ESS 533/ATMS512 Dynamics of Ice Masses

Homework on Ice Sheets

Vialov Ice-sheet Profile

1. The Vialov solution for an ice-sheet profile has the form

$$h^{2+\frac{2}{n}} = K\left(L^{1+\frac{1}{n}} - x^{1+\frac{1}{n}}\right), \tag{1}$$

where

$$K = \frac{2(n+2)^{\frac{1}{n}}}{\rho g}\left(\frac{c}{2A}\right)^{\frac{1}{n}}, \tag{2}$$

and x is horizontal distance, h is ice thickness, L is the span of the ice sheet, c is accumulation rate, ρ and g are density and gravitational acceleration, and A and n are flow-law parameters.

(a) What assumptions were required to derive this profile?
(b) Show how conservation laws for mass and momentum allow you to write a differential equation for *h(x)*, with the assumptions in (a).
(c) Verify that the solution in equations (1) and (2) above for *h(x)* satisfies your differential equation in (b), subject to these assumptions.

Changes in Ice Sheets

2. Using the Vialov model, predict the fractional change in the thickness at the center of the Greenland Ice Sheet caused by the following changes. *In all cases, please comment on the relative changes in the forcing in relation to the relative changes in the response.*

(a) Accumulation rate increases by a factor of 2. (Accumulation rate increased more than a factor of 2 at the termination of the last glacial maximum.)
(b) The 1/2 width of the ice sheet decreases by 50 km. (This may have been the case at the end of the last glacial maximum when sea level rose.)
(c) The mean temperature of the ice close to the bed increases from about -20ºC to -10ºC. (This is the order of the temperature change at the end of the last glacial maximum, which is still now affecting the base of the ice.)

Weertman-Paterson Ice-sheet Profile

3. A steady ice-sheet with span L on a flat bed has uniform accumulation rate c from $x=0$ to $x=R$, and uniform ablation rate a from $x=R$ to $x=L$. The ice sheet has the form of a long ridge at right angles to the x axis, so that it deforms in plane strain (2-D). With the SIA, the solution for the surface profile is

$$\left(\frac{h}{H}\right)^{2+2/n} + \left(1+\frac{c}{a}\right)^{1/n}\left(\frac{x}{L}\right)^{1+1/n} = 1 \qquad 0 \le x \le R \tag{3}$$

$$\left(\frac{h}{H}\right)^{2} = \left(1+\frac{a}{c}\right)^{1/n+1}\left(1-\frac{x}{L}\right) \qquad R < x \le L \tag{4}$$

This is called the Paterson-Weertman profile.

- Find a relationship among *R, L, c,* and *a*, based on mass conservation in steady state.
- Derive the differential equations that Paterson solved to find the solution in (3) and (4).
- Demonstrate that the solution does in fact satisfy your differential equations.
- Find the relationship that exists among *H, R, L, c, a, A,* ρ and g to make this be true.