**ESS 524**
**Introduction to Heat and Mass Flow Modeling in Earth Sciences**

## Transient Diffusion

In HW#3, you wrote a finite-volume solver for the steady-state diffusion equation (1),

$$\frac{d}{dx}\left(\Gamma(x)\frac{d}{dx}\phi(x)\right) + S(x,\phi) = 0 \qquad (1)$$

on the interval $0 \le x \le 1$, with conductivity $\Gamma(x)$, source strength $S(x)$, and boundary conditions given by

$$\begin{aligned} \Gamma(x) &= \left(2x^2 - 2x + 1\right)^{-1} \\ S(x) &= 100\left(2x^3 - 3x^2 + x\right) \\ \left[\frac{d\phi}{dx}\right]_0 &= 2 \\ \phi(1) &= 5 \end{aligned} \qquad (2)$$

1) Now, modify your code to solve the transient problem,

   $$\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial x}\left(\Gamma(x)\frac{\partial}{\partial x}\phi(x,t)\right) + S(x,\phi) \qquad (3)$$

   with the same conductivity, source, and boundary conditions in (2), using fully implicit time steps $\Delta t$. You will need to modify your coefficients $a_P$ *etc* to incorporate the time derivative, and then include a time-step loop.

2) For a transient problem, you will need an initial condition that also satisfies the boundary conditions in (2). Try

   $$\phi(x,0) = 6 + 2x - 3x^2 \qquad (4)$$

   and run your model until the profile reaches the steady-state solution that you expect from HW#2 and #3.

3) Now try different initial conditions and show that you can still reach the same steady state. The function

   $$\psi(x) = a\cos\left(\left(\frac{2n+1}{2}\right)\pi x\right) \qquad (5)$$

   has zero slope at $x$=0, and zero amplitude at $x$=1, for any $a$ and for any integer $n$. That means that you can add any multiple of $\psi(x)$ in (5) to the initial condition (4) and still satisfy the boundary conditions in (2). Try, for example, $a$=0.5, and $n$=4.

4) Describe in prose any insights or knowledge that you learned about transients in numerical methods from the exercise (suggested length approx 1 page). For example, how did you select your space intervals $\Delta x$ and time intervals $\Delta t$, and why?