ESS 533/ATMS 512 Dynamics of Snow and Ice

Problem Set - Glen Flow Law for Ice Slabs

1. Deformation rates in inclined ice slabs

A uniform slab of ice sits on a long planar sloping surface. The x_1 axis is directed down-slope along the ice surface, which has a slope angle of α, and the x_2 direction is downward and normal to the sloping surface. The surface slope and the ice thickness are independent of x_1 and x_3. The density of ice is ρ, and gravitational acceleration is g. There is no accumulation or ablation on the top surface of the ice, and no basal melting or accretion.

- Would you expect flow speed to vary in the x_1 or x_3 directions? Why or why not?
- Would you expect stress to vary in the x_1 or x_3 directions? Why or why not?
- Starting from the momentum conservation equations, find expressions for all components of the stress tensor σ_{ij}, the deviatoric stress tensor τ_{ij}, and the deformation rate tensor $\dot{\varepsilon}_{ij}$ as functions of depth x_2. Explain in words how you get your results.

2. Effects of stretching

A uniform slab of ice sits on a long planar sloping surface. The x_1 axis points down-slope along the ice surface, which has a slope angle of α, and the x_2 direction is downward and normal and to the sloping surface. Surface slope and ice thickness are independent of x_1 and x_3. The density of ice is ρ, and gravitational acceleration is g. The slab is also stretched uniformly at rate $\dot{e}$ in the x_1 direction with a longitudinal deviatoric stress $\tau_{11}(x_2)$. A uniform accumulation rate b (ice equiv.) on the top surface of the ice, is just the right amount to prevent the slab from getting thinner or thicker over time.

- Would you expect flow speed to vary in the x_1 or x_3 directions? Why or why not?
- Starting from the momentum conservation equations, find expressions for all components of the stress tensor σ_{ij}, the deviatoric stress tensor τ_{ij}, and the deformation rate tensor $\dot{\varepsilon}_{ij}$ as functions of depth x_2. Explain in words how you get your results.
- Find the extensile strain rate $\dot{e}$ in terms of slab shape and climate.
- With extensile strain rate $\dot{e}$ uniform at all depths, what is the maximum value τ_0 that $\tau_{11}(x_2)$ takes, and where does it take that value?
- Outline how you could solve for $\tau_{11}(x_2)$ (or alternatively, $x_2(\tau_{11})$).
- Now do it. Use the value of $A = 3.5\times10^{-25}$ Pa^{-3} s^{-1} at -10 C (from Cuffey and Paterson p. 75).

3. The same but different

These two ice slabs look rather similar. With ρ=900 kg m^{-3}, g=9.8 m s^{-2}:

- Find the longitudinal strain rate $\dot{e}$ for 5 different glaciers or ice sheets in the Table below.
- Using your solution in Question (2), plot $\sigma(x_2)$ for each glacier.
- Find the relative impact of $\sigma(x_2)$ on the bed-parallel shear strain rate $\dot{\varepsilon}_{12}$ at the surface, at mid-depth, and at the bed for each glacier.
- Fill in the Table, and discuss implications of your findings in terms of glacier flow.

Property	Glacier	j=1	2	3	4	5
Thickness	H (m)	100	100	300	1000	3000
Accumulation rate	b (m a^{-1})	0	2.0	1.0	0.5	0.2
Slope	α (deg)	-3°	-3°	-2°	-0.5°	-0.2°
Strain rate $\dot{\varepsilon}_{11}$	$\dot{e}$					
τ_{11} *at surface*	σ_0					
Shear strain enhancement	$\dot{\varepsilon}_{12}(j)/\dot{\varepsilon}_{12}(1)$	1				
τ_{11} *at mid-depth*	$\sigma(H/2)$					
Shear strain enhancement	$\dot{\varepsilon}_{12}(j)/\dot{\varepsilon}_{12}(1)$	1				
τ_{11} *at bed*	$\sigma(H)$					
Shear strain enhancement	$\dot{\varepsilon}_{12}(j)/\dot{\varepsilon}_{12}(1)$	1				