Assignment for **Day 5 – April 25, 2007**

Read §2.8 and §§3.1–3.3 *Pay particular attention to the proof in §2.8, which we will review together in class*.

Study Questions

1. The theorem that is proved in §2.8 is a special case of a theorem found in every calculus book. It is worthwhile trying to grasp the basic ideas of the proof. One way to do this is to recall, again, the notion of a "Plot Outline" for a proof. It was to ask yourself, generally: What does the proof start with? What happens?, *etc.* More particularly, try to find the *four* phases:

    Preparation. Notation and general strategy are laid out. The ground is prepared.

    Narrowing the problem. The general statement of the theorem is reduced to a more limited statement.

    The nub or crux. The more limited statement is proved in all its details.

    Wrap-up or closure (optional). We go back out from the nub or crux to the general statement.

    a. See how well you can fit the proof for existence of area under the graph of a monotonic function to this scheme.

    b. The "crux" of this proof takes place somewhere around p. 94. Can you summarize what happens there? What makes the proof there work? What's the main idea here?

2. How important in the proof in §2.8 is the assumption that $f(x)$ is monotonic? Is there any possibility of slightly modifying the proof, so that we can prove the same theorem for *any and all* functions?

3. The graph of the function $y = \sin x$ is monotonic from x = 0 to $\pi/2$. Using the ideas of the proof in §2.8, you should be able to get two fairly close estimates of the area under the graph of $y = \sin x$ is monotonic from x = 0 to $\pi/2$, one that is too high and one that is too low. Discuss what you would get in this way by dividing the interval vide the interval 0 to $\pi/2$ into 8 equal pieces. You might even try doing this on a spreadsheet and increasing the number of segments you divide the interval into.

4. Consider the proof of the statement that Suiseth's series has 2 as a sum. First give your own summary of the proof. Do you find it convincing? Does the statement that "2 is the sum of this series" make sense to you? Why/why not? Is this a "real and true" statement, or is it "hypothetical" or "theoretical" in some way? Does it seem "questionable" to you in some other way?

5. Everyone seems to agree that there is something questionable about the number π, because, as they say, we cannot write out its decimal expansion. But in the early 18th Leibniz established the existence of an infinite series that enables us to approximate π to as many decimal places as we like. Subsequent mathematicians have devised infinite series that enable us to do this much more quickly than Leibniz's series. Then why is π any different from $\sqrt{2}$? Both have infinite non-repeating decimal expansions. Why is π *less real* than $\sqrt{2}$? For that matter, you can never write out the decimal expansion for 1/3. So, why is it more real than π or $\sqrt{2}$?

6. After reading §3.3, summarize your own answers to the three questions on p.114. In particular, what is *your* answer to Question 1? Then answer the inevitable question: Where do you stand on the issue of whether or not the decimal system is "real?" Do infinite decimals "exist?" Do some of them exist and others not? Another question that comes to mind: Is an infinite decimal an approximation, in some sense?

7. Can you imagine presenting any of the material in §3.3 to students in the 8th grade—even very bright ones?

E-Post Questions for Day 5

In your E-Post, you should give your responses to the following questions:

You have been invited to give a talk to a high school Honors Math Club. They want to know what's involved in proving that we can define the area under any monotonic function over a finite interval $x=a$ to $x=b$. Is there any way you can imagine presenting the basic idea of the proof in §2.8 to them? What would they find particularly difficult? What issues in the proof in §2.8 would you want to particularly expand on? ***Be as specific as you can.***

Weekly Writing Assignment for Day 5

Imagine drawing a vertical line down the middle of your paper and carefully writing out on the left hand side the steps of the proof (by Compression) about the area of a circle given in §2.4. Then imagine writing the proof of the theorem about area under a monotonic function given in §2.8. What analogies or correspondences would exist between the two proofs? What elements of one of the two proofs would have no counterpart in the other? ***Be specific.***