Andrew Hoffman

Class summary for Monday May 7

The surface/skin temperature of the earth can be much difference than the surface temperature of the air. Surface air temperature is typically measured at 1.5 – 2m where the log-layer in the boundary layer is likely stable.

If T(z,t) is the surface temperature, Fourier's law of heat conduction says that

$$H_g = -k\frac{\partial T}{\partial z}$$

Where k is the thermal conductivity of the surface and H_g is the downward ground heat flux into the subsurface medium. Thermal energy conservation implies that

$$\rho c\frac{\partial T}{\partial t} = -\frac{\partial H_g}{\partial z}$$

Combining these equations and assuming the surface structure is homogeneous, so the material constants do not depend on z, we find derive the diffusion equation

$$\frac{\partial T}{\partial t} = -\frac{k}{\rho c}\frac{\partial^2 T}{\partial z^2}$$

If we assume that deep in the ground $T(z \to \infty) = \bar{T}$ and take $T(z \to 0) = \bar{T} + A\cos(\omega\, t)$. We can look for solutions to the diffusion equations that are sinusoidal with the same frequency:

$$T(z,t) = \bar{T} + Re(a(z)e^{i\omega t})$$

where a(z) is a complex valued function of z. To satisfy our diffusion equation:

$$i\omega a = -\frac{k}{\rho c}\frac{d^2 a}{dz^2}$$

And then to satisfy our boundary conditions:

$$a(0) = A, a(z \to \infty) = 0$$

So that our solution becomes

$$a(z) = A\, e^{-(1+i)z\sqrt{\frac{\rho\, c\, \omega}{2\, k}}}$$

$$T(z,t) = \bar{T} + e^{-z\sqrt{\frac{\rho\, c\, \omega}{2\, k}}}\cos\left(\omega\, t - z\sqrt{\frac{\rho\, c\, \omega}{2\, k}}\right)$$

This solution shows how the surface signal is damped with depth and lags the surface temperature by a phase $z\sqrt{\frac{\rho\, c\, \omega}{2\, k}}$, which will increase with depth. We can think of $\sqrt{\frac{\rho\, c\, \omega}{2\, k}}$ as the dampening depth.

Going back to our expression for the ground heat flux, we can solve this expression and find how the ground heat flux lags the temperature signal.

$$H_g = -k\frac{\partial T}{\partial z} = -\rho c\sqrt{\frac{k}{\rho c}\omega}\, A\, Re(1+i)e^{i\omega t} = -\rho c\sqrt{\frac{k}{\rho c}\omega}\, A\cos\left(\omega\, t + \frac{\pi}{4}\right)$$

This is a real signal that we can observe at the surface:

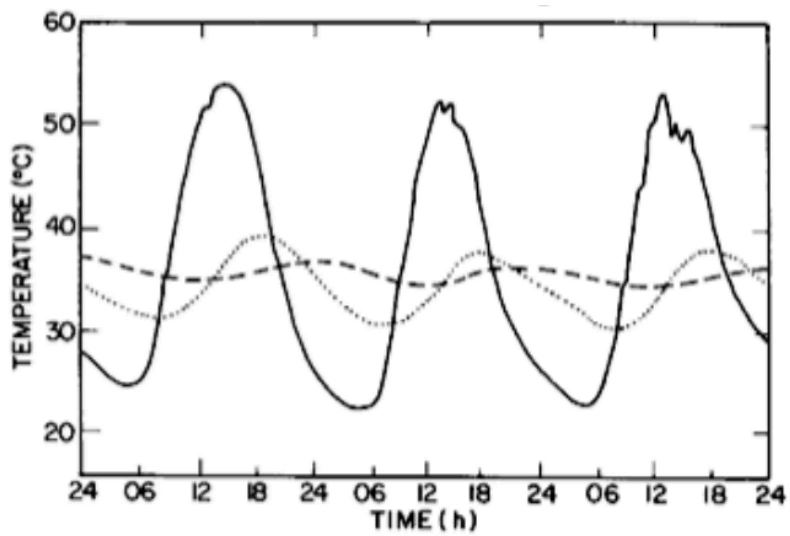

Fig. 4.1 Observed diurnal course of subsurface soil temperatures at various depths in a sandy loam with bare surface. ——, 2.5 cm; ·····, 15 cm; ---, 30 cm. [From Deacon (1969); after West (1952).]

We also talked about Dorthe's paper very briefly and how temperature profiles measured at depth can tell us about the history of surface temperature that has diffused into the ice.

Deacon (1969)

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