ESS 524

Introduction to Heat and Mass Flow Modeling in Earth Sciences

My First Numerical Solution in Matlab!

1. Analytical solution to steady heat equation

The steady heat equation (1) with spatially variable sources and conductivity can be solved analytically when *S(x)* is a polynomial in *x*, and 1/*k(x)* is a polynomial in *x*.

$$\frac{d}{dx}\left(k(x)\frac{d}{dx}\phi(x)\right) + S(x) = 0 \qquad (1)$$

- Write an m-file to evaluate the solution for generic *S(x)* and 1/*k(x)* polynomials.
- Find the analytical (polynomial) solution on [0,1] when

$$\frac{1}{k(x)} = 2x^2 - 2x + 1$$

$$S(x) = 100\left(2x^3 - 3x^2 + x\right)$$

$$\left[\frac{d\phi}{dx}\right]_0 = 2$$

$$\phi(1) = 5$$

- Using 3 graphs, plot *k(x)*, *S(x)* and your polynomial solution for $\phi(x)$.

Hints:

- Look for functions **polyint, polyval**, and **conv.**
- The solution will require 2 integrations. You should be able to work the boundary conditions naturally into your polynomial solution, if you integrate once from *x*=0, and once from *x*=1.

2. Numerical Solution by Finite Differences

- Write a Matlab m-file to solve the above equation using ***finite differences***.
- Determine when your grid interval *dx* is small enough by comparing your numerical answer to your analytical answer in Problem 1.
  Try using 5 nodes -> 100 nodes.
- Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing.

3. Numerical Solution by Finite Volumes

- Write a Matlab m-file to solve the above equation using ***finite volumes***.
- Determine when your grid interval *dx* is small enough by comparing your numerical answer to your analytical answer. Try using 5 nodes -> 100 nodes.
- Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing.