ESS 533 Class Summary 4/23
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We finished looking at the shallow ice approximation, which is applicable when the thickness to length ratio of a glacier or ice sheet is very small (δ = H/L << 1). First, we compared and discussed the implications of using the Hutter formulation (where $X = x\delta$, $Z = z$, and $T = t\delta$) and the Fowler formulation (where $X = x\delta$, $Z = z$, and $T = t$). The vertical ice velocities are amplified in the Hutter formulation ($W = 1/\delta w$). In contrast, the horizontal velocities appear attenuated in the Fowler formulation as the vertical velocity remains unchanged ($W = w$). Consequently, the vertical and horizontal velocities have comparable magnitudes in both cases. We then wrote U and σ as a power series in δ, which can be substituted back into the equations of momentum conservation and the constitutive equation. τ and $\dot{\varepsilon}$ can be derived from these expressions to similarly be expressed as a power series in δ. In the resultant momentum and conservation equations, the terms within brackets must sum to zero in order for the expression to be valid for any δ. Solving this series of equations results in what we know as the shallow ice approximation.

We then discussed a simple model that treats ice as perfectly plastic to derive an ice thickness profile (*h(x)*). Starting with the shear stress, $\tau_{xz} = \rho g(h - z)sin(\alpha)$, we solved this expression at z = 0 to produce $\tau_0 = \rho g h dh/dx$, integrated this equation, and found an expression for the thickness profile, $h(x) = \sqrt{\frac{\tau_0}{2\rho g}(L - x)}$. We briefly discussed how Reeh used a similar equation along various paths of the Greenland Ice Sheet to reconstruct historical thickness profiles. We ended the day's lecture by addressing how to derive an analytic solution the velocity profile *u(z)*.