Assignment for Day 3 – April 11, 2007

Reading: Re-read §1.6 and §1.7. Then read §2.1–2.3 in Volume II of *Making Sense of Calculus*

Study Questions

[The Study Questions on Chapter 1 are slightly modified from last week.]

1. In your own words, explain what "The Question you never asked" is, exactly. How do mathematicians answer this question? Also explain how it might be helpful to our project of putting a sound foundation under calculus.

2. A student once wrote that "The Completeness Property tells us that whenever a walk 'chokes off,' then this walker has a wall. Without this axiom, the walker might fall into a hole." Does this make sense? How would you explain or elaborate it?

3. Suppose we consider Sally's walk which we now know is bounded (p.31). Adopting the Completeness Property as an axiom, tells us that every bounded set has a least upper bound. So, there is a spot which is the least upper bound of Sally's positions. Why is this spot the one that Sally's Wall will appear at? After all, to say that she has a Wall at a particular location is to say that she surpasses every place to the left of it.

4. What does it mean, exactly, to say that we will "adopt an axiom?" We will now adopt the Completeness Property as an axiom. Does this seem "natural," "self-evident?" Does it seem more or less "natural" than the Archimedean Property? By adopting this axiom, are we saying that it is "true?"

5. What does it mean to say that "The Completeness Property doesn't hold in the world of the rational numbers?" If you lived in the world of rational numbers, in what sense would it be true to say that: "There are smooth curves which don't always have tangent lines at every point?

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6. A student once raised the following question: "When we are given an increasing sequence of numbers, how do we compute its least upper bound?" A person might say that in the reading this week, you are being shown how to compute the least upper bound of a sequence of areas of polygons and using this to define the area of a circle. But others might strongly disagree. Part of the confusion probably arises from the term "compute" that is being used here. Perhaps, one might say that in the proof by Exhaustion, "we *obtain* a least upper bound of a sequence of areas of polygons and using this to define the area of a circle." What's the difference between *compute* and *obtain*?

7. It's important that you have an overall sense of the proof being presented in §2.3, and that you know how to fill in most of the details. What is being proved, exactly? What is being assumed?

   One way to begin to grasp a proof is to be able to right a kind of "plot summary." What does it start with? What happens? Like many proofs in mathematics, this particular proof can be seen as having *four* phases:

   a. Preparation. Notation and general strategy are laid out. The ground is prepared.
   b. Narrowing the problem. The general statement of the theorem is reduced to a more limited statement.
   c. The nub or crux. The more limited statement is proved in all its details.
   d. Wrap-up or closure (optional). We go back out from the nub or crux to the general statement.

   See how well you can fit the proof of Exhaustion into this scheme? What is the nature of the argument at the nub or crux?

   We will probably go over the Proof by Exhaustion in class. You should review the proof closely enough to join in this work.

8. Look at the questions raised by the *Interrogator* in the Exhaustion proof. Restate these question in your own words. Do you have questions of your own of this type? If so, what are they?

E-Post Question for Day 3

In your E-Post, you should give your responses to the following question:

> People who study mathematics have various strategies for reading—and *understanding*—mathematical proofs. Very few read a proof ONCE straight through, from beginning to end, and just know what its about. What's your strategy? How do you get the overall idea of a proof and connect it to the details?

Weekly Writing Assignment for Day 3

It's often said that the Greeks invented a way of determining the area of a circle and that the way they did it is to make regular polygons inside a circle with 4 sides, then 8 sides, then 16 sides, then 32, and so forth. Looking at the pictures of this, as in the book, anyone can see that these polygons quickly fill up the circle. If this is true, what is the laborious work about on pages 48–58? First adopt the position of a *skeptical reader*, like the Interrogator. What questions of such a reader might be raised? Why not just say: "The polygons get closer and closer to filling the circle, so we can use them to determine the area of the circle."? In what way is all the labor in these 10 pages answering such a question? *WHAT'S THE POINT OF ALL THIS MATH?*