ESS 411/511 Geophysical Continuum Mechanics Class #18

Highlights from Class #17 – Abigail Thienes
Today's highlights on Friday – _______________
Wednesday is Veteran's Day)

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. https://courses.washington.edu/ess511/NOTES/notes.shtml

- Stein and Wysession 5.7.2
- Stein and Wysession 5.7.3/4
- Raymond notes on failure

Also see slides about current topics

- Failure and Mohr's circles – slides

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas

- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

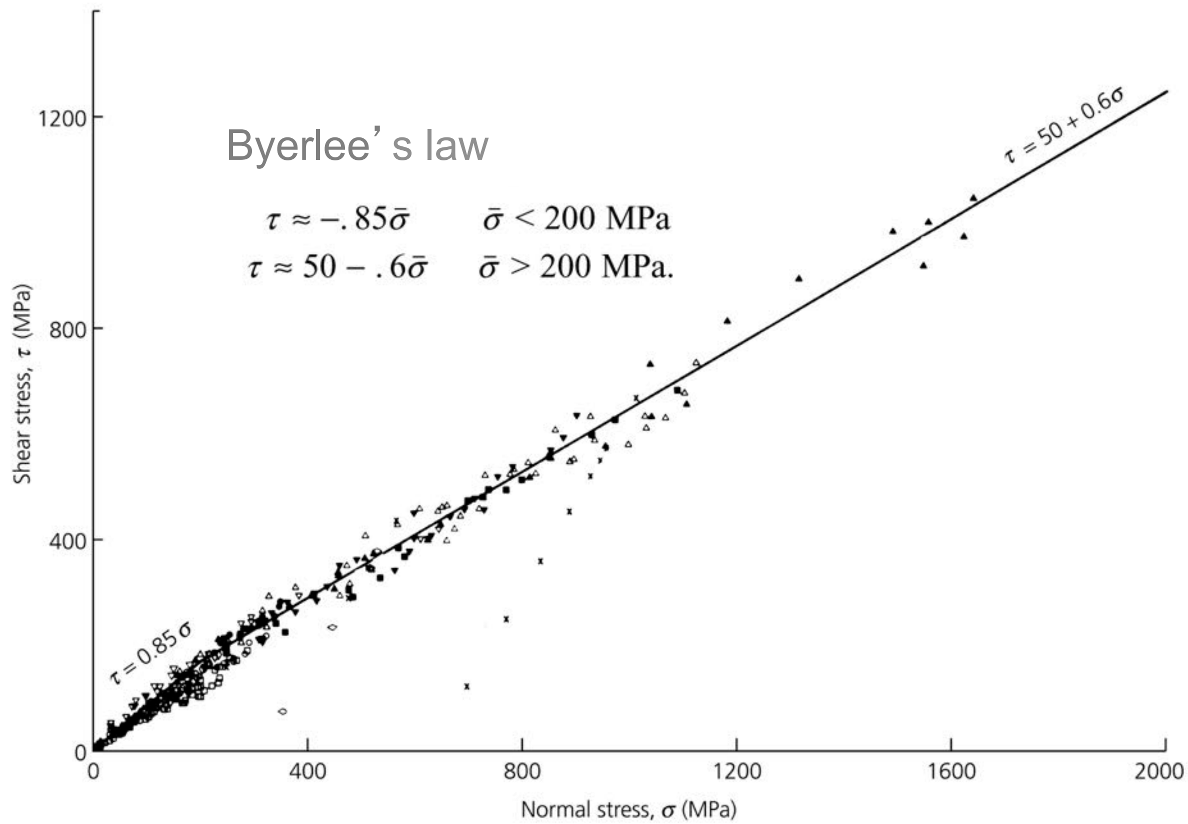
- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Warm-up (Break-out rooms)
Hydrostatic stress and Overburden stress

$P_w \approx 1000$ kg m^{-3}
$\rho_r \approx 2700$ kg m^{-3}
$g \approx 10$ m s^{-2}
$z = 8$ km

At 8 km depth:

- Estimate hydrostatic stress $p_w = \rho_w gz$ in MPa
- Estimate overburden stress $p_r = \rho_r gz$ in MPa
- Relate p_r at 8 km to Byerlee's Law



Hydrostatic stress and Overburden stress

$P_w \approx 1000 \text{ kg m}^{-3}$
$\rho_r \approx 2700 \text{ kg m}^{-3}$
$g \approx 10 \text{ m s}^{-2}$
$z = 8 \text{ km}$

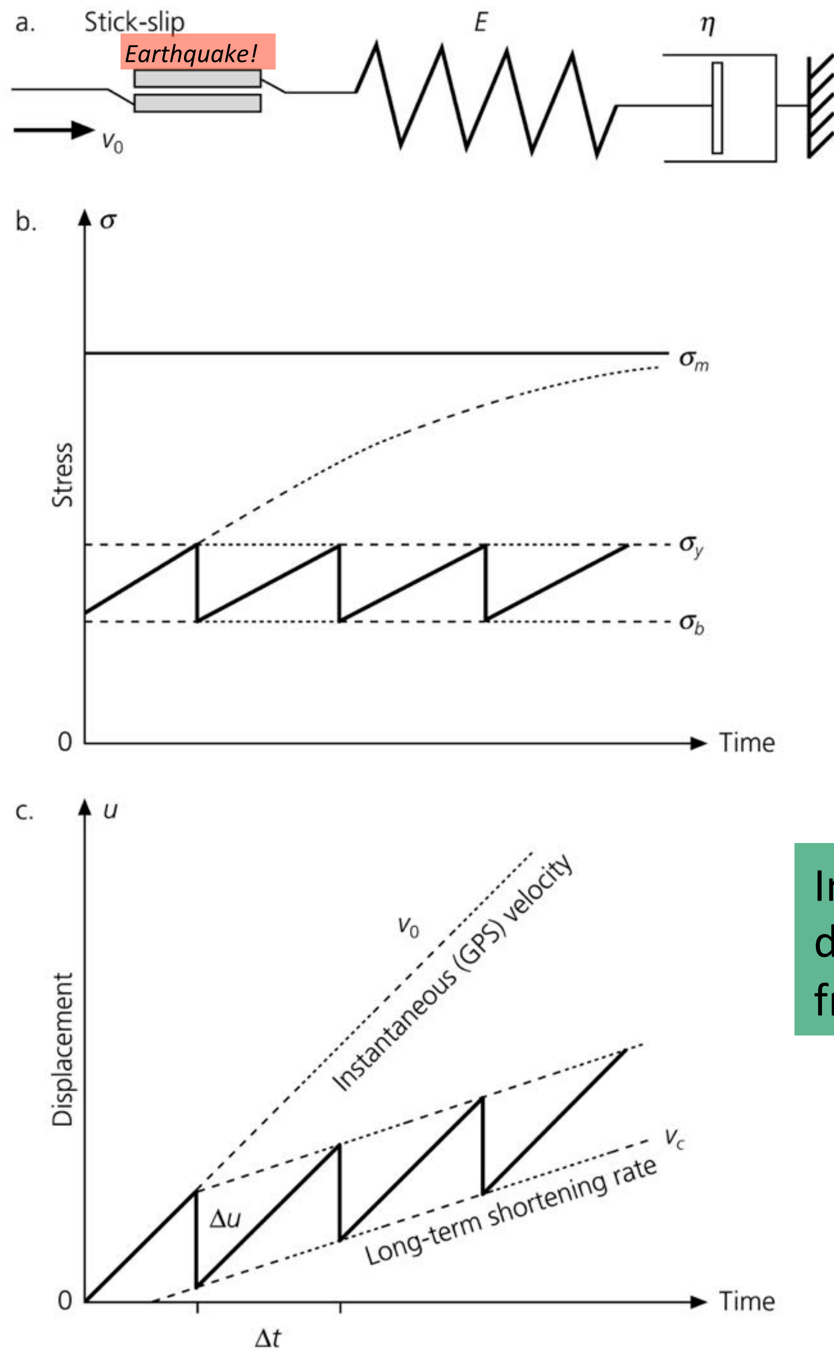
$$\begin{aligned} p_r &= \rho_r g z \\ &\approx 2700 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 8\times10^3 \text{ m} \\ &= 216\times10^3 \text{ kg m s}^{-2}/\text{m}^2 \\ &= 216 \text{ MPa} \end{aligned}$$

$$\begin{aligned} p_w &= \rho_w g z \\ &\approx 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 8\times10^3 \text{ m} \\ &= 80\times10^3 \text{ kg m s}^{-2}/\text{m}^2 \\ &= 80 \text{ MPa} \end{aligned}$$

This assumes that water is connected to the free surface
But p_w can be

- less if rocks are dry
- More if water is trapped in formations and pressurized by p_r

Figure 5.7-28: Modeling the deformation of South America with a viscoelastic-plastic crust.



Instantaneous rate of deformation may differ from long-term rate

Class-prep questions for today (break-out rooms)

Influence of pore pressure p_f on fault slip

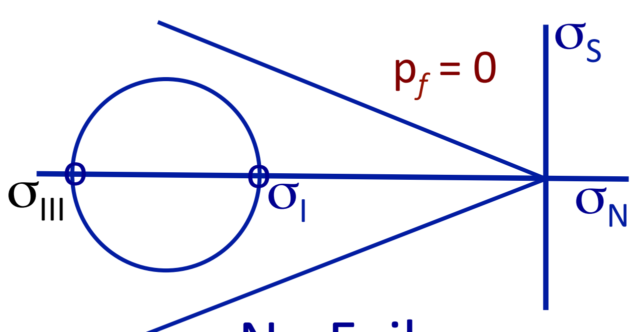
Fluid in rock pores and cracks is a lubricant, and fluid pressure p_f is a non-negative quantity.

The frictional failure criterion is modified when pore fluid is present.

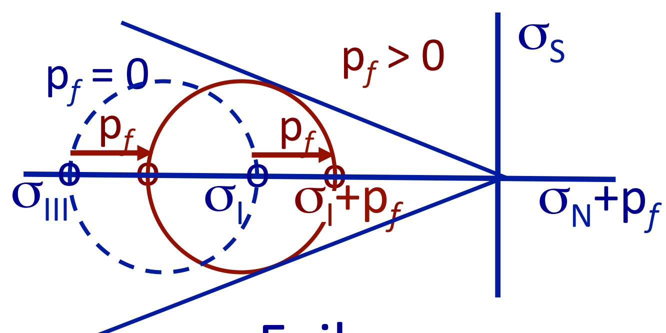
$$\sigma_S = \tau_0 - m\,(\sigma_N + p_f)$$

- ○ τ_0 is cohesion on the fault (see Byerlee's Law for $\sigma_N > 200$ MPa)
- ○ μ is coefficient of friction
- ○ p_f is pore pressure (***not*** the mean stress $p=-\sigma_{ii}/3$)

- What is the fluid doing at the microscale to enhance slip? (Think about the asperities)
- Explain how the Mohr's circles below illustrate the role of pore pressure.



No Failure

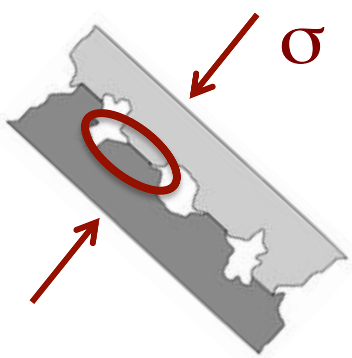


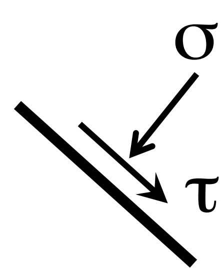
Failure

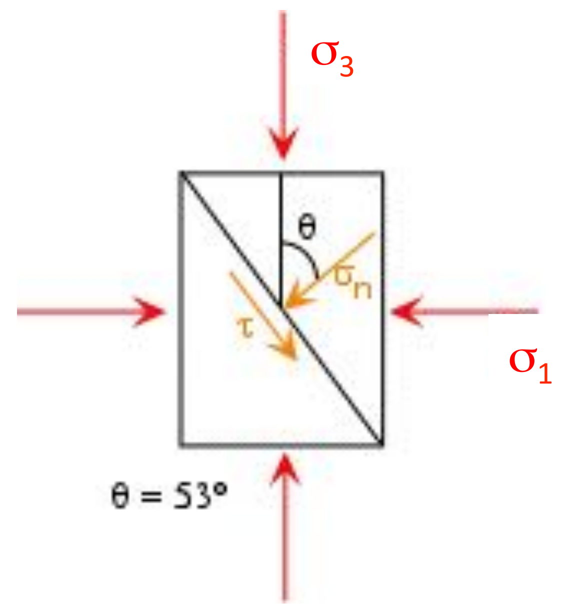
Coulomb stress and rock fracture

- Notion of friction:
  - More shear stress τ is needed to overcome increase in normal stress σ and cause a fault to slip – Byerlee's law is an example
- Coulomb stress
  - $\sigma_S = \tau_0 - \mu (\sigma_N + p_f)$
  - where μ is intrinsic coefficient of friction, p_f is pore pressure
    (***not*** the mean stress $p=-\sigma_{ii}/3$, need to be careful of context)

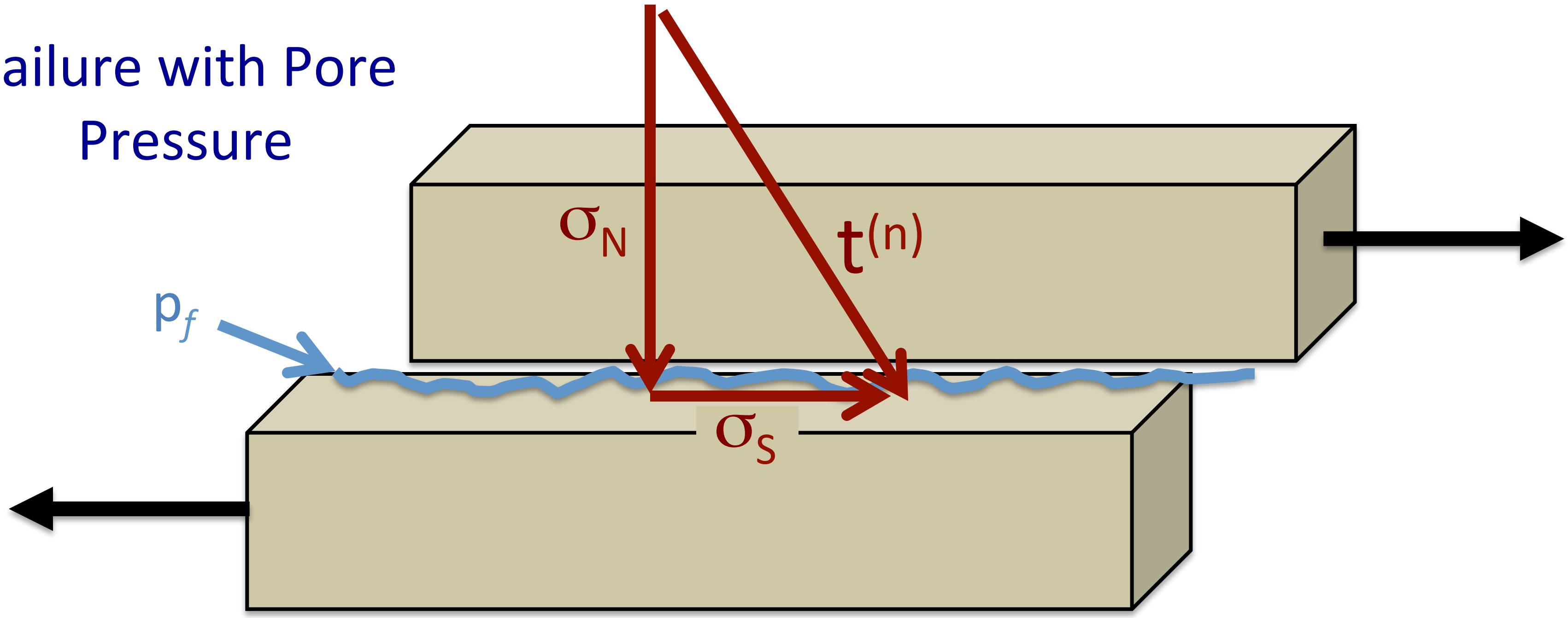
- The real area of contact (much smaller than apparent area) is controlled by normal stress
  - deformation of asperities in response to normal stress
    increases contact area
  - harder to over-ride asperities at higher normal stress







Failure with Pore Pressure



Friction

$\sigma_S = \tau_0 - \mu\,(\sigma_N + p_f)$

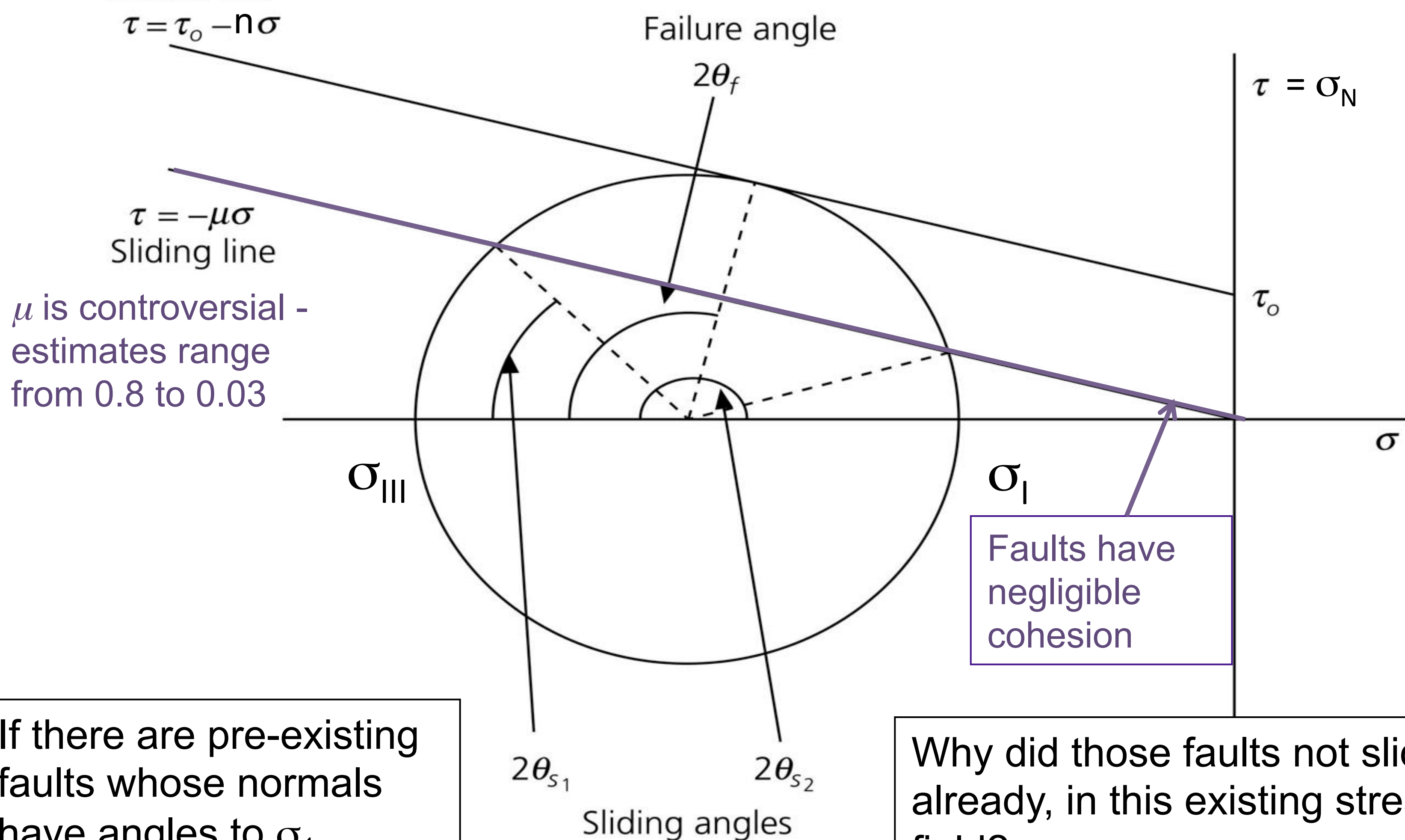
- μ is ***coefficient of friction*** for sliding on an existing fault
- τ_0 is cohesion of the fault (generally small)
- p_f is fluid pore pressure

Fracture

$\sigma_S = \tau_0 - n\,(\sigma_N + p_f)$

- n is ***coefficient of internal friction*** for fracture on a new fault
- τ_0 is cohesion of the material in absence of any confining stress σ_N
- p_f is fluid pore pressure

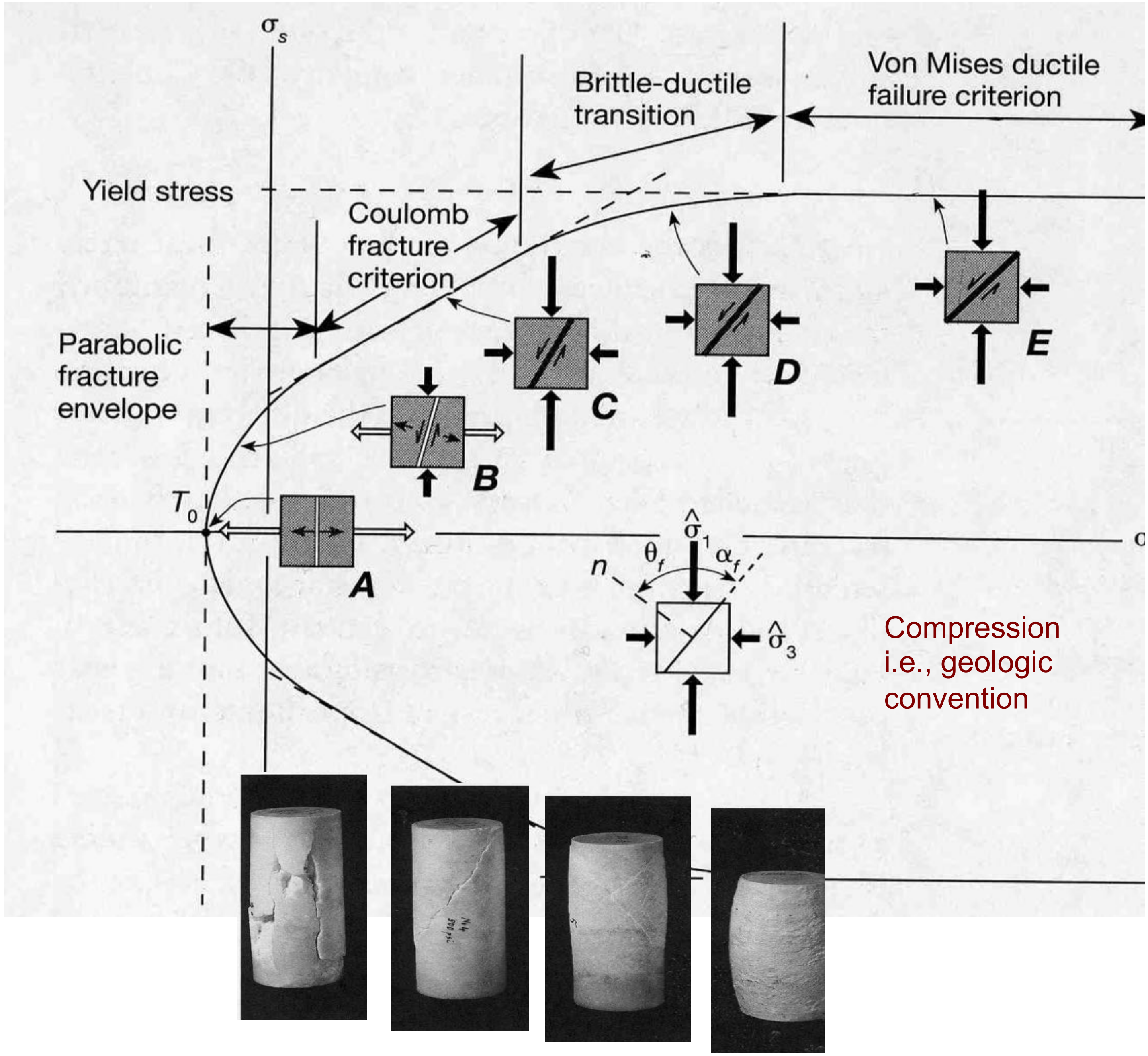
Old fault, new fault …



If there are pre-existing faults whose normals have angles to σ_1 between θ_{S1} and θ_{S2} those faults will slide before new fracture can occur.

Why did those faults not slide already, in this existing stress field?

- A recent earthquake could have just raised the differential stress (σ_I-σ_{III})



σs
Brittle-ductile transition
Von Mises ductile failure criterion
Yield stress
Coulomb fracture criterion
Parabolic fracture envelope
T0
A
B
C
D
E
σ̂1
θf
αf
n
σ̂3
σ
Compression
i.e., geologic
convention

A *B* *C* *D*

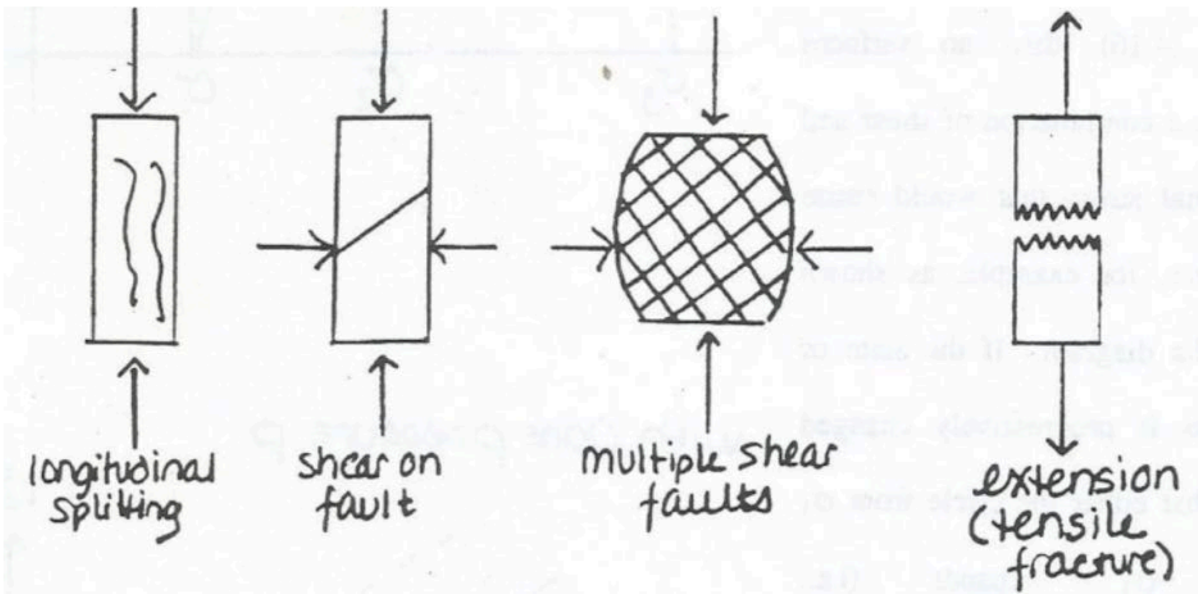
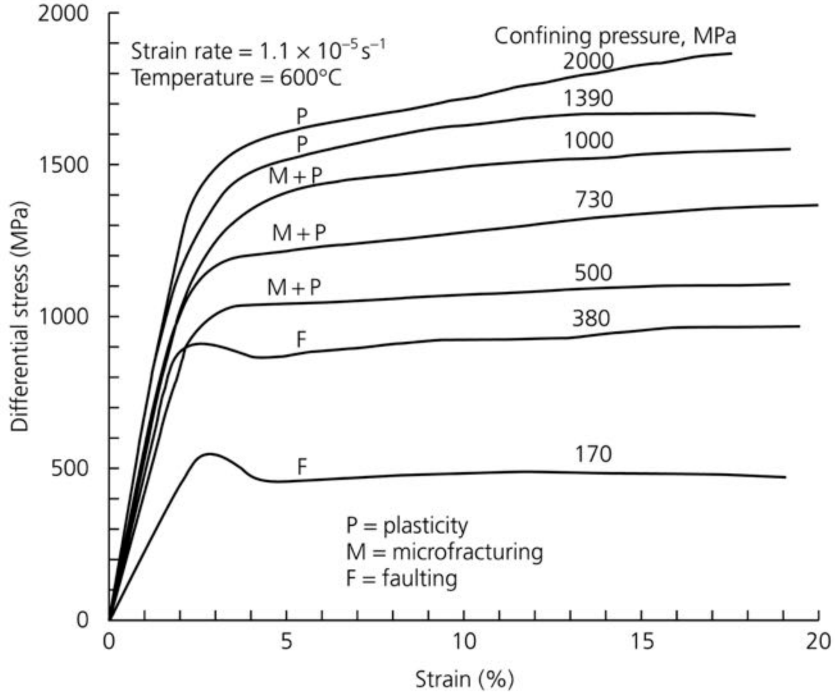
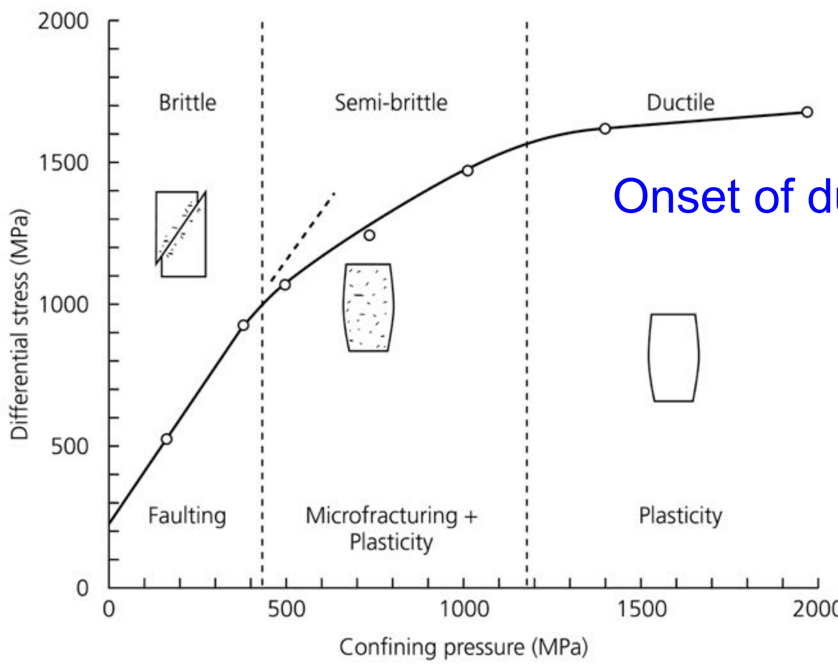

longitudinal splitting
shear on fault
multiple shear faults
extension (tensile fracture)

Figure 5.7-3: Rheology of rocks subjected to large compressive stresses.





Onset of ductility

Stress-strain relation for ductile flow

Laboratory experiments on minerals find ductile flow to be:

$$\frac{de}{dt} = \dot{e} = f(\sigma)\ \underbrace{A\exp[-(E^* + PV^*)/RT]}_{\text{viscosity}^{-1}}$$

T = temperature

R = the gas constant

P is pressure

$f(\sigma)$ = function of the stress difference $|\sigma_1 - \sigma_3|$

A = a constant

E^*, V^* = activation energy and volume (effects of T and P)

mineral-specific

In terms of the principal stresses,

$$f(\sigma) = |\sigma_1 - \sigma_3|^n$$

$$\dot{e} = |\sigma_1 - \sigma_3|^n \; A \exp[-(E^* + PV^*)/RT]$$

The rheology of such fluids is characterized by a power law.
If n = 1 the material is called *Newtonian*, whereas a non-Newtonian fluid with n = 3 is often used to represent the mantle.

The viscosity depends on both temperature and pressure

$$\eta = (1/2A) \exp[(E^* + PV^*)/RT]$$

The viscosity decreases exponentially with temperature, and increases exponentially with pressure!

Example: a common flow law for dry olivine is:

$$\dot{e} = 7 \times 10^4 \, |\sigma_1 - \sigma_3|^3 \exp\left(\frac{-0.52 \text{ MJ/mol}}{RT}\right) \qquad \text{for } |\sigma_1 - \sigma_3| \leq 200 \text{ MPa}$$

$$= 5.7 \times 10^{11} \exp\left[\frac{-0.54 \text{ MJ/mol}}{RT}\left(1 - \frac{|\sigma_1 - \sigma_3|}{8500}\right)^2\right] \qquad \text{for } |\sigma_1 - \sigma_3| > 200 \text{ MPa}$$

where $\dot{e}$ is in s^{-1}.

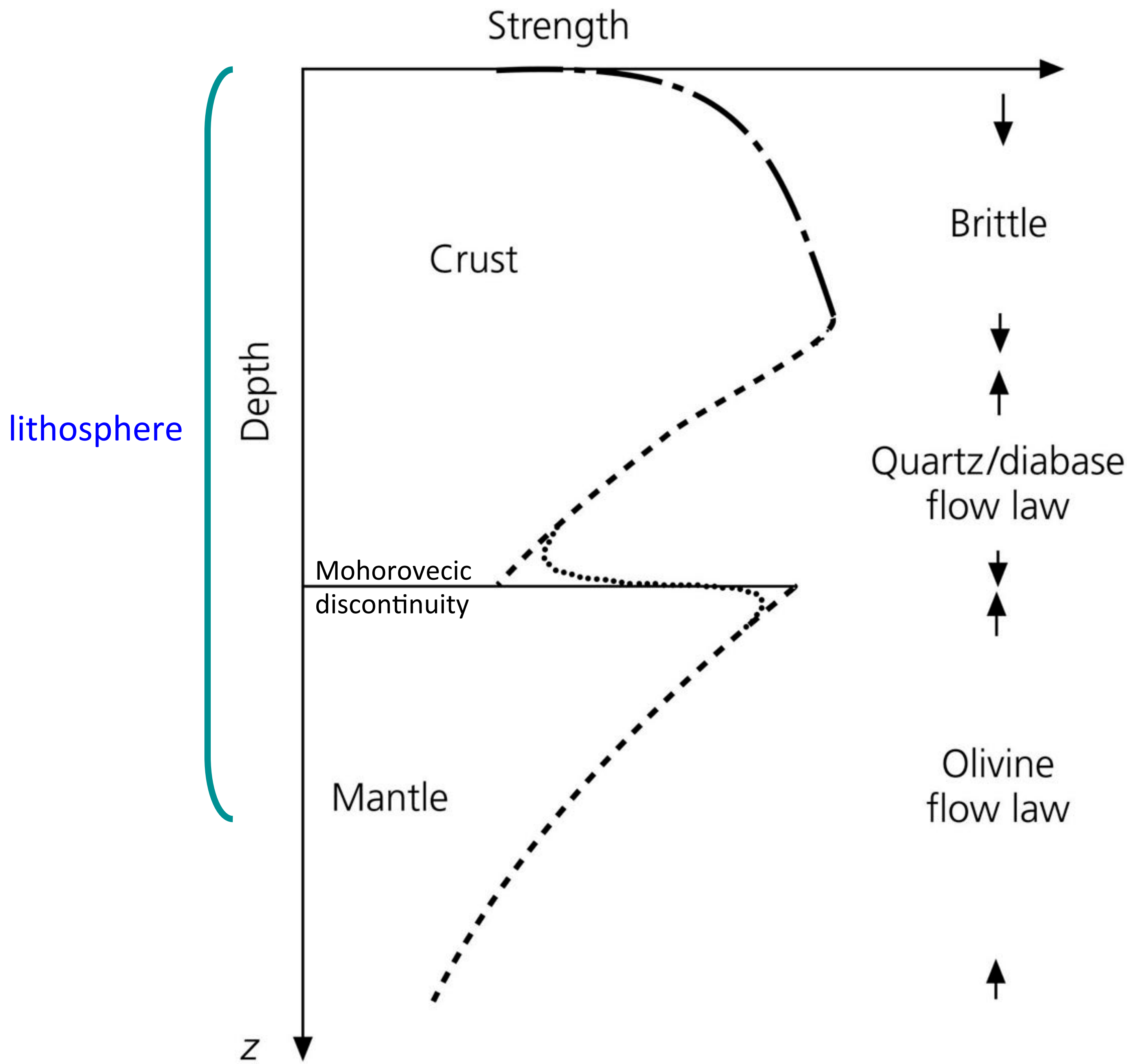
A flow law for quartz is:

$$\dot{e} = 5 \times 10^6 \, |\sigma_1 - \sigma_3|^3 \exp\left(\frac{-0.19 \text{ MJ/mol}}{RT}\right) \qquad \text{for } |\sigma_1 - \sigma_3| < 1000 \text{ MPa}$$

At a given strain rate, quartz is much weaker than olivine!

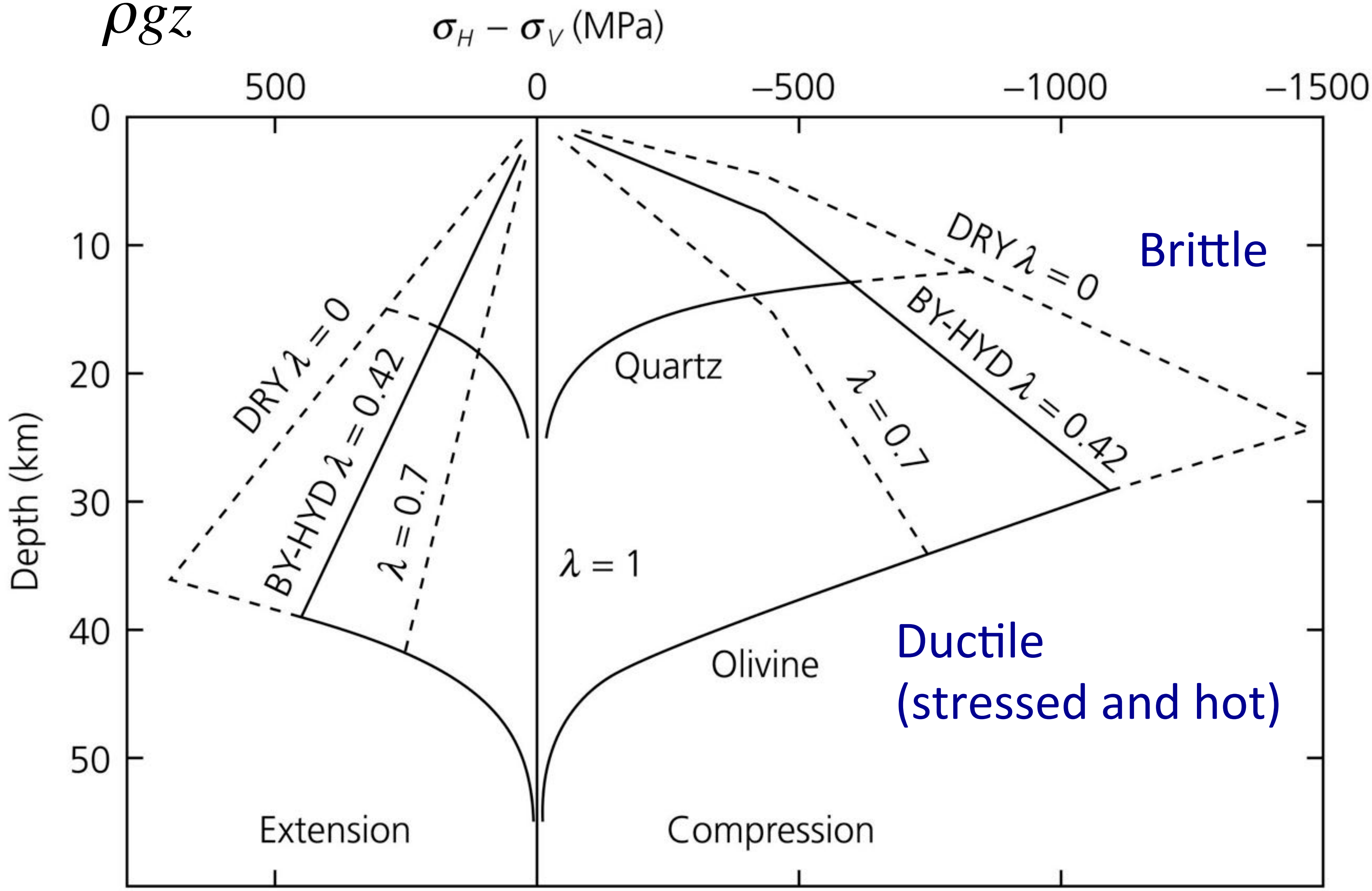
The quartz-rich continental crust is weaker than the olivine-rich oceanic crust.

Figure 5.7-17: Schematic strength envelope for continents.



$$\lambda = \frac{p_w}{\rho g z}$$

Strength of the Lithosphere



Strength envelopes of olivine in aging and cooling oceanic plate