

**ESS 590 B Spring 2008**  
**Introduction to Heat and Mass Flow Modeling in Earth Sciences**

**My First Numerical Solution in Matlab!**  
**Due Thursday April 24**

**1. Analytical solution to steady heat equation**

The steady heat equation (1) with spatially variable sources and conductivity can be solved analytically when  $S(x)$  is a polynomial in  $x$ , and  $1/k(x)$  is a polynomial in  $x$ .

$$\frac{d}{dx} \left( k(x) \frac{d}{dx} \phi(x) \right) + S(x) = 0 \quad (1)$$

- Write an m-file to evaluate the solution for generic  $S(x)$  and  $1/f(x)$  polynomials.
- Find the analytical (polynomial) solution when

$$\frac{1}{k(x)} = 2x^2 - 2x + 1$$

$$S(x) = 100(2x^3 - 3x^2 + x)$$

$$\left[ \frac{d\phi}{dx} \right]_0 = 2$$

$$\phi(1) = 5$$

- Using 3 graphs, plot  $k(x)$ ,  $S(x)$  and your polynomial solution for  $\phi(x)$ .

Hints:

- Look for functions **polyint**, **polyval**, and **conv**.
- The solution will require 2 integrations. You should be able to work the boundary conditions naturally into your polynomial solution, if you integrate once from  $x=0$ , and once from  $x=1$ .

**2. Numerical Solution by Finite Differences**

- Write a Matlab m-file to solve the above equation using *finite differences*.
- Determine when your grid interval  $dx$  is small enough by comparing your numerical answer to your analytical answer. Try using 5 nodes -> 100 nodes.
- Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing.

**3. Numerical Solution by Finite Volumes**

- Write a Matlab m-file to solve the above equation using *finite volumes*.
- Determine when your grid interval  $dx$  is small enough by comparing your numerical answer to your analytical answer. Try using 5 nodes -> 100 nodes.
- Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing.