

Assignment for Day 7 – May 9, 2007

Read §4.1–4.3, pp. 167–204.

Study Questions

1. Before getting started in this reading, you should reflect on what your own understanding is of the concept of a continuous function. Suppose you *suddenly* found yourself in front of a high school calculus class and you needed to give an explanation of this concept; what would you say?
2. The existence of a function like $g(x)$ defined on the lower half of page 168 was a serious shock to the mathematical world in the 19th century. Does this function fit into any notion you have of the concept of function—why or why not? What do you think we will have to do in order to exclude it from our considerations as we build a rigorous theory of calculus?
3. Before reading the dialogue in §4.2, describe any weaknesses or confusions that you see in the standard textbook definition of a continuous function presented on p. 173-4.
4. Suppose you came into the discussion presented in the Dialogue in §4.2. How would you define the notions of jump and gap? What differences and connections do you see between these two concepts?
5. What are the various ideas and difficulties around the notion of a jump as it is discussed by these students in the first 3 or 4 sections (pages 176–184) of the Dialogue? How are they dealt with? Be specific here: What ideas does α (Alf) have? How are they dealt with by the other students; etc, etc?
6. Describe the insight announced by β (Beth) in the last section of the Dialogue. Does this *solve* their problem, or does it just make it more complicated?
7. Summarize in your own words each of the issues outlined at the end of §4.2.
8. The concept of a jump in a monotonic function is defined in the middle of §4.3. You should paraphrase for yourself what this definition means. Illustrate how you understand this idea with the aid of examples of “reasonable” functions, using some that do have jumps and others that don’t have jumps.

9. We will carefully review in class the propositions in §4.3 that describe the relationship between the concepts of a jump and a gap in a monotonic function. You should review the details of these propositions carefully.
10. Paraphrase in your own words what it means to say that a function that is defined on some interval $[a, b]$ and monotonic there is continuous. How does this definition relate to the definition of a continuous given at the beginning of this chapter?

E-Post Questions for Day 7

In your E-Post, you should give your responses to the following question:

In reading §4.2, you probably found yourself plunged into yet another strange world in this course. How would you describe your experience of this new world? How does it compare to other worlds you have entered in this book? Is it stranger than the world of infinite walks? How about the world of the Supreme Court of Mathematics where the concept of “infinite sum” was discussed?

Weekly Writing Assignment for Day 7

Write at least a page (typed) in which you describe, more or less informally, the concept of a jump in a monotonically increasing function. What is the difficulty in defining this concept? That is, why do we have to labor so hard to arrive at a definition? How does this concept relate to the concept of a gap?