Q5: Are we confident that quantum spin microscopy will work?

A5: Product-sum representations provide a vital QMOR technique

ALGORITHMS FOR NUMERICAL ANALYSIS IN HIGH DIMENSIONS*

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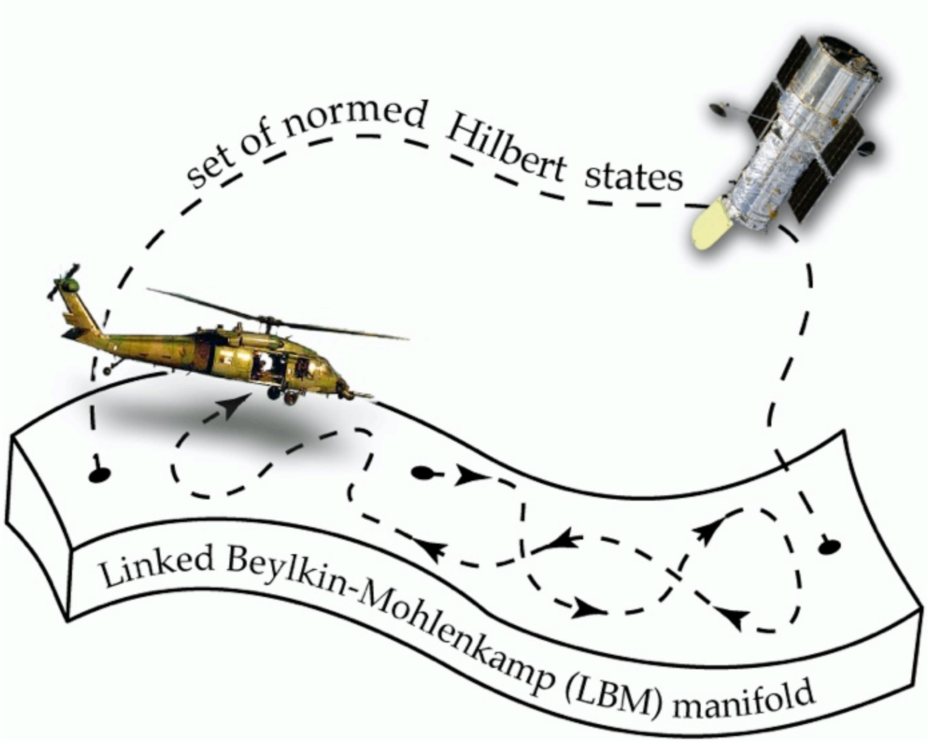
Abstract. Nearly every numerical analysis algorithm has computational complexity that scales exponentially in the underlying physical dimension. The separated representation, introduced previously, allows many operations to be performed with scaling that is formally linear in the dimension. In this paper we further develop this representation by: (i) discussing the variety of mechanisms that allow it to be surprisingly efficient; (ii) addressing the issue of conditioning; (iii) presenting algorithms for solving linear systems within this framework; and (iv) demonstrating methods for dealing with antisymmetric functions, as arise in the multiparticle Schrödinger equation in quantum mechanics. Numerical examples are given.



$$|\psi\rangle = \left\{ \begin{array}{l}
\begin{bmatrix} c_1^{1,1} \\ c_2^{1,1} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,2} \\ c_2^{1,2} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,3} \\ c_2^{1,3} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,4} \\ c_2^{1,4} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,5} \\ c_2^{1,5} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,6} \\ c_2^{1,6} \end{bmatrix} \otimes \begin{bmatrix} c_1^{1,7} \\ c_2^{1,7} \end{bmatrix} \cdots, \\
\begin{bmatrix} c_1^{2,1} \\ c_2^{2,1} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,2} \\ c_2^{2,2} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,3} \\ c_2^{2,3} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,4} \\ c_2^{2,4} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,5} \\ c_2^{2,5} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,6} \\ c_2^{2,6} \end{bmatrix} \otimes \begin{bmatrix} c_1^{2,7} \\ c_2^{2,7} \end{bmatrix} \cdots, \\
\begin{bmatrix} c_1^{3,1} \\ c_2^{3,1} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,2} \\ c_2^{3,2} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,3} \\ c_2^{3,3} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,4} \\ c_2^{3,4} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,5} \\ c_2^{3,5} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,6} \\ c_2^{3,6} \end{bmatrix} \otimes \begin{bmatrix} c_1^{3,7} \\ c_2^{3,7} \end{bmatrix} \cdots, \\
\begin{bmatrix} c_1^{4,1} \\ c_2^{4,1} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,2} \\ c_2^{4,2} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,3} \\ c_2^{4,3} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,4} \\ c_2^{4,4} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,5} \\ c_2^{4,5} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,6} \\ c_2^{4,6} \end{bmatrix} \otimes \begin{bmatrix} c_1^{4,7} \\ c_2^{4,7} \end{bmatrix} \cdots, \\
\begin{bmatrix} c_1^{5,1} \\ c_2^{5,1} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,2} \\ c_2^{5,2} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,3} \\ c_2^{5,3} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,4} \\ c_2^{5,4} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,5} \\ c_2^{5,5} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,6} \\ c_2^{5,6} \end{bmatrix} \otimes \begin{bmatrix} c_1^{5,7} \\ c_2^{5,7} \end{bmatrix} \cdots
\end{array} \right\}$$

- **Separated representations provide a "JPEG format" for compressing quantum state trajectories**
- **They efficiently compress all Hilbert states *except* the high-rank states employed in quantum computation**
- **They are well-suited to quantum system engineering**