Assignment for Day 2 – April 4, 2007

Read: Note to Reader, Prologue and Chapter 1 (pp. 1-44)

Study Questions

1. A classroom scene is described in the Prologue which has *some* similarity to the discussion in this class on the first day. How similar and how different was your experience in this class to the scene described in the Prologue? Be specific about the similarities and the differences. The Prologue then goes on to make an analogy with the struggles that the mathematicians of the 19^{th} century went through on the same issues. What would you say are the main differences between what mathematicians went through and what took place in our class?

2. Most of us would agree that Wally "gets closer and closer to the wall," as in the discussion in §1.2. He also "does not get beyond the wall." But these same two statements could also be made about a second wall that is 1 framit beyond (to the right) of Wally's wall. So, you might say that there is something very special in the relationship between the Wall at 1 framit and Wally's walk that would not be true of any other wall that might be placed along his path. How would you describe this special relationship?

3. The Archimedean Property is the first of several properties that we will want to take as axioms for the real numbers if we are to establish firmly grounded "truths" for calculus. How would you describe this property in everyday language, for instance, to a high school student? Is it surprising to you in any way that we need to state this property as a basic assumption? Does it seem "natural," "obvious," "obscure?" What role does the property play, exactly, in the discussion of infinite walks? [Note that axioms play a very particular role in rigorous mathematics. You might examine your experience of other mathematical subjects, such as geometry, to recall what this role is.]

4. There are four propositions about the relationship between step-sizes and positions of infinite walks in §1.3. They are stated in a formal language, which is on the way to a rigorous study of infinite processes in a number of areas of mathematics. You should summarize what each of these propositions says and also write out a clear and understandable proof for each in your own words. Use as simple and ordinary language as you can in doing this, while being as rigorous as you can be. ***Bring your writings on these propositions to class, so that you can work with others producing proofs of your own.***

5. An argument is given in §1.4 to the effect that Alice's walk with step-sizes given by the Harmonic Sequence is unbounded. Suppose that someone you know, who is mathematically experienced, but is not in this class, understands that Wally's walk chokes off, but doesn't see how it could be, then, that Alice's walk does not. First, give a sketch in your own words of the proof that Alice's walk is unbounded. Then

explain to this person how this could possibly happen. How can we say that one walk is bounded and the other is not, when we are talking about something that "happens at infinity?" "Where's the catch?" What makes it work?

6. In §1.5, two more walks are introduced: Harold's walk and Sally's walk. What do we learn from these walks? What's the *crux* of their behavior? What do they tell us about infinite walks?

7. Using some examples from these infinite walks, explain in your own words what it means to say that an increasing sequence converges to a value A. Explain in your own words what it means for a decreasing sequence to converge to 0. Suppose that the increasing sequence converges to the value A. Is it possible that A is actually one of the terms of the sequence?

8. In your own words, explain what "The Question you never asked" is, exactly. How do mathematicians answer this question? Also explain how it might be helpful to our project of putting a sound foundation under calculus.

   A student once wrote that "The Completeness Property tells us that whenever a walk 'chokes off,' then this walker has a wall. Without this axiom, the walker might fall into a hole." Does this make sense? How would you explain or elaborate it?

9. What does it mean, exactly, to say that we will "adopt an axiom?" We will now adopt the Completeness Property as an axiom. Does this seem "natural," "self-evident?" If it doesn't, then how can we adopt it? By adopting this axiom, are we saying that it is "true?"

E-Post Question for Day 2

In your E-Post, you should give your responses to the following question:

As you begin to read the early sections in *Zeno and the Infinite Walks*, you may find yourself plunging into a strange new world. As you do this, what is your overall sense of this world, its use of language, of logic, etc? Are there other experiences of entering a new world that you can compare and contrast against this world, as a way of understanding this experience? Is it like studying a new language; arriving in a strange place, learning a new skill, or any such experience?

What qualities of this world do you find most striking and unusual, e.g., the use of language, the formal reasoning, its fantastical nature, etc, etc?

Weekly Writing Assignment for Day 2

The following terms have been used in Chapter 1 to describe an increasing sequence of numbers: Bounded, convergent, "chokes off." Are these all the same? Are there important differences among them? Suppose Willy takes an infinite walk in which his step-size decrease and converge to 0. Does this mean that Willy has a wall? *EXPLAIN.*