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ESS 533 – Ice Dynamics
Class Summary for 4/4/18

On Wednesday, we continued our development of a flow law for crystalline ice. For the most part, we are still following Alley (1992). Throughout, Alley emphasizes that there is no perfect flow law for ice, but he says that in the most generalized case the creep parameter, n, is often thought to be 3. He derives a theoretical framework that shows this is the case for dislocation glide along the basal plane. This basal glide is likely the dominant mechanism of ice flow in most cases because other glide mechanisms are ~100-1000 times more resistant to flow. Mechanisms other than glide are possible in some scenarios, such as grain boundary sliding in temperate ice or dislocation climb at high stresses, and are discussed by Alley but I will not consider them here.

To derive the creep exponent in the ice flow law for basal glide, Alley defines the bulk strain rate of ice as

$$\dot{\epsilon} = \alpha \rho b v \tag{1}$$

where $\alpha$ is a geometric factor, $\rho$ is the density of dislocations, $b$ is the Burger's vector (defining the motion of a dislocation), and the velocity of dislocations is

$$v = \gamma \sigma \exp\left(\frac{-Q}{kT}\right) \tag{2}$$

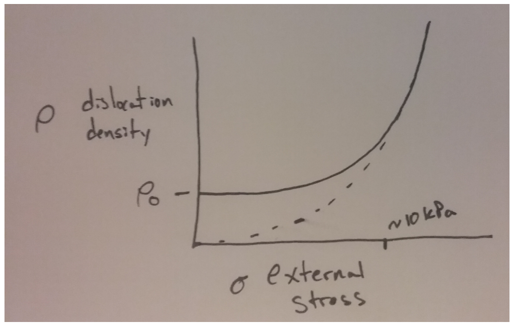
where $\gamma$ is a constant, $\sigma$ is the applied stress, $Q$ is an activation energy, $k$ is Boltzmann's constant, and $T$ is the temperature of the ice. Dislocation density is dependent on stress because more dislocations are created when the crystal is put under stress,

$$\rho \approx \left(\frac{\beta \sigma}{\mu b}\right)^2 \tag{3}$$

Combining equations (1-3) we come up with something that looks very familiar to Glen's original flow law (Glen, 1955),

$$\dot{\epsilon} = \frac{\alpha \beta^2 \gamma}{\mu^2 b} \exp\left(\frac{-Q}{kT}\right) \sigma^3 \tag{4}$$

The constant term in front of the exponential is most commonly wrapped up into the rate factor, $A_0$. Through this derivation, we see that there is a theoretical justification for assuming that the creep exponent, $n$, on the applied stress is equal to 3 for dislocation glide along the basal plane.




Experiments show that $n = 3$ is not the case for all (or possibly even most) glacial ice. In fact, Glen's original work showed that it is closer to 4. Alley dedicates significant effort toward explaining the physical reasons why this might be. Here, I will point out one example at low stresses. Dislocation density, as defined above in equation 3, is stress-dependent and will approach zero dislocations as the applied stress goes to zero (see the dashed line in the above figure). However, in reality there is no such ice that is 'dislocation-free.' Instead, ice at low stresses approaches a constant dislocation density, $\rho_0$, at low stresses. What this means for the flow law is that the $\sigma^2$ term from equation (3) disappears. Now the strain rate is only dependent on stress linearly. This mechanism of flow is diffusional, where molecules in the crystalline lattice are simply diffusing away from high stresses and toward lower stresses. We might find this near the surface of an ice sheet where stresses are low.

**References**


Alley, R. B. (1992). Flow-law hypotheses for ice-sheet modeling. *Journal of Glaciology*, *38*(129).

Glen, J. W. (1955). The Creep of Polycrystalline Ice. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, *228*(1175), 519–538. Retrieved from http://rspa.royalsocietypublishing.org/content/228/1175/519.abstract