ESS 411/511 Geophysical Continuum Mechanics Class #26

Highlights from Class #25 – Andrew Gregovich
Today's highlights on Friday – Madie Mamer

Problem Set #7

- is posted. We will do just the first 2 questions.

Midterm

- Has been marked, and I will return annotated versions to you later today.
- We can discuss results on Thursday.

For Friday

- Read (*https://courses.washington.edu/ess511/NOTES/*)
  - Ed's note on volume elements
  - Ed's note on conservation laws
  - Ed's note on constitutive relations

- ESS 511 60-second project updates on Friday

ESS 411/511 Geophysical Continuum Mechanics Class #26

For Problems Lab tomorrow

- Read (*https://courses.washington.edu/ess511/NOTES/*)
  - Raymond notes on stress and moments
  - Turcotte and Schubert Section 3.9

- For Friday class
- Read (*https://courses.washington.edu/ess511/NOTES/*)
  - Ed's note on volume elements
  - Ed's note on conservation laws
  - Ed's note on constitutive relations

ESS 411/511 Geophysical Continuum Mechanics

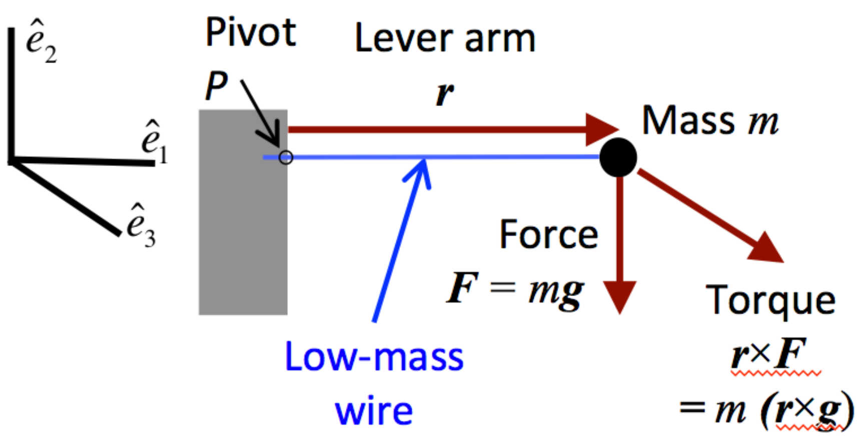
Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

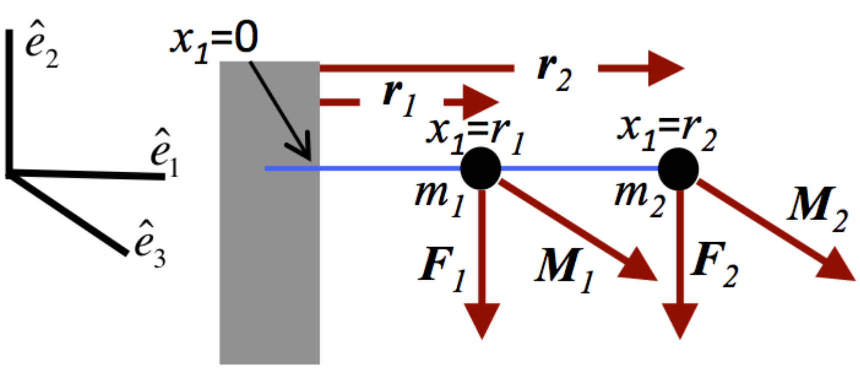
Class-prep: Please give me a moment …

Please Read *Raymond notes on Stress and Moments* on class web site
https://courses.washington.edu/ess511/NOTES/CFR_CHAPTERS/CFR_stress_notes.pdf

"Moment" and torque" are often used interchangeably, although technically a moment is a static concept, and torque applies to a rotation motion. Both involve a vector cross product between a force and a lever arm (or *moment arm*), so the units are Newton meters (N m). The first figure illustrates the moment exerted around the point P by a point mass *m* on the end of a stiff low-weight wire and subjected to gravity.



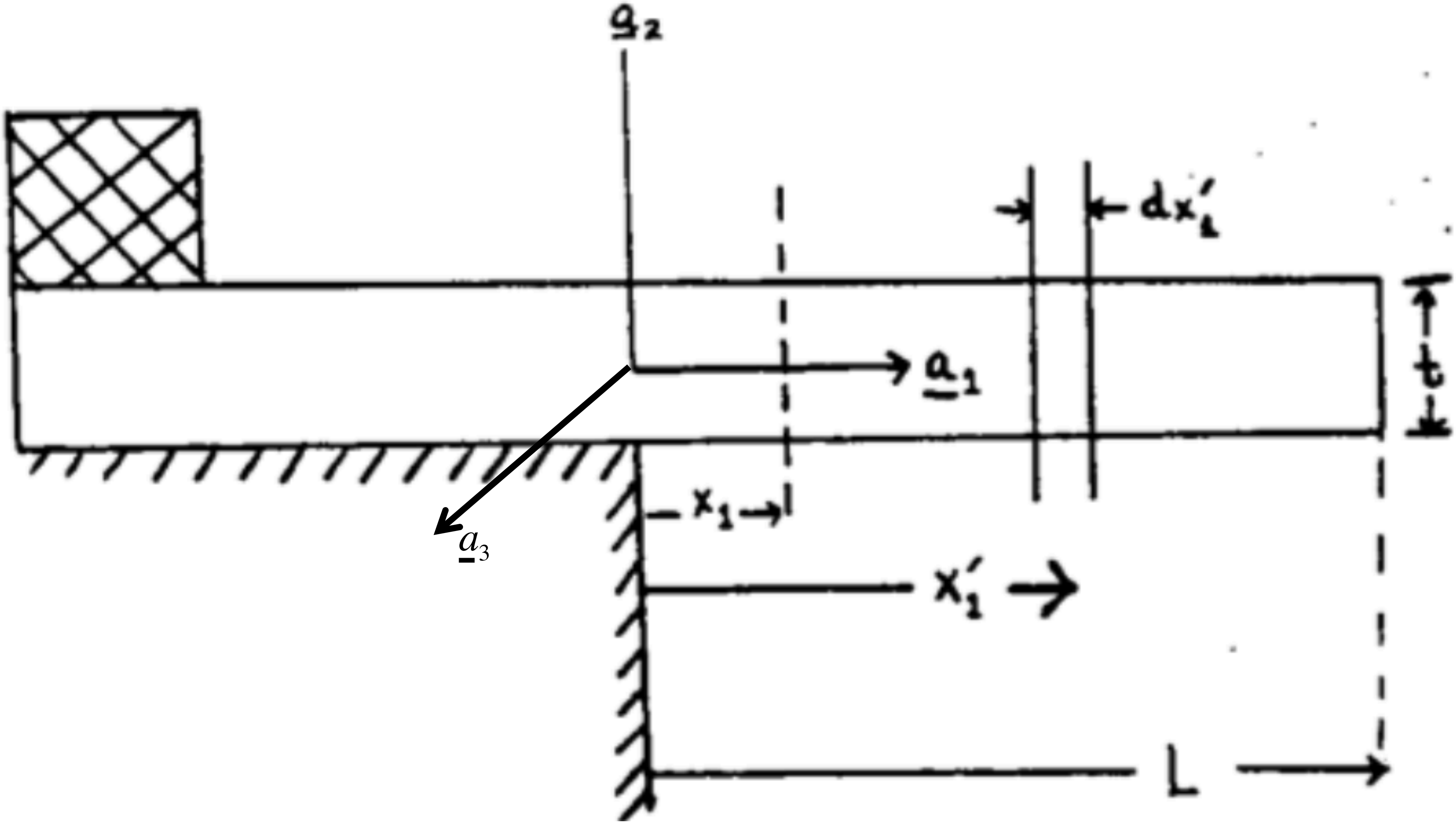
Now let's put a second mass on the wire, and see how the moments work.



Assignment

- What is the moment exerted by mass m_1 at point x_1=0?
- What is the moment exerted by mass m_2 at point x_1=0?
- What is the total moment exerted by both masses m_1 and m_2 at point x_1=0?
- What is the moment exerted by mass m_2 at point x_1= r_1?
- Suppose that we keep adding more discrete masses on the wire until we effectively have a solid steel bar. In a couple of sentences, explain how you could still find the net moment exerted at any particular point, such as x_1=0. (Consider that integral calculus could help.)

A hanging plate



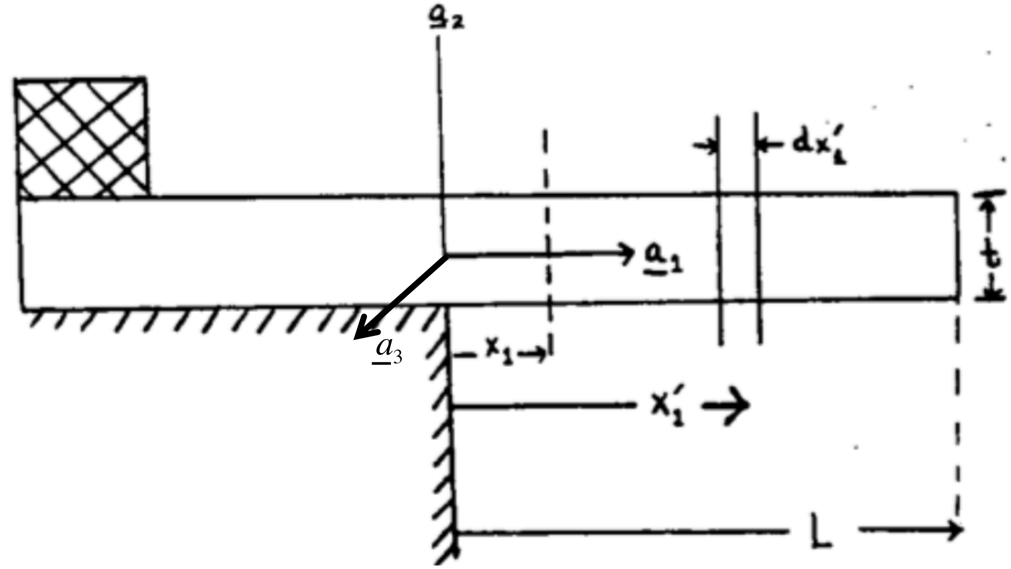
Average stress across the beam (per unit width)

Plane at x_1 must support the weight of all material to the right.
e.g. $\langle\sigma_{12}\rangle t$ is vertically directed force per unit width in x_3

$$\langle\sigma_{11}\rangle t = \rho g_1 (L - x_1)\, t = 0$$

$$\langle\sigma_{12}\rangle t = \rho g_2 (L - x_1)\, t = 0$$

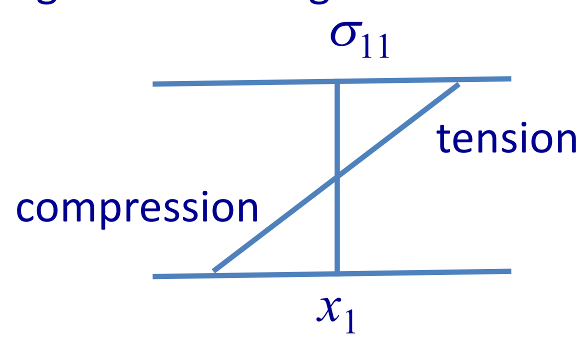
$$\langle\sigma_{13}\rangle t = \rho g_3 (L - x_1)\, t = 0$$



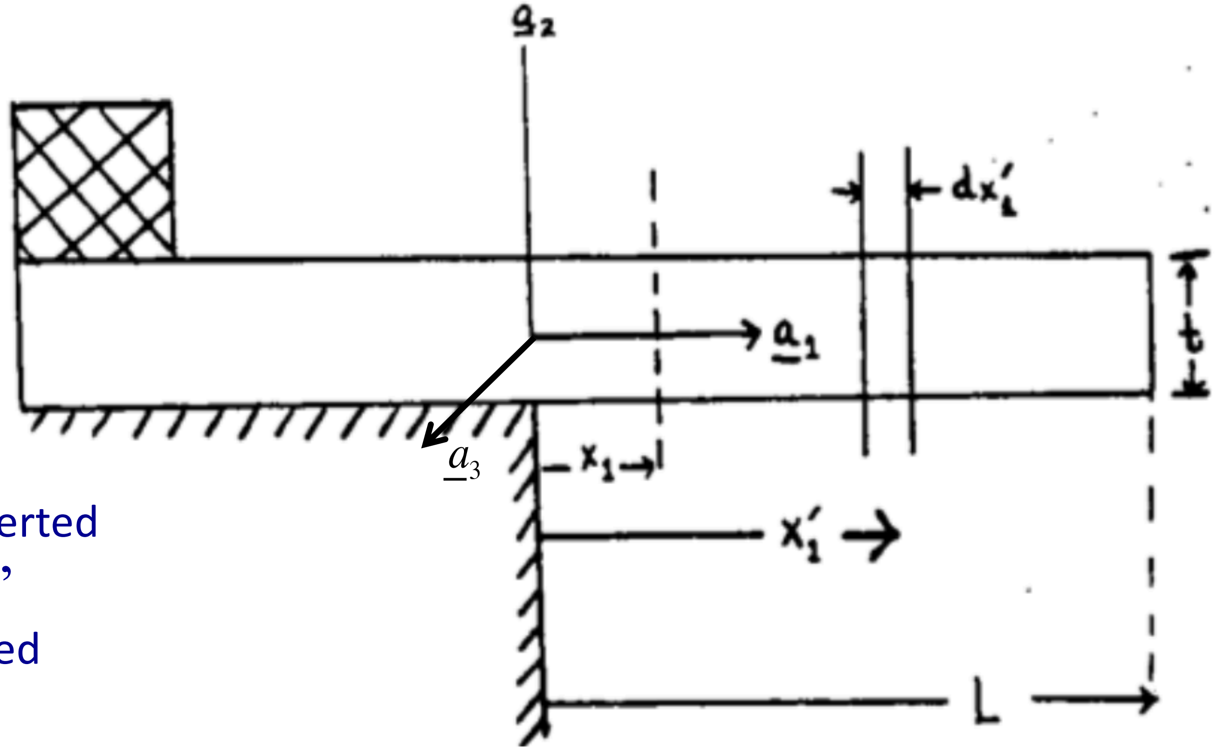
Although $\langle\sigma_{11}\rangle=0$, there must be

- tension ($\sigma_{11}>0$) in the upper part and
- compression ($\sigma_{11}<0$) in the lower part,

in order to prevent the material to the right from falling down.



Incremental moment at x_1 due to outboard weight



Incremental moment at x_1 exerted by thin slice dx_1' of slab at x_1'
e.g. <σ_{12}>t is vertically directed force per unit width in x_3

$$dM_g = -\rho g t\, dx_1'(x_1' - x_1)$$

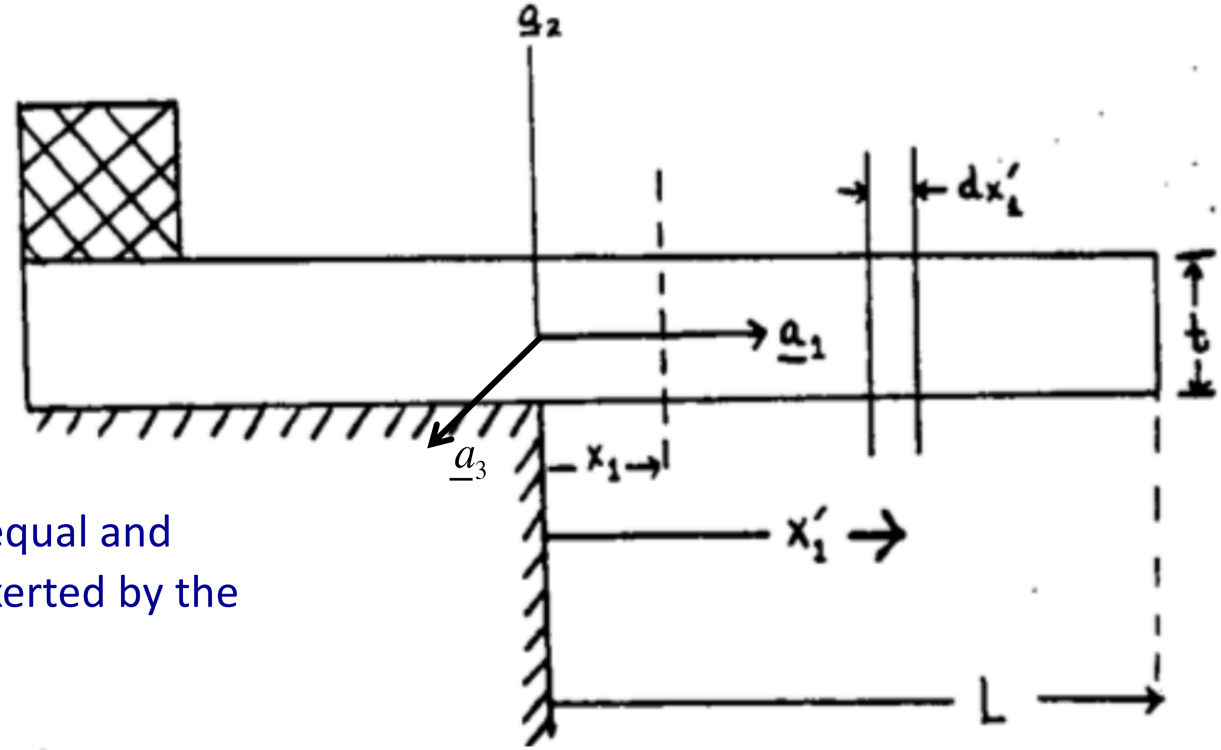
Now add up the incremental moments at x_1 due to *all* thin slices dx_1' to the right of x_1

$$M_g = \int_{x_1}^{L} \rho g t(x_1 - x_1')dx_1' = -\frac{1}{2}\rho g t(L - x_1)^2$$

The force per unit width: ρg t dx_1'

The lever arm: (x_1 - x_1')

Incremental moment at x_1 due to stress in the beam

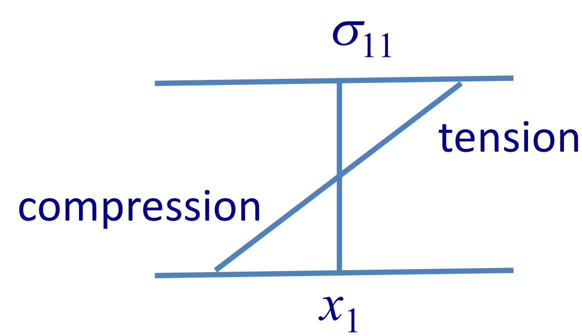


M_g must be balanced by an equal and opposite moment M_t at x_1 exerted by the stress state there.

$$M_t = + \int_{-t/2}^{t/2} \sigma_{11}(x_1, x_2')\, x_2'\, dx_2'$$

The force per unit width: $\sigma_{11}(x_1, x_2')\, dx_2'$

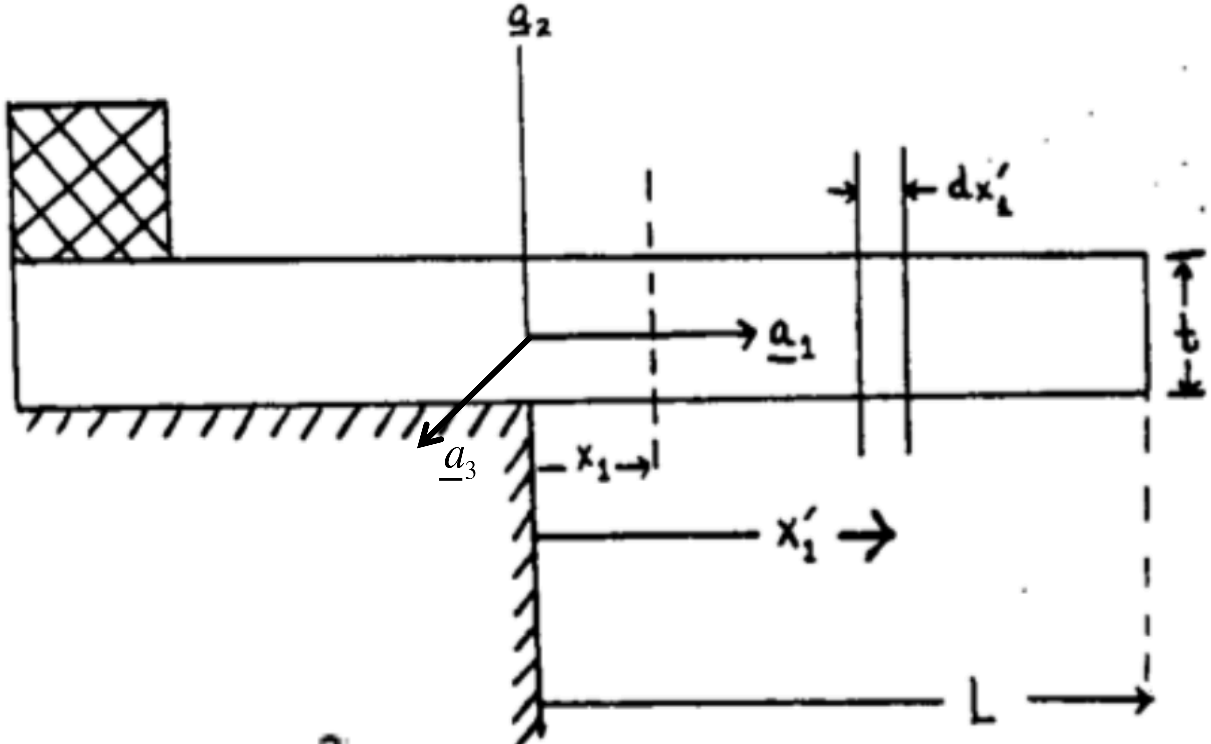
The lever arm: x_2'



Shear stresses at x_1 don't contribute because they have no moment arm around x_1

σ_{11} is assumed to be linear, but it is a very good assumption.

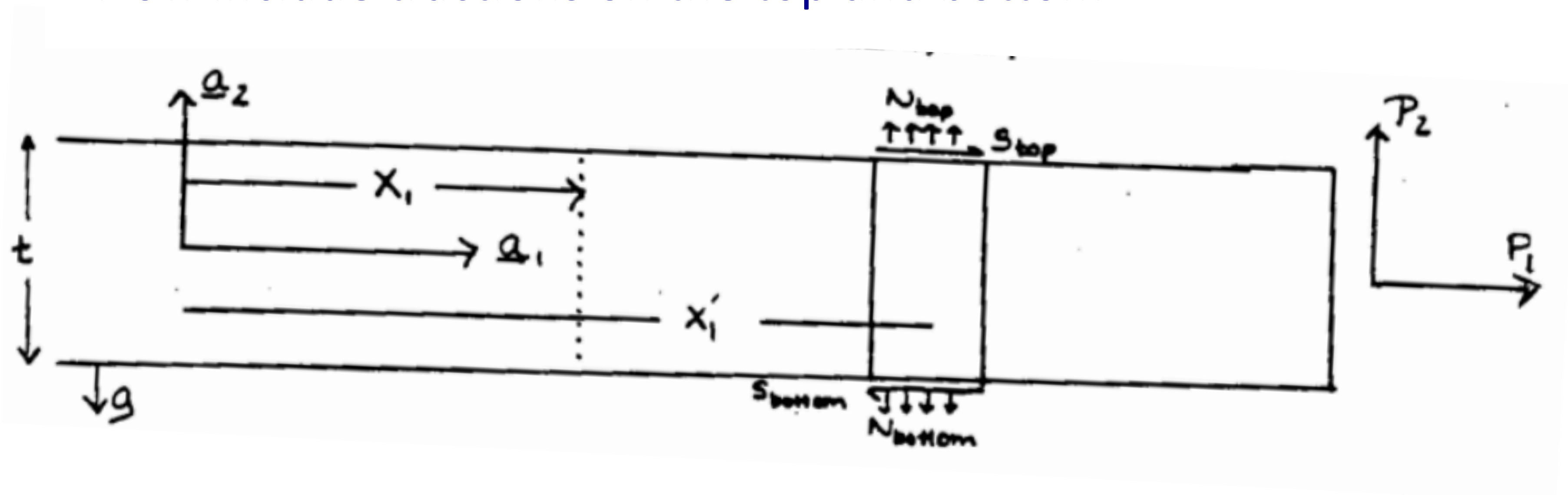
Putting it together -



$M_t + M_g = 0$

$$\int_{-t/2}^{t/2} \sigma_{11}(x_1, x_2')x_2' dx_2' = \frac{1}{2}\rho g t\,(L - x_1)^2$$

Now include tractions on the top and bottom

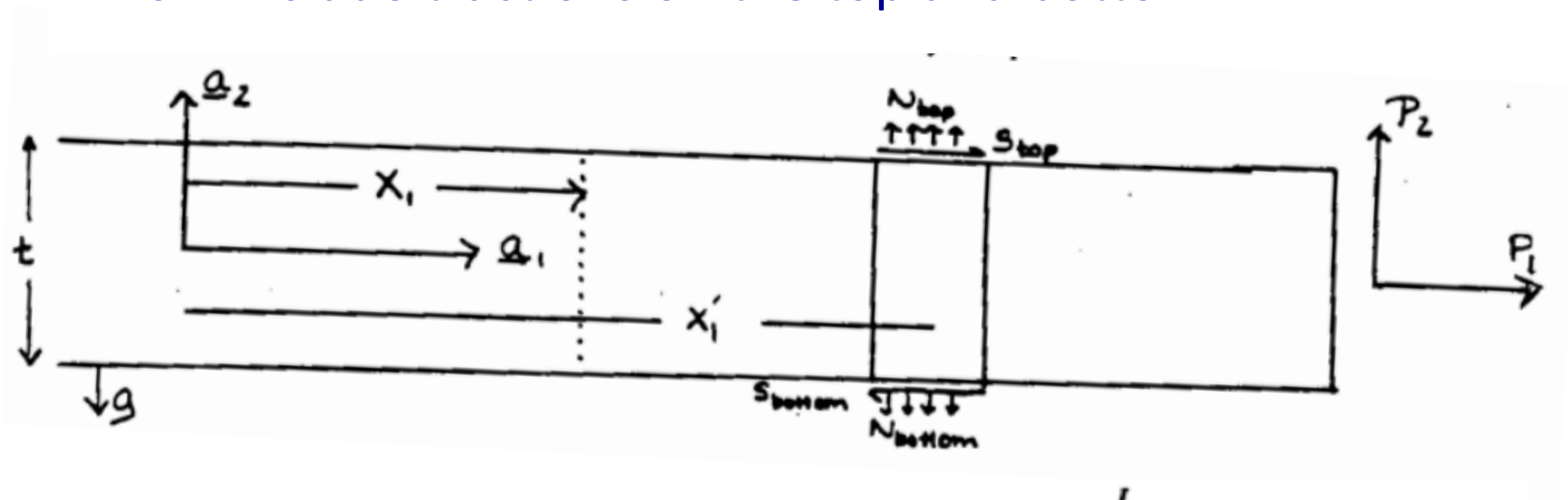


$$F_2(x_1') = -\rho g t + N_{top} - N_{bottom}$$

$$F_1(x_1') = S_{top} - S_{bottom}$$

Putting it together

Now include tractions on the top and bottom



$$F_2(x_1') = -\rho g t + N_{top} - N_{bottom}$$

$$F_1(x_1') = S_{top} - S_{bottom}$$

$$<\sigma_{11}>t + \int_{x_1}^{L} F_1(x_1')dx_1' + P_1 = 0$$

$$<\sigma_{11}> = \frac{1}{t}\{P_1 + \int F_1(x_1')dx_1'\}$$

Putting it together

$$<\sigma_{12}>t + \int_{x_1}^{L} F_2(x_1')dx_1' + P_2 = 0$$

$$<\sigma_{12}> = \frac{1}{t}\{P_2 + \int_{x_1}^{L} F_2(x_1')dx_1'\}$$

$$M_L = \int_{x_1}^{L} F_2(x_1')(x_1' - x_1)dx_1' + P_2(L - x_1) - P_1(u_2(L) - u_2(x_1))$$

$$M_t = + \int_{-t/2}^{t/2} \sigma_{11}(x_1, x_2')x_2' dx_2'$$

$$M_t + M_L = 0$$

$$M_t(x_1) = -\int_{x_1}^{L} F_2(x_1')(x_1' - x_1)dx_1' - P_2(L - x_1) + P_1(u_2(L) - u_2(x_1))$$