Earth and Space Sciences 411/511

Geophysical Continuum Mechanics

Autumn 2021 Midterm

This is a take-at-home test with a time limit of 2.5 hours. If you need more time, please draw and annotate a line on your answer sheets after 2.5 hours to indicate how far you got in 2.5 hours, then continue. It should be taken without access to books, or the web, or other people. (A few notes with some of your favorite equations or definitions is fine.)

Each of the 4 questions is worth 10 points.

Turn in your answers on Canvas by 10:00 PM on Saturday Nov 13.

Be sure to explain **in words** what you are doing at each step, and read the **Tips for Writing an Exam** on the class web site.

Problem 1. Mohr's circles for stress

Explain in prose (use of neat diagrams to illustrate your prose is good):

(a) what Mohr's circles for stress represent about a stress state,

(b) how they can be constructed, and

(c) why they are useful for investigating fault failures.

Problem 2. Strength of the crust

Assuming the crust has numerous preexisting faults in all directions, use Mohr circles to examine the strength of the crust, which is defined by the maximum $\sigma_1 - \sigma_3$ that the crust can support before faulting occurs to relieve stress. Assume that σ_1 and σ_3 are both negative, with $\sigma_1 > \sigma_3$ (the mathematical/engineering convention in Mase, Smelser, and Mase) and $\sigma_2 = \sigma_3$. Draw the sliding line $\sigma_S = -\mu\, \sigma_N$ with coefficient of sliding friction $\mu = \tan\alpha$ and angle of sliding friction $\alpha = 35°$. (Note faults have negligible cohesion.) Mark the lithostatic stress (vertical normal stress) associated with a depth of 6 km, that is, ρgz, with $\rho = 2800$ kg/m^3 and $g = 9.8$ m/s^2 (ignore possible pore pressure). Near Earth's surface, one of the principal stresses must be vertical; the others will be horizontal. Draw the Mohr circle for σ_1 vertical and strength (σ_1-σ_3) as large as possible. Then draw the Mohr circle for σ_3 vertical and strength (σ_1-σ_3) as large as possible. Carefully estimate the ratio of the strength of the crust in compression to its strength in extension. Sketch the strength diagram down to 6 km for this sliding line.

Problem 3.

In a continuum, the stress field relative to axes $Ox_1x_2x_3$ is given by

$$[t_{ij}] = \begin{bmatrix} x_1^2 x_2 & x_1(1-x_2^2) & 0 \\ x_1(1-x_2^2) & \frac{1}{3}(x_2^3 - 3x_2) & 0 \\ 0 & 0 & 2x_3^2 \end{bmatrix}$$

Determine

(a) the body force distribution if the equilibrium equations are satisfied through-out the field,

(b) the principal stresses at point P = (a, 0, 2a$^{1/2}$),

(c) the maximum shear stress at P,

(d) the principal deviator stresses S_{ij} at P.

Problem 4: **Heat flow in anisotropic rocks**
Heat flow $\boldsymbol{q}$ (in W m^{-2}) is given by $q_i = -k_{ij}T_{,j}$ where k_{ij} is the thermal-conductivity tensor (W m^{-1} deg^{-1}), and $T_{,j}$ is the temperature gradient $\partial T/\partial x_j$. The bulk thermal conductivity can be written as a tensor when the rocks are composed of a stack of layers of different rock types, and when we view the rock on scales larger than the layer thicknesses. For example, in a stack of rocks with alternating high and low intrinsic conductivities, it is easier for heat to flow parallel to the layers than across the layers, because of the difficulty in getting through the low-conductivity layers. Suppose that the principal directions of the conductivity tensor are oriented normal to and parallel to the rock layering. Suppose further that the bulk conductivity normal to layering is half the bulk conductivity in the direction of the layering, and that there is no preferred direction in the plane of the layering.

- If the rocks are not lying flat, but are tilted at 30º to horizontal, and if the temperature gradient is aligned with vertical, find the angle from vertical at which heat is actually flowing through the rocks on the scale at which the continuum approximation is valid.
- NOTE THAT THE ANGLE OF TILT DIFFERS FROM THE ANGLE IN THE STUDY QUESTION.