FE 320

MATH REVIEW

"We need more to be reminded than instructed" - Samual Johnson

"Engineers must deal quantitatively with a wide scope of scientific information, much of which is imperfectly recollected" - Mott Souders

Contents

TRIGONOMETRIC FUNCTIONS

DEFINITION OF TRIGONOMETRIC FUNCTIONS FOR A RIGHT TRIANGLE

Triangle ABC has a right angle (90^0) at C and sides of length a,b,c. The trigonometric functions of angle A are defined as follows:

$$\text{sine of } A = \sin A = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

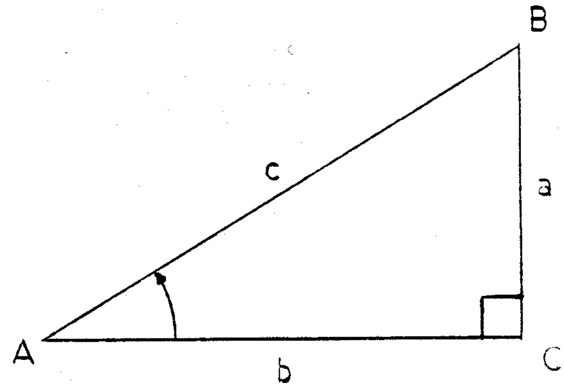
$$\text{cosine of } A = \cos A = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } A = \tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cotangent of } A = \cot A = \frac{b}{a} = \frac{\text{adjacent}}{\text{opposite}}$$

$$\text{secant of } A = \sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{cosecant of } A = \csc A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite}}$$



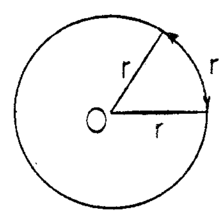
RELATIONSHIP BETWEEN DEGREES AND RADIANS

A radian is that angle subtended at the center (O) of a circle by an arc (MN) equal in length to the radius (r).

Since 2π radians = 360^0 we have:

$$1 \text{ radian} = 180^0/\pi = 57.29577^0$$

$$1^0 = \pi/180 \text{ radians} = 0.1745 \text{ radians}$$



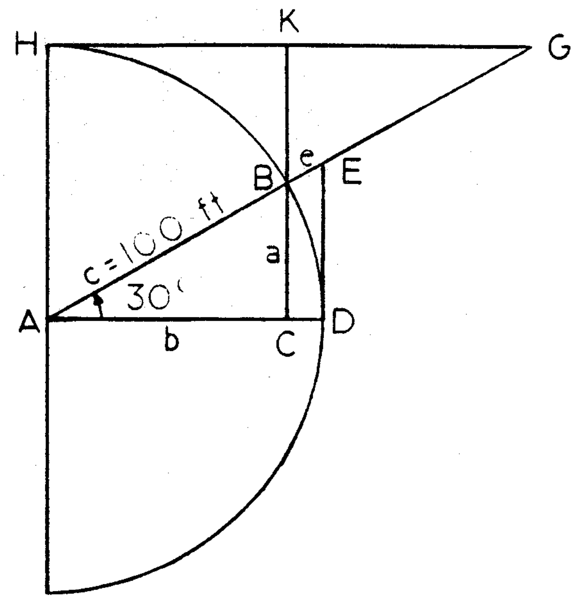
RELATIONSHIPS AMONG TRIGONOMETRIC FUNCTIONS

$$\tan A = \frac{\sin A}{\cos A} \qquad \sin^2 A + \cos^2 A = 1$$

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \qquad \sec^2 A - \tan^2 A = 1$$

$$\sec A = \frac{1}{\cos A} \qquad \csc^2 A - \cot^2 A = 1$$

$$\csc A = \frac{1}{\sin A}$$



c = 100' Angle BAC = 30^{0}

Sin A = $\frac{a}{c}$; a = (sin 30^{0})(100) = 50.00'

Cos A = $\frac{b}{c}$; b = (cos 30^{0})(100) = 86.60'

Tan A = $\frac{a}{b}$ = $\frac{DE}{c}$; DE = (tan 30^{0})(100) = 57.70'

Cotan A = $\frac{b}{a}$ = $\frac{HG}{c}$; HG = (cotan 30^{0})(100) = 173.21'

Cosec A = $\frac{c}{a}$ = $\frac{AG}{c}$; AG = (cosec 30^{0})(100) = 200.00'

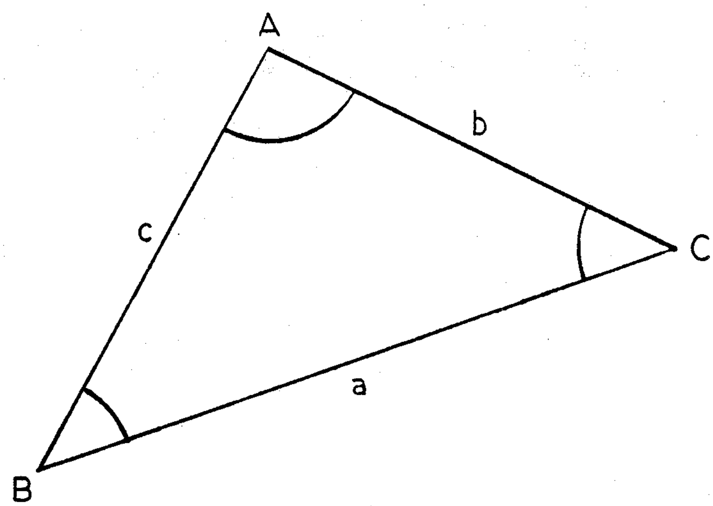
Sec A = $\frac{c}{b}$ = $\frac{AE}{c}$; AE = (sec 30^{0})(100) = 115.47'

Versin A = $\frac{CD}{c}$; CD = (vers 30^{0})(100) = 13.40'; [versin A = (1-cos A)]

Exsec A = $\frac{BE}{c}$; BE = (exsec 30^{0})(100) 15.47'; [(exsecA = $\frac{1-\cos A}{\cos A}$)]

Tan½ A = $\frac{CD}{BC}$ = $\frac{\text{vers } A}{\sin A}$ = $\frac{d}{a}$

RELATIONSHIPS BETWEEN THE SIDES AND ANGLES OF A PLANE TRIANGLE



The following results hold for any plane triangle ABC with sides a,b,c, and angles A,B,C:

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

with similar relations involving the other sides and angles.

Law of Tangents

$$\frac{a + b}{a - b} = \frac{\tan\frac{1}{2}(A + B)}{\tan\frac{1}{2}(A - B)}$$

with similar relations involving the other sides and angles.

$$\sin A = \frac{2}{bc}\sqrt{s(s - a)(s - b)(s - c)}$$

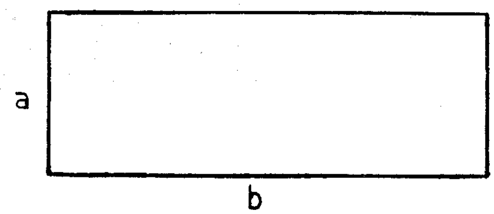
where $s = \frac{1}{2}(a + b + c)$ is the semiperimeter of the triangle. Similar relations involving angles B and C can be obtained.

GEOMETRIC FORMULAS

RECTANGLE OF LENGTH b AND WIDTH a

Area = ab

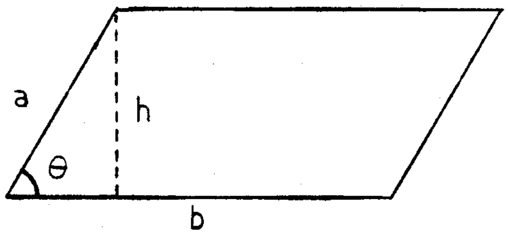
Perimeter = $2a + 2b$



PARALLELOGRAM OF ALTITUDE h AND BASE b

Area = $bh = ab \sin\theta$

Perimeter = $2a + 2b$



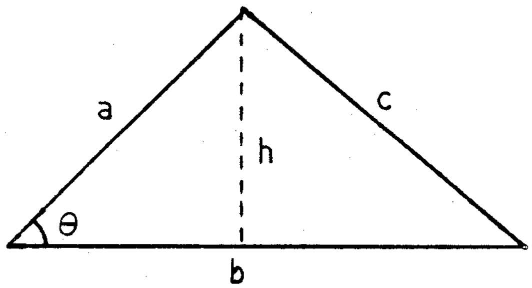
TRIANGLE OF ALTITUDE h AND BASE b

Area = $\frac{1}{2}bh = \frac{1}{2}ab \sin$

$= \sqrt{s(s - a)(s - b)(s - c)}$

where $s = \frac{1}{2}(a + b + c)$ = semiperimeter

Perimeter = $a + b + c$

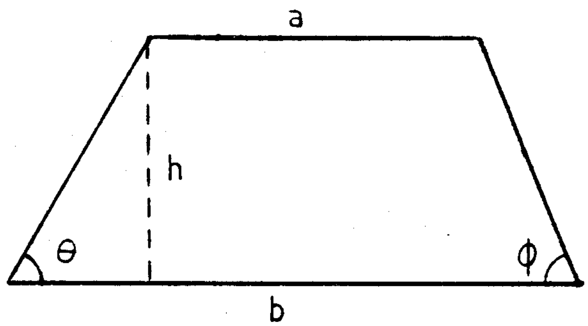


TRAPEZOID OF ALTITUDE h AND PARALLEL SIDES a AND b

Area = $\frac{1}{2}h(a + b)$

Perimeter = $a + b + h(\frac{1}{\sin\theta} = \frac{1}{\sin\phi})$

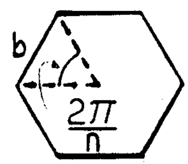
$= a + b + h(\csc\theta + \csc\phi)$



REGULAR POLYGON OF n SIDES EACH OF LENGTH b

Area = $\frac{1}{4}nb^2\cot\frac{\pi}{n} = \frac{1}{4}nb^2\frac{\cos(\pi/n)}{\sin(\pi/n)}$

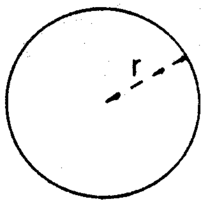
Perimeter = nb



CIRCLE OF RADIUS r

Area = πR^2

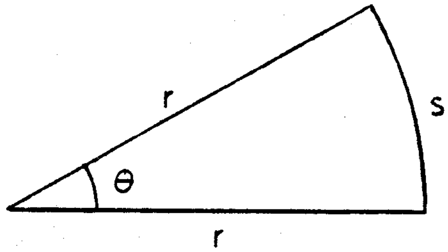
Perimeter = $2\pi r$



SECTOR OF CIRCLE OF RADIUS r

Area = $\frac{1}{2} r^2 \theta$ [θ in radians]

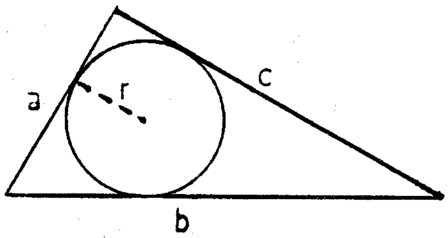
Arc length $s = r\theta$



RADIUS OF CIRCLE INSCRIBED IN A TRIANGLE OF SIDES a,b,c

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

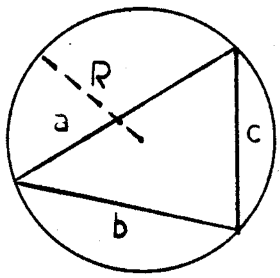
where $s = \frac{1}{2}(a + b + c)$ = semiperimeter



RADIUS OF CIRCLE CIRCUMSCRIBING A TRIANGLE OF SIDES a,b,c

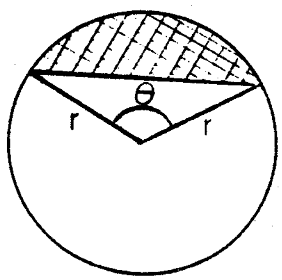
$$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where $s = \frac{1}{2}(a + b + c)$ = semiperimeter



SEGMENT OF CIRCLE OF RADIUS r

Area of shaded part = $\frac{1}{2} r^2(\theta - \sin\theta)$



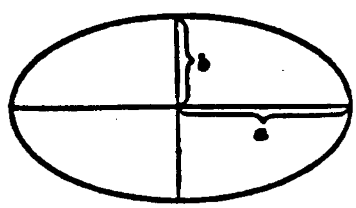
ELLIPSE OF SEMI-MAJOR AXIS a AND SEMI-MINOR AXIS b

Area = πab

Perimeter = $4a\int_0^{\pi/2}\sqrt{1 - k^2 \sin^2\theta}\, d\theta$

$= 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$ [approximately]

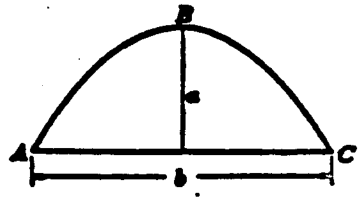
where $k = \sqrt{a^2 - b^2/a}$



SEGMENT OF A PARABOLA

Area = $2/3\ ab$

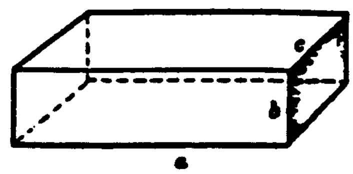
Arc length ABC = $\frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b}{8a}\ln\left(\frac{4a + \quad b^2 + 16a^2}{b}\right)$



RECTANGULAR PARALLELEPIPED OF LENGTH a, HEIGHT b, WIDTH c

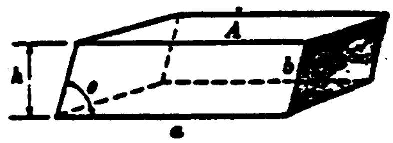
Volume = abc

Surface area = $2(ab + ac + bc)$



PARALLELEPIPED OF CROSS-SECTIONAL AREA A AND HEIGHT h

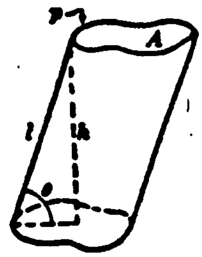
Volume = $Ah = abc \sin\theta$



CYLINDER OF CROSS-SECTIONAL AREA A AND SLANT HEIGHT l

Volume = $Al = \frac{Ah}{\sin\theta} = Ah \csc\theta$

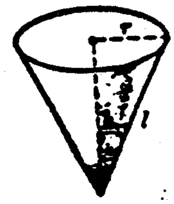
Lateral surface area = $pl = \frac{ph}{\sin\theta} = ph \csc\theta$



RIGHT CIRCULAR CONE OF RADIUS r AND HEIGHT h

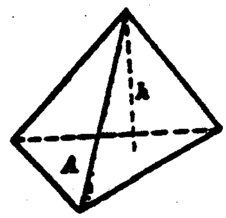
Volume = $1/3\ \pi r^2 h$

Lateral surface area = $\pi r \sqrt{r^2 + h^2} = \pi r l$



PYRAMID OF BASE AREA A AND HEIGHT h

Volume = $1/3\ Ah$

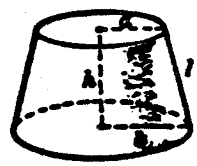


FRUSTRUM OF RIGHT CIRCULAR CONE OF RADII a,b AND HEIGHT h

Volume = $1/3\pi h(a^2 + ab + b^2)$

Lateral surface area = $\pi(a + b)\sqrt{h^2 + (b - a)^2}$

$= \pi(a + b)l$



SOLUTIONS OF ALGEBRAIC EQUATIONS

QUADRATIC EQUATION: $ax^2 + bx + c = 0$

Solutions: $$x = \frac{-b \pm \sqrt{b^2 = 4ac}}{2a}$$

If a,b,c are real and if $D = b^2 - 4ac$ is the discriminant, then the roots are:

(i) real and unequal if D 0

(ii) real and equal if $D = 0$

(iii) complex conjugate if D 0

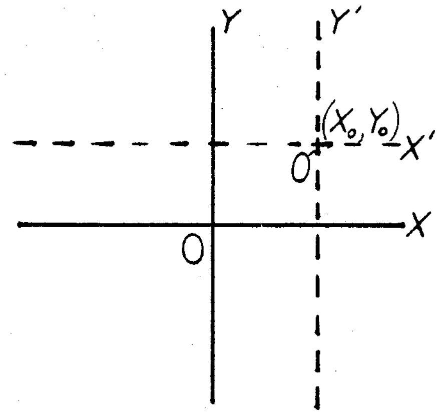
If x_1, x_2 are the roots, then $x_1 + x_2 = -b/a$ and $x_1x_2 = c/a$

FORMULAS FROM PLANE ANALYTIC GEOMETRY

TRANSFORMATION OF COORDINATES INVOLVING PURE TRANSLATION

$$x = x' + x_0 \qquad \text{or} \qquad x' = x - x_0$$
$$y = y' + y_0 \qquad\qquad\quad y' = y - y_0$$

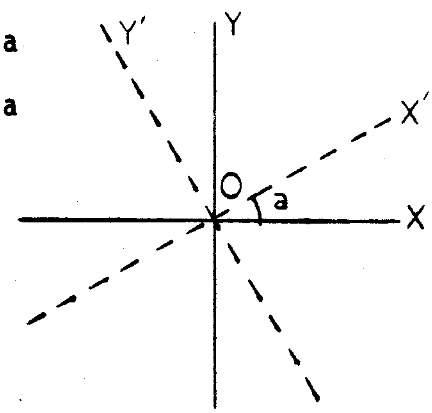
where (x,y) are old coordinates [i.e., coordinates relative to xy system], (x',y') are new coordinates [relative to x'y' system] and (x_0,y_0) are the coordinates of the new origin O' relative to the old xy coordinate system.



TRANSFORMATION OF COORDINATES INVOLVING PURE ROTATION

$$x = x' \cos a - y' \sin a \qquad \text{or} \qquad x' = x \cos a + y \sin a$$
$$y = x' \sin a + y' \cos a \qquad\qquad y' = y \cos a - x \sin a$$

where the origins of the old [xy] and new [x'y'] coordinate systems are the same but the x' axis makes an angle a with the positive x axis.



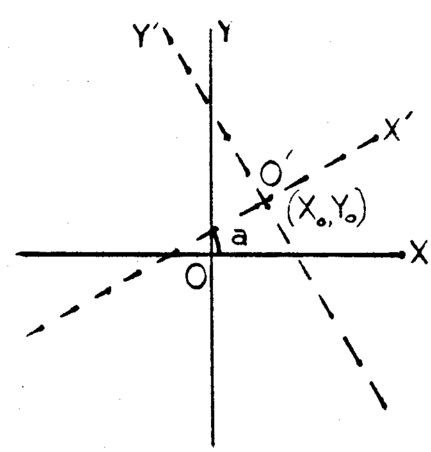
TRANSFORMATION OF COORDINATES INVOLVING TRANSLATION AND ROTATION

$$\begin{cases} x = x' \cos a - y' \sin a + x_0 \\ y = x' \sin a + y' \cos a + y_0 \end{cases}$$

or

$$\begin{cases} x' = (x - x_0) \cos a + (y - y_0) \sin a \\ y' = (y - y_0) \cos a - (x - x_0) \sin a \end{cases}$$

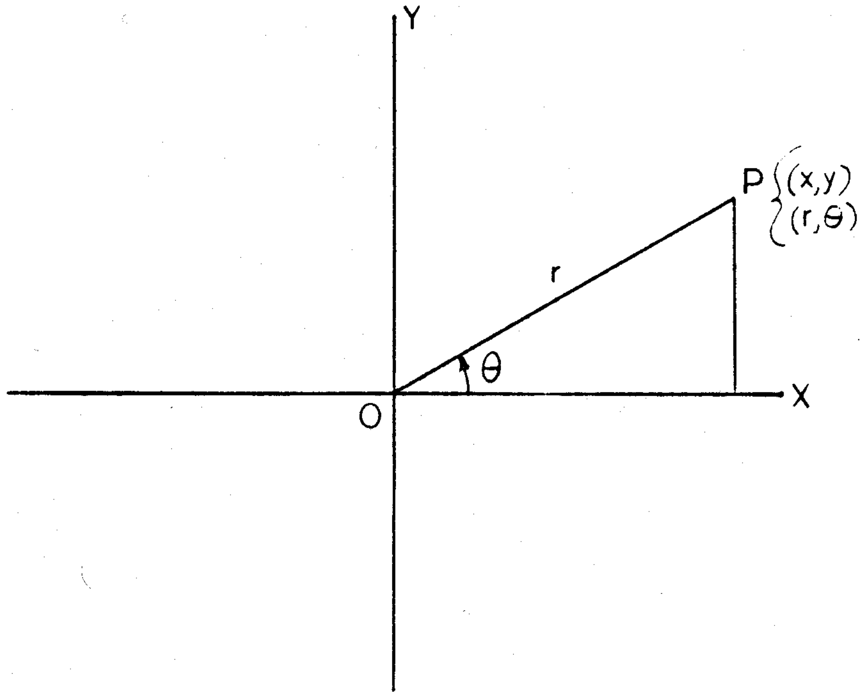
where the new origin O' of x'y' coordinate system has coordinates $(x_0 y_0)$ relative to the old xy coordinate system and the x' axis makes an angle a with the positive x axis.

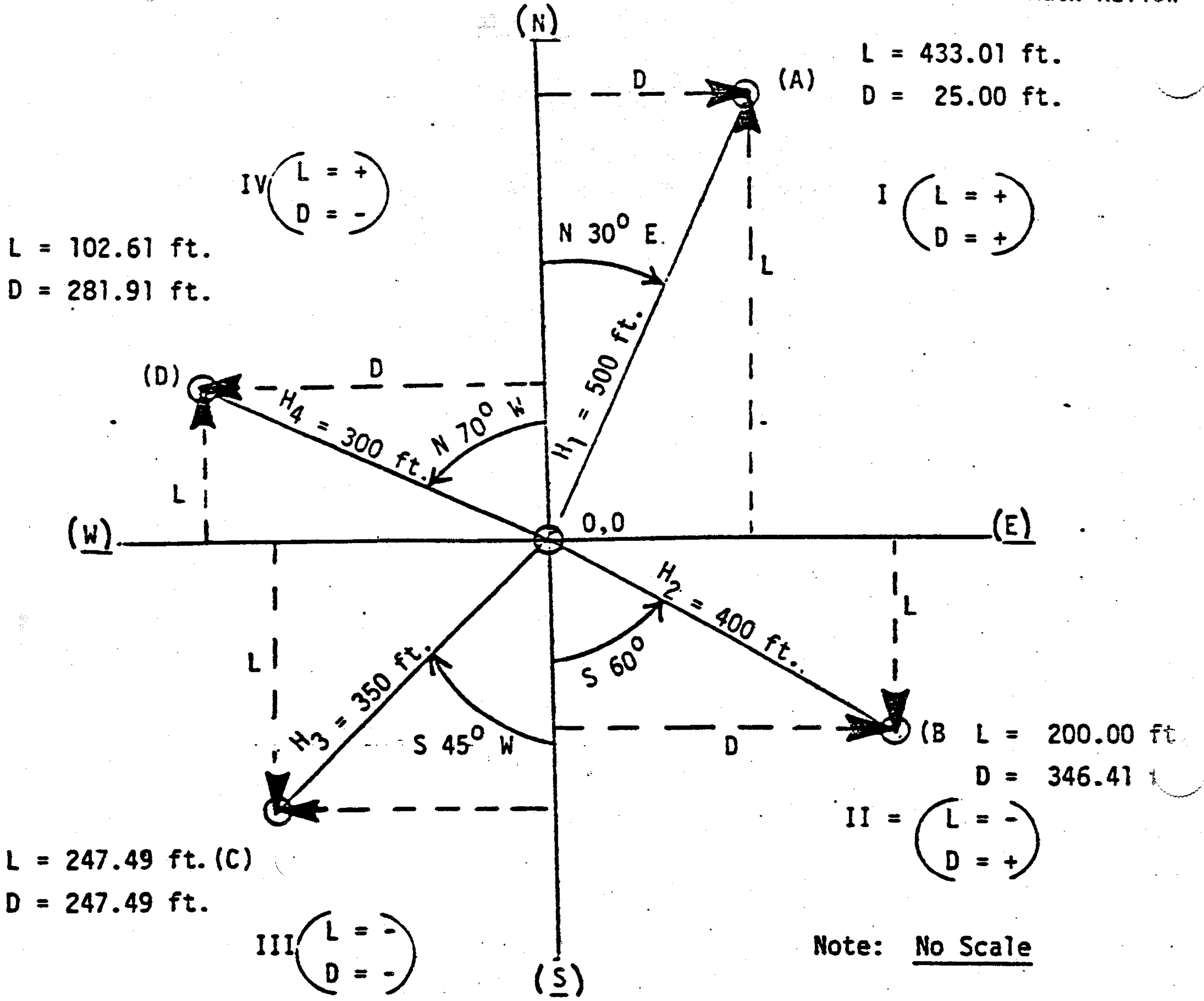


POLAR COORDINATES (r,)

A point P can be located by rectangular coordinates (x,y) or polar coordinates (r,). The transformation between these coordinates is

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{or} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}(y/x) \end{cases}$$



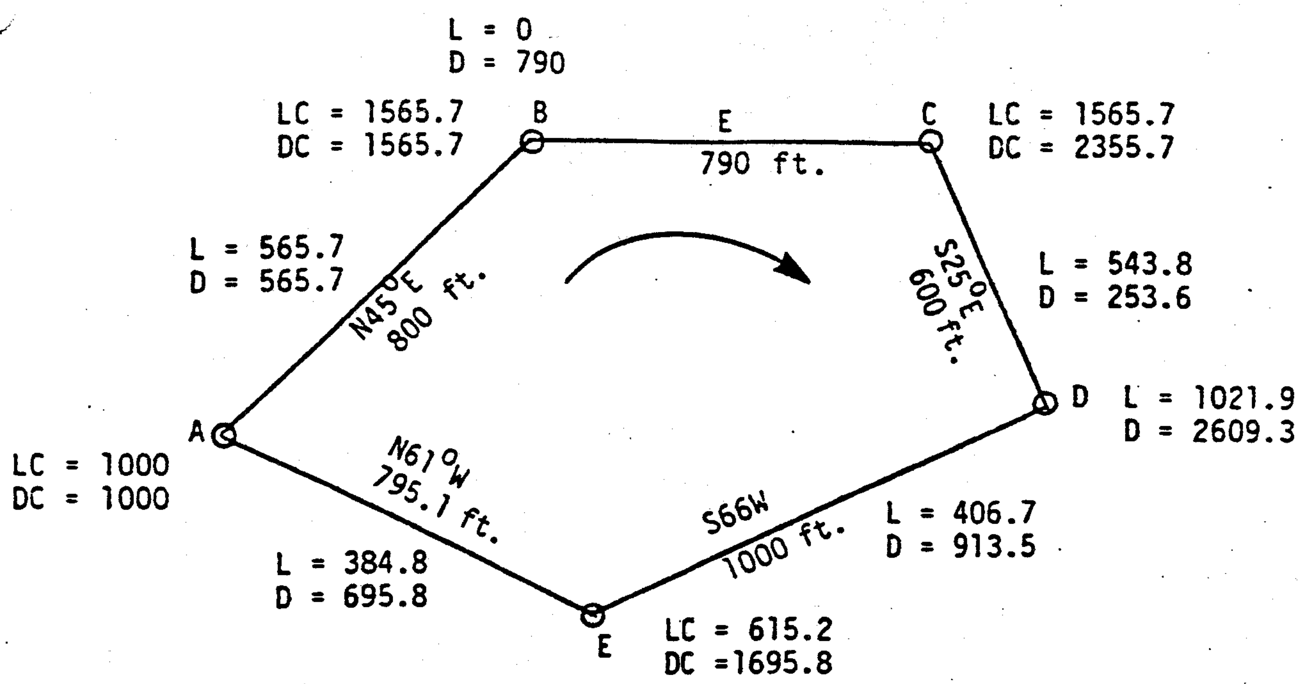


Formulas :

Latitude, (L) = $H_n \cos \measuredangle$

Departure, (D) = $H_n \sin \measuredangle$

Latitude & Departure

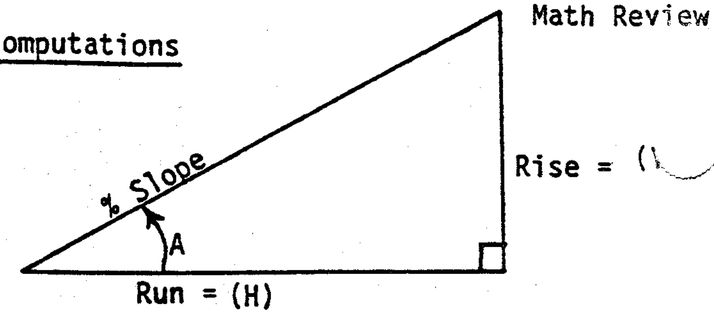


Area By Coordinate Method

A	B	C	D	E	A
$\frac{1000}{1000}$	$\frac{1565.7}{1565.7}$	$\frac{1565.7}{2355.7}$	$\frac{1021.9}{2609.3}$	$\frac{615.2}{1695.8}$	$\frac{1000}{1000}$

$$\begin{array}{r} 11{,}687{,}538.52 \\ \underline{7{,}275{,}596.88} \\ 4{,}411{,}941.64 \end{array} \div 2 = \frac{2{,}205{,}970.82 \text{ ft}^2}{43{,}560 \text{ ft/Ac}} = \underline{\underline{50.64 \text{ acres}}}$$

Percent Slope Computations



Equation:

$$\% \text{ Slope} + \frac{(H)}{(V)}(100\%)$$

$$\% \text{ Slope} = (100\%) \tan A$$

Example:

Run = 50 ft., Rise = 5 ft.

$$\% \text{ Slope} = \frac{5 \text{ ft.}}{50 \text{ ft.}}(100\%) = \underline{\underline{10\%}}$$

Divider Settings to Peg Grades on Contour Maps

Equation:

$$\text{Divider Setting (inches)} = \frac{\text{C.I. in ft}}{\% \text{ Grade}} \times 100 \times \frac{1}{\text{map scale in ft./in.}}$$

C.I. = Map Contour Interval

Example:

Given: C.I. = 20 ft.
Map Scale = 4 inches/mile

Find: Divider Setting in inches to peg a 5% Grade on a Topog Map

Solution:

(1) Map Scale in ft./in. $= \frac{5280 \text{ ft.}}{4 \text{ in.}} = 1320$ ft./in.

(2) Divider Setting $= \frac{20 \text{ ft.}}{5\ \%} \times 100 \times \frac{1}{1320 \text{ ft./in.}} = \underline{\underline{0.30\overline{3} \text{ in.}}}$

(3) Check: $\frac{\text{C.I.} \times 100}{(1320)(0.303)} = \frac{(20)(100\%)}{400} = 5.0\%$

Note: For map scales corresponding to engineers scale, use the following procedure:

Ex: map scale 1 in. = 400 ft.
C.I. = 20 ft.
Grade = 8%

Divider setting $= \frac{(20)(100)}{8} = 250$ ft.

Set dividers at 250 ft. using the 40 scale.