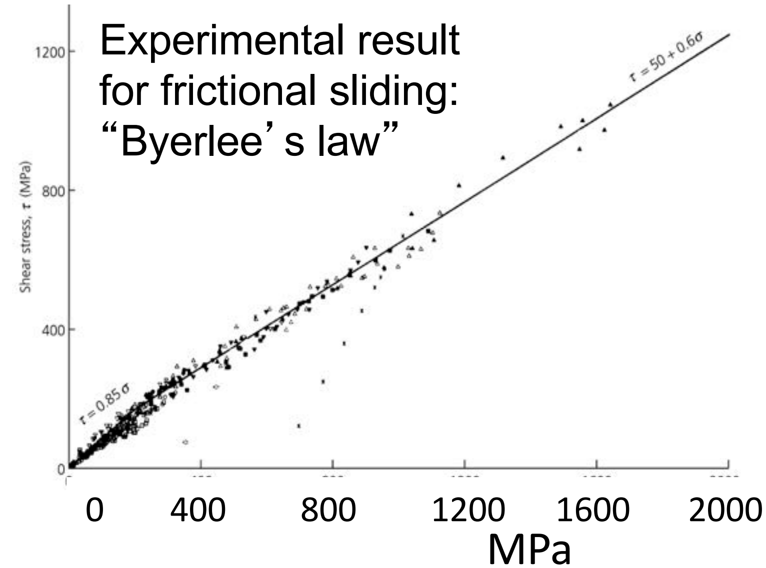
$p = \rho g z$
$= 2700\ \mathrm{kg\ m^{-3}} \times 10\ \mathrm{m\ s^{-2}} \times 8\ 10^{3}\ \mathrm{m}$
$= 216\ \mathrm{MPa}$

8 km is roughly the depth at which Byerlee's Law changes slope, indicating nonzero cohesion and reduced coefficient of friction.

Hydrostatic pore pressure, or Hydrostatic stress

p = $\rho_w g z$

ρ_w = 1000 kg m^{-3}
g ~ 10 m s^{-2}
z = 8 km



Estimate pore pressure at 8 km depth

p = $\rho g z$
= 1000 kg m^{-3} × 10 m s^{-2} × 8 10^3 m
= 80 MPa

Break-out questions

Influence of pore pressure p_f on fault slip

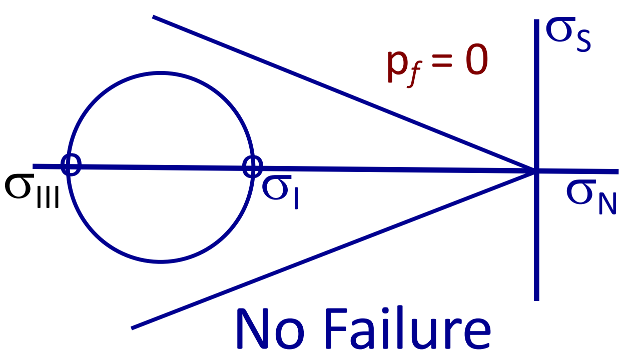
Fluid in rock pores and cracks is a lubricant, and fluid pressure p_f is a non-negative quantity.

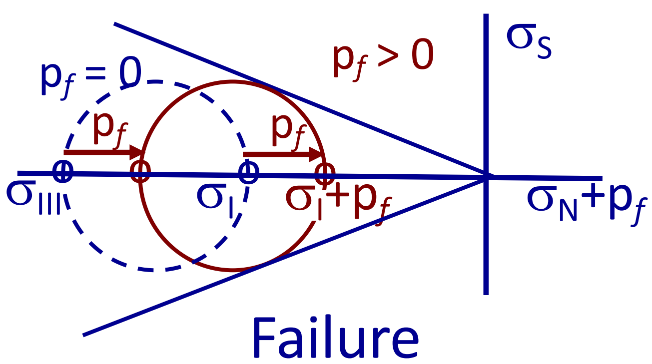
The frictional failure criterion is modified when pore fluid is present.

$$\sigma_S = \tau_0 - \mu\,(\sigma_N + p_f)$$

- τ_0 is cohesion on the fault (see Byerlee's Law for $\sigma_N > 200$ MPa)
- μ is coefficient of friction
- p_f is pore pressure (***not*** the mean stress $p=-\sigma_{ii}/3$)

- What is the fluid doing at the microscale to enhance slip? (Think about the asperities)
- Explain how the Mohr's circles below illustrate the role of pore pressure.





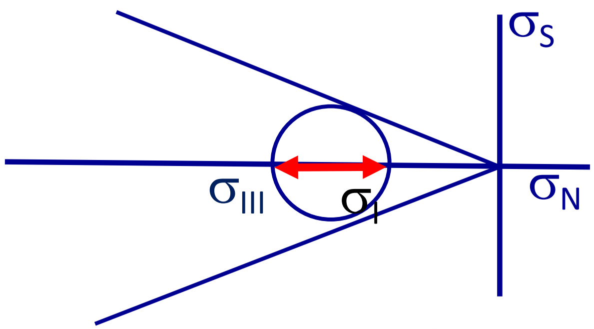
Crustal strength

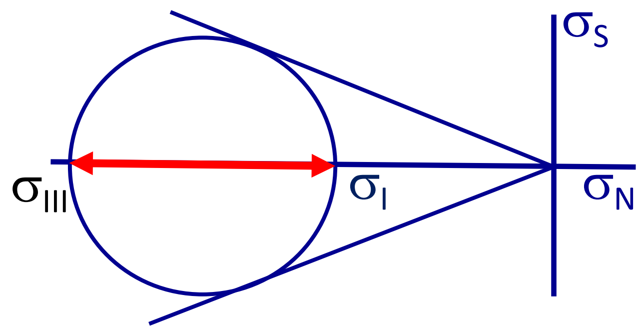
Strength of the crust under *in situ* stress is estimated by the largest differential stress that it can sustain without failure.

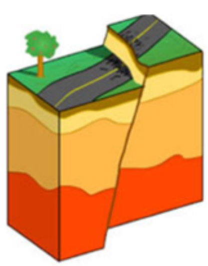
- The surface of the Earth is a principal plane for stress.
- One of the principal stresses is vertical and lithostatic, $\sigma_V \sim \rho g z$

If $\sigma_V = \sigma_{III}$
- There is horizontal extension
- Strength $(\sigma_I - \sigma_{III})$ is low

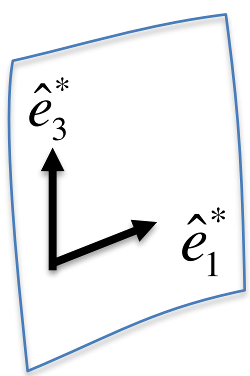
If $\sigma_V = \sigma_I$
- There is horizontal compression
- Strength $(\sigma_I - \sigma_{III})$ is high

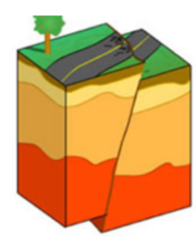




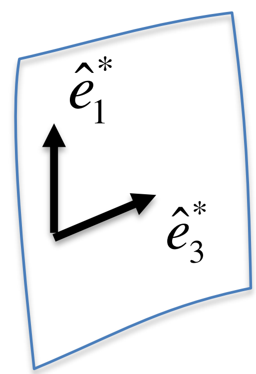


Normal fault





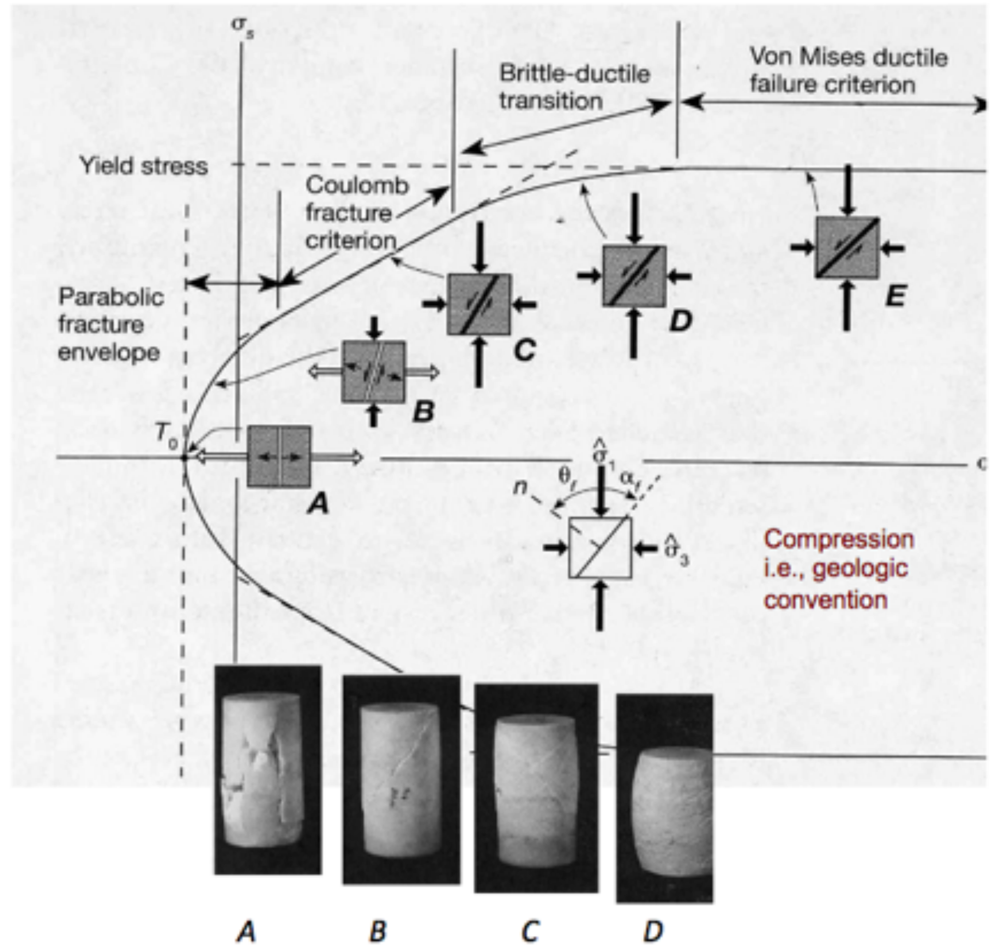
Reverse fault



Class-prep questions for
Wednesday Class_16

Style of Failure under Various Normal Stresses σ_N

The figure shows the failure envelope and failure modes in stress space, based on experiments on rocks subjected to a range of normal stresses σ_N. Note that these authors used the convention that compression is positive (yuck …)



- Describe in words what is happening in this generalization of the failure envelopes that we have discussed in class.
- In a sentence or two for each, describe characteristics of the failure mode in each of the 5 stress regimes *A, B, C, D*, and *E*. The regime names, the angles of the failure planes, and the visual states of the samples after the experiments ended may be helpful.

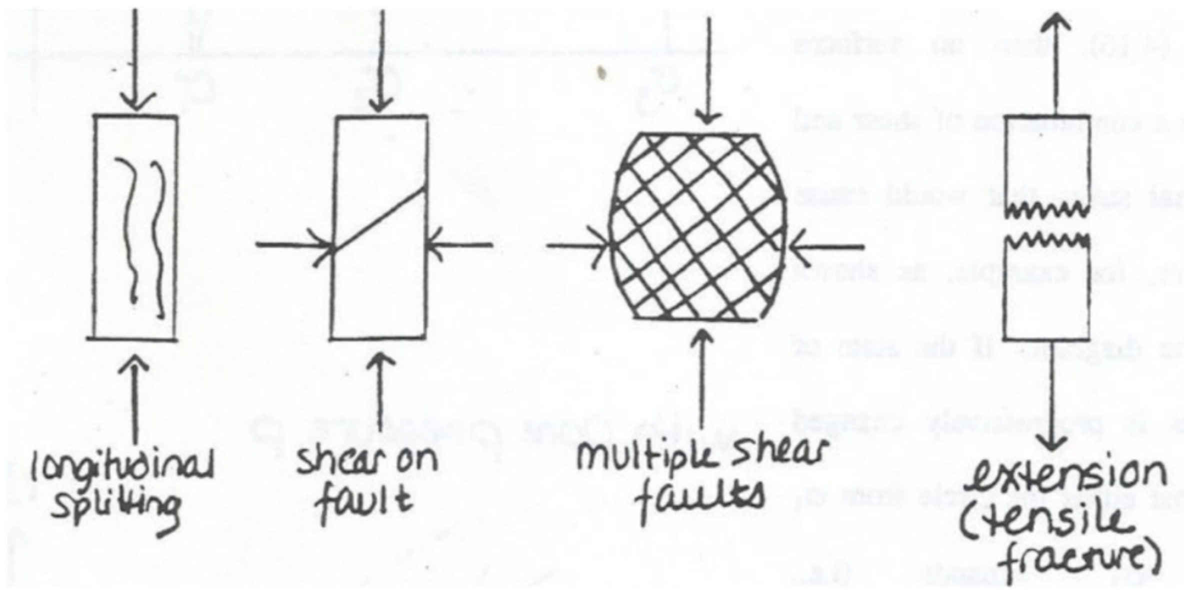
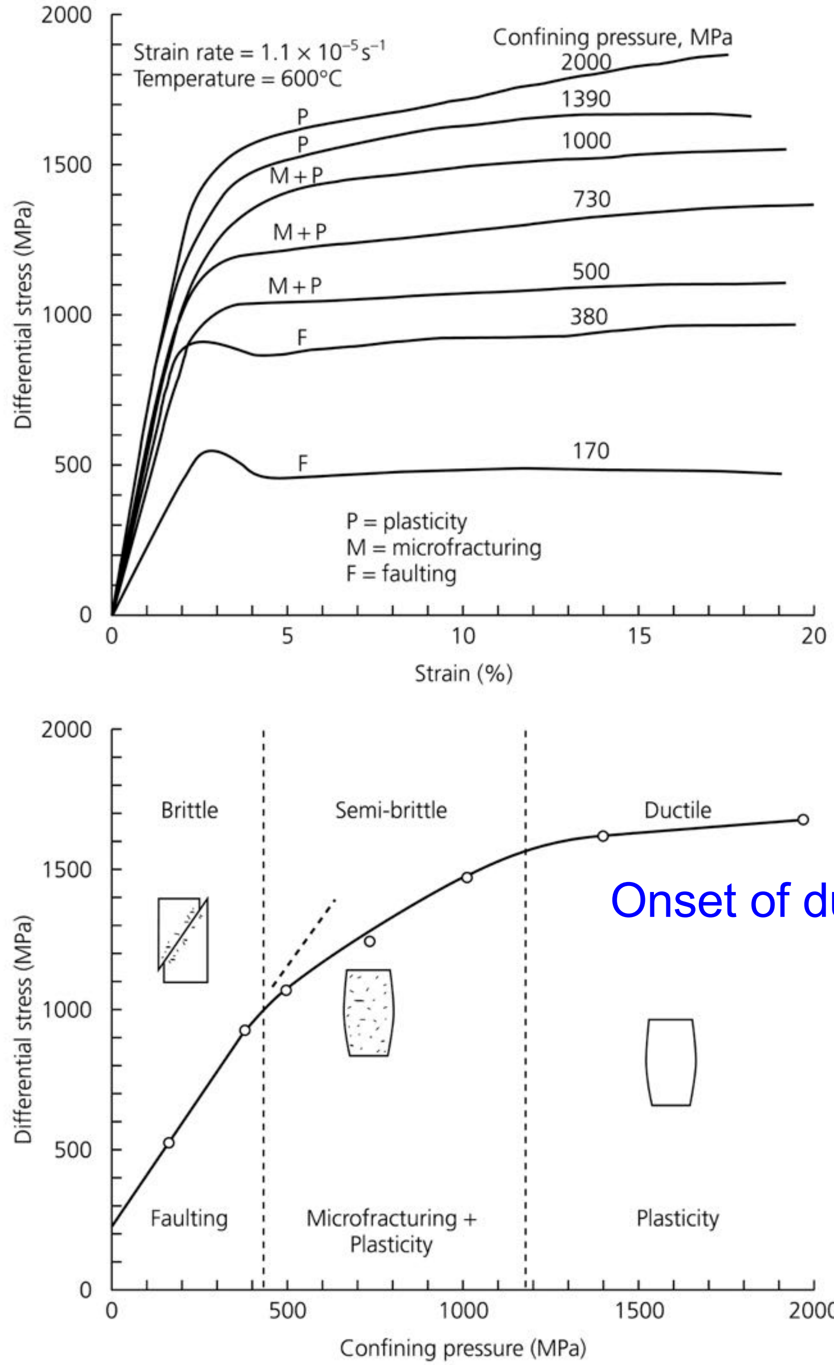

longitudinal splitting
shear on fault
multiple shear faults
extension (tensile fracture)

Figure 5.7-3: Rheology of rocks subjected to large compressive stresses.



Onset of ductility

Stress-strain relation for ductile flow

Laboratory experiments on minerals find ductile flow to be:

$$\frac{de}{dt} = \dot{e} = f(\sigma)\ \underline{A \exp[-(E^* + PV^*)/RT]}$$

viscosity^{-1}

T = temperature

R = the gas constant

P is pressure

$f(\sigma)$ = function of the stress difference $|\sigma_1 - \sigma_3|$

A = a constant

E^*, V^* = activation energy and volume (effects of T and P)

mineral-specific

In terms of the principal stresses,

$$f(\sigma) = |\sigma_1 - \sigma_3|^n$$

$$\dot{e} = |\sigma_1 - \sigma_3|^n \; A \exp[-(E^* + PV^*)/RT]$$

The rheology of such fluids is characterized by a power law.
If $n = 1$ the material is called *Newtonian*, whereas a non-Newtonian fluid with $n = 3$ is often used to represent the mantle.

The viscosity depends on both temperature and pressure

$$\eta = (1/2A)\exp[(E^* + PV^*)/RT]$$

The viscosity decreases exponentially with temperature, and increases exponentially with pressure!

Example: a common flow law for dry olivine is:

$$\dot{e} = 7 \times 10^4 \, |\sigma_1 - \sigma_3|^3 \exp\left(\frac{-0.52 \text{ MJ/mol}}{RT}\right) \qquad \text{for } |\sigma_1 - \sigma_3| \leq 200 \text{ MPa}$$

$$= 5.7 \times 10^{11} \exp\left[\frac{-0.54 \text{ MJ/mol}}{RT}\left(1 - \frac{|\sigma_1 - \sigma_3|}{8500}\right)^2\right] \qquad \text{for } |\sigma_1 - \sigma_3| > 200 \text{ MPa}$$

where $\dot{e}$ is in s^{-1}.

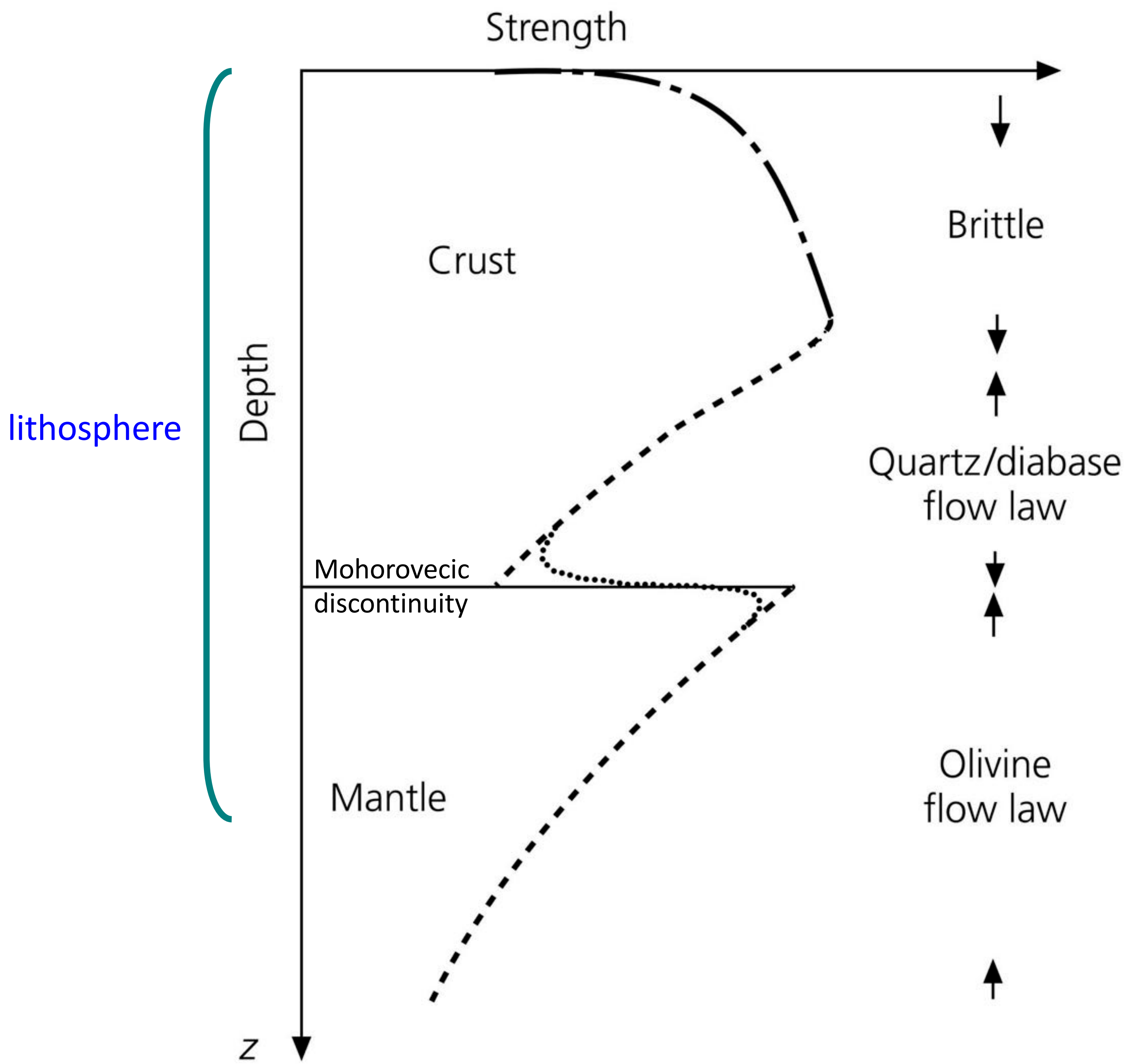
A flow law for quartz is:

$$\dot{e} = 5 \times 10^6 \, |\sigma_1 - \sigma_3|^3 \exp\left(\frac{-0.19 \text{ MJ/mol}}{RT}\right) \qquad \text{for } |\sigma_1 - \sigma_3| < 1000 \text{ MPa}$$

At a given strain rate, quartz is much weaker than olivine!

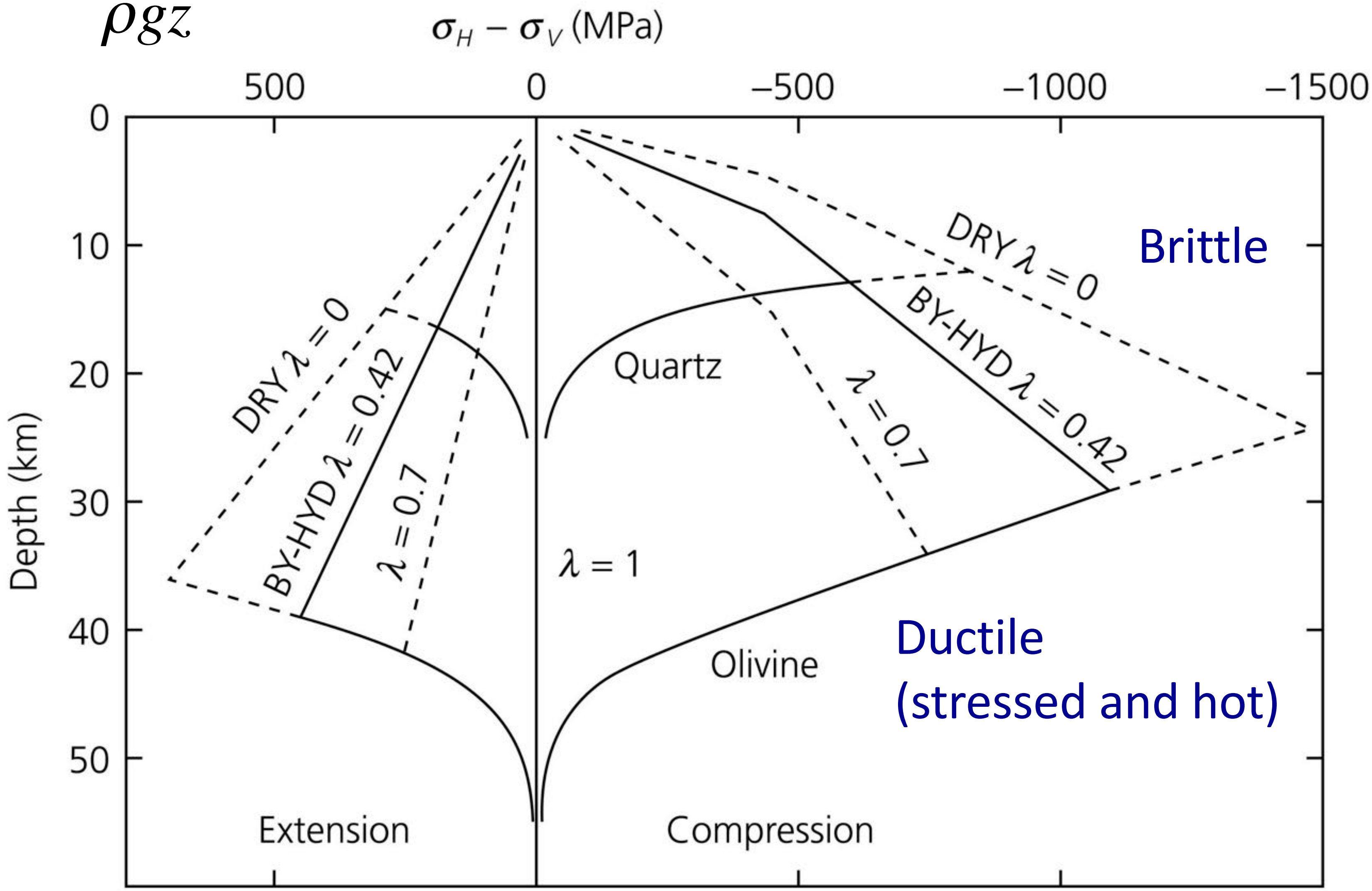
The quartz-rich continental crust is weaker than the olivine-rich oceanic crust.

Figure 5.7-17: Schematic strength envelope for continents.



$$\lambda = \frac{p_w}{\rho g z}$$

Strength of the Lithosphere



Strength envelopes of olivine in aging and cooling oceanic plate