## *Assignment for* Day 4 – April 17, 2007

Read §§2.4–2.7, pp 59-86 (top) [*See notes below about what to emphasize.*]

### Study Questions

1. How do you explain, in your own words, why we have two different proofs about the area of a circle. Is one *better* than the other? Is one *more correct* than the other? What, exactly, was the shortcoming or weakness of the Proof by Exhaustion? How is this overcome in the Proof by Compression? Do you find one of these two proofs *more convincing* (somehow) than the other?
2. Recall the notion of a "Plot summary" from the last assignment. Do something of this kind for the Proof by Compression? ***We will probably go over this proof in class***. Make sure you think about the question raised by the Interrogator in this section.
3. The properties of the geometric sequence, 1/2, 1/4, 1/8, …, play an enormously important role in both of the proofs about area in this chapter. State in your own words what this role is.
4. In reading §2.5, you should ask yourself the following questions:
   a. Why are being given another definition of convergence? What was wrong with the one we already had? What is gained by a new one?
   b. In everyday language, what do Propositions 1–5 say? What is it you are being told here?
   c. The proofs! What actually happens here? Why are they so complicated?

   In order to answer these questions, focus on one or two of the propositions and their proofs. Try to take in and understand their proofs. Then go back to the above questions, and so forth. ***For the sake of having a class discussion about proofs, we will probably spend some time on the proof of Proposition 1.***
5. Consider the following sequence: 1/2, 5/4, 7/8, 17/16, 31/32, 65/64, …. In a more formal language it is given by:

   $$a_n = 1 + (-1)^n/2^n.$$

   Is there a value $A$ to which this sequence converges? If so, what is it? Can you give a formal proof for your answer?
6. What *exactly* is being proved in §2.6? How is it similar to and different from what was done in earlier sections in this chapter? What is it about perimeter that makes this proof so much more complicated than those about area?
7. The first paragraph in §2.7 begins to present what it means to say that "a circle is a limiting case of polygons." The key statement is in the middle of the paragraph where it says that "[this] is a way of transferring knowledge

we already have about polygons to their 'infinite relatives,' circles." In Proposition 6, knowledge we have about relationships among areas of certain polygons is *transferred* to circles. What is the knowledge we start with about polygons? What is the resulting knowledge about circles? *How exactly is this knowledge TRANSFERRED*?

Proposition 7 also transfers knowledge from polygons to circles. Again, tell what knowledge is being transferred here. The same is true about Proposition 8. Again, give the specifics.

## E-Post Questions for Day 4

In your E-Post, you should give your responses to the following questions:

An *Interrogator* might say of the two proofs about area of a circle in this chapter: "Ok, I see. What you are showing here is that in the long run, when you include enough sides, a polygon is effectively a circle." Is there some truth in this statement? What weaknesses do you find in it?

## Weekly Writing Assignment for Day 6

Here is an excerpt from a paper written by a student in this course a few years ago. Please write your own responses to it or to the issue the student was writing about.

**Limits versus Approximation**
It is difficult for me to get over the notion that limits and approximations are not the same thing.

In the area of a circle problem, the [Exhaustion] method would always still be an approximation. For some reason, the compression method can be accepted as giving a complete answer. I do not see how. If you take two polygons and compress them until the area between them gets closer and closer to zero, you supposedly get the area of the circle. This raises a number of questions: at what point do you stop compressing? Will there not always be space between the two polygons? If this is so, isn't the compression method still an approximation? Likewise, if you have an infinite walk, your walk may converge to a numbered wall, but will you ever reach the wall? How can you name that wall if you cannot reach it?

Thus, the limit exists just so that we can say, "Yes, this is the method by which we can get an exact answer if we converge forever." It is there for definition, rather than truth. Do you really know that something is there? Does the limit serve any other purpose than this?