Assignment for Day 9 – May 23, 2007

Read §5.1–5.3, pp. 237–257.

Study Questions

1. Check your understanding of the process by which the tangent line to the graph of a function $f(x)$ at $x=p$ is derived by going through this process for various functions, such as $f(x) = x^2$ and $g(x) = 4x^3 + 5x$.

2. Now that you have gone several times through this process of developing the formula for the derivative of a function, explain what it is about this process that led to so much controversy between Newton and his critics. This all seems fairly straightforward; what is the difficulty exactly? You might discuss this in terms of the function $y = \sin x$.

3. Suppose the function $f(x)$ is continuous on the interval $[a, b]$. Does this necessarily tell us that the function has a derivative on that interval?

4. In the text, there is a proof of the theorem that says that if function $f(x)$ has a derivative at the value $x=p$, then it is also continuous there. You should be able to explain in a fairly straightforward manner why this theorem is true. What, if anything, does this theorem tell us about the absolute value function $y = |x|$?

5. The main work of this week's assignment is to master the proof of the statement that if a function is continuous on the interval $[a, b]$, then the area under the graph of this function over the interval $[a, b]$ exists. Before beginning the reading, you should go back and find your proof of the corresponding theorem for a monotonic function. In particular, you should make sure you understand the proof of the earlier theorem as a series of logical steps, especially in parallel with the proof by compression of the area of a circle. *Also note that there is an outline of the proof in this present section on page 246. You should start by matching the main steps of this outline to what you did for monotonic functions.*

6. Lower and upper sums are defined on p.248 for any partition of the interval $[a, b]$. How are these different from the lower and upper sums defined in the earlier proof for monotonic functions? Similarly, upper and lower approximations to the area are defined on p.250. How are these related to similar values in the earlier proof?

7. On p.252 the not very surprising fact that the lower approximation to the area is less than or equal to the upper approximation. What's the main idea behind the proof of this fact?

8. On p.253 we actually assume for the first time in this section that the function $f(x)$ is continuous on the interval $[a, b]$. What have we been assuming about $f(x)$ until now? How has it been possible to not use continuity until this point? Specifically, how is the continuity of the function $f(x)$ used in the proof on p. 253-4?

9. On p.253 the heavy lifting of this proof really begins. Before getting into the details, look over the proof and make sure you have a rough idea of how it goes. In what way is we do here different from or similar to what was done with monotonic functions?

10. The Convergent Subsequence Theorem from Chapter 4 is used on p.255. It also provides the crucial step in the three main propositions §4.6. What is the connection between these various uses of that theorem? What does it do for us in these various proofs?

11. We will go over the entire proof given in this section (not just the theorem on p.254). Make sure you understand as much as you can on your own. If you arrive at a point at which you can't follow what is going on, write out in clear English what your problem is. *See the writing assignment below*.

12. What does the material in this section have to do with the notion of an integral that you learned in your elementary calculus course?

E-Post Questions for Day 9

In your E-Post, you should give your responses to the following question:

> One student might say about the material in §5.3, "I get the big picture of this proof, but I can't follow all the details." Another might say, "Well, I can handle the details, but what are they all about? How do they fit together?" Other reactions are also possible. What do you get about the §5.3? Where do you need the most immediate help?

Weekly Writing Assignment for Day 8

Your fairy godmother will answer all your questions on the proof in §5.3, *BUT ONLY IF YOU WRITE THEM OUT VERY CLEARLY FILLING OUT AT LEAST ONE PAGE*. Write out one or two such questions, giving a complete discussion for each of what you know of the material related to this question and, *precisely*, what your question is. E.g., say something like "I understand this fact (or line) and that fact (or line), but then when we get down to $%^#*, I can't follow why that is true."