Section A. Multiple choice and simple computation. [5 points each] (Version A)

(1-4) Four independent projects, each with risk free cash flows, have the following B/C ratios:

<table>
<thead>
<tr>
<th>Project</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>B/C</td>
<td>5</td>
<td>1</td>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

1. *Circle all of the projects that have NPV > 0?
   a) A  b) B  c) C  d) D  e) can’t tell

2. *Circle all of the projects that have NPV < 0?
   a) A  b) B  c) C  d) D  e) can’t tell

3. *With an unlimited budget, in which project(s) would you invest?
   a) A  b) B  c) C  d) D  e) can’t tell

4. *Suppose each project is mutually exclusive and has the same PV of costs. With an unlimited budget, in which project(s) would you invest?
   a) A  b) B  c) C  d) D  e) can’t tell

(5-7) You’ve developed a new medical device that recently received FDA approval. You would like invest in forming a manufacturing company to produce and sell the device. Based on a hurdle rate of 15% and the large estimated profits you will earn into the foreseeable future, you calculate a positive NPV.

5. Which of the following must be true?
   a) Investing now is optimal since the estimated Net Present Value is positive.
   b) Before investing you should invest in market research since there is a positive real option value.
   c) There is an IRR of at least 15% on the estimated cash flows.
   d) The estimated payback period is negative.

6. *If the hurdle rate precisely captures all opportunity costs, which of the following must be true?
   a) Investing now is optimal since the estimated Net Present Value is positive.
   b) Before investing you should invest in market research since there is a positive real option value.
   c) There is an IRR of at least 15% on the estimated cash flows.
   d) The estimated payback period is negative.

7. *If the hurdle rate doesn’t capture the opportunity cost of delay, which of the following must be true?
   a) Investing now is optimal since the estimated Net Present Value is positive.
   b) Before investing you should invest in market research since there is a positive real option value.
   c) There is an IRR of at least 15% on the estimated cash flows.
   d) The estimated payback period is negative.
8. You purchased an at the money Call option on IBM three months ago that matures today. If the return on IBM stock was -110% over this period, what is the value of the option to you today?

a) \((-1.1)K\)  
b) \(0\)  
c) \(K\)  
d) \((1.1)K\)  
e) can’t tell

9. You wrote an at the money Call option on IBM three months ago that matures today. If the return on IBM stock was 230% over this period, what is the value of the option to you today?

a) \((-2.3)K\)  
b) \(0\)  
c) \(-K\)  
d) \((2.3)K\)  
e) can’t tell

10. You purchased an in the money Call option on IBM three months ago that matures today. If the return on IBM stock was -10% over this period, what is the value of the option to you today?

a) \((-0.1)K\)  
b) \(0\)  
c) \((0.1)K\)  
d) \((0.9)K\)  
e) can’t tell

(11-12) Starbucks recently issued the following, Grade A, corporate bonds:
A: 3 year, $1,000 face value, 10% coupon, \(P_A = $1,000\)
B: 3 year, $100 face value, 8% coupon, \(P_B = $96\)
C: 4 year, $1,000 face value, 0% coupon, \(P_C = $850\)
D: 3 year, $100 face value, 0% coupon, \(P_D = $86.11\)
E: 3 year, $100 face value, 4% coupon, \(P_E = ?\)

11. What is the price, \(P_E\), of E?

a) $90  
b) $94  
c) $96  
d) $100  
e) none of the above

12. What is the 3 year spot rate on Grade A corporate bonds?

a) 1.4%  
b) 3.6%  
c) 4.1%  
d) 5.1%  
e) none of the above

(13-15) Assume Revenue is linear in \(q\), Cost is convex in \(q\), and \(q\) is non-negative.

13. If \(MR(q) > MC(q)\) then which of the following is true?

a) \(q^* < q\)  
b) \(q^* = q\)  
c) \(q^* > q\)  
d) \(F > 0\)  
e) Profit is linear in \(q\)

14. You’re considering two production technologies, (1) and (2), neither of which affect revenue. If you prefer (1) to (2) when each is at its optimal production level, \(q_i^*\), which of the following must be true?

a) \(MC(q_1^*) < MC(q_2^*)\)  
b) \(MC(q_1^*) = MC(q_2^*)\)  
c) \(MC(q_1^*) > MC(q_2^*)\)  
d) none of the above

15. If \(TC_1(q) = 7 + 2q^2\) and \(TC_2(q) = 16 + q^2\) then the break-even volume

a.) \(q_b < 3\)  
b.) \(q_b = 3\)  
c.) \(q_b > 3\)  
d.) does not exist
Simple computations

16. If you make payments of $50 per month on a loan with 6% nominal interest, compounded bi-annually, and no fees, what is the effective monthly interest rate?

\[ r = \left( 1 + \frac{0.06}{2} \right)^{1/6} - 1 = 0.494\% \]

17. Suppose you make the payments in (16) for a total of 10 years. How much did you borrow?

\[ P = 120 \times (P \setminus A, 0.08, 5) + 30 \times (P \setminus G, 0.08, 5) \]

\[ P = 120 \times \frac{(1 + 0.08)^5 - 1}{0.08(1 + 0.08)^5} + 30 \times \frac{(1 + 0.08)^5 - 0.08 \times 5 - 1}{0.08^2(1 + 0.08)^5} \]

\[ P = 479.12 + 221.17 \]

\[ P = \$700.29 \]

18. On a certain piece of machinery, it is estimated that the maintenance will be $100 at end of year 2 and increase by $100 per year through the end of year 5. What is the Equivalent (uniform) Annual maintenance cost if the interest rate is 6% compounded annually?

\[ P = 50 \times (P \setminus A, 0.00494, 120) = \frac{50}{0.00494} \left[ 1 - \frac{1}{(1 + 0.00494)^{120}} \right] = \$4,518.71 \]

19. You have purchased a new car and want to deposit enough money in the bank to pay for maintenance on the car for the first 5 years. Maintenance will cost $120 for the first year and increase by $30 per year. Assume the maintenance costs occur at the end of each year and the bank pays 8% interest. How much should you deposit in the bank?

\[ A = 100 \times (A \setminus G, 0.06, 5) \]

\[ A = 100 \times \left[ \frac{1 - \frac{n}{i(1 + i)^n}}{1 - \frac{n}{i(1 + i)^n}} \right] = 100 \times \left[ \frac{1}{0.06} - \frac{5}{(1 + 0.06)^5 - 1} \right] = \$188.36 \]

20. You want to be a millionaire by the time you turn 60 and believe that you can obtain a 15% rate of return on your investments. You decide to contribute annually to a mutual fund beginning on your 30th birthday and ending on your 59th birthday. How much money should you invest each year to reach your goal?

\[ A = 1,000,000 \times (A \setminus P, 0.15, 30) \]

\[ A = 1,000,000 \times \frac{0.15 \times (1 + 0.15)^5}{(1 + 0.15)^5 - 1} \]

\[ A = \$171,017.05 \]
Section B. Short Answer [100 points total]

1. [40] You are evaluating a cash flow for a large, privately owned, engineering firm with a B grade credit rating.

a) Given the set of investment opportunities available to you, as described below, what is the most you would be willing to pay today for the cash flow? (amounts given in thousands of dollars)

b) How much are you paying (earning) in risk premiums?

You will have to raise capital to cover the negative cash flows. The positive cash flow in year 2 is from a US government contract; whereas, the cash flow in year 3 is from a contract a with new start-up company. You’re worried that this company will not follow through on their contractual obligations (i.e., you do not consider them to be as reliable as your own company). There are no floating costs for issuing a bond.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-100</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>-50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Opportunities</th>
<th>Term</th>
<th>1 yr</th>
<th>2 yrs</th>
<th>3 yrs</th>
<th>4 yrs</th>
<th>5 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds/Grade</td>
<td>US Gvt.</td>
<td>5.5%</td>
<td>6.1%</td>
<td>6.5%</td>
<td>6.8%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Rates</td>
<td>Corporate/A</td>
<td>6.0%</td>
<td>7.0%</td>
<td>7.5%</td>
<td>8.0%</td>
<td>8.5%</td>
</tr>
<tr>
<td></td>
<td>Corporate/B</td>
<td>6.5%</td>
<td>8.0%</td>
<td>8.5%</td>
<td>9.0%</td>
<td>9.5%</td>
</tr>
<tr>
<td></td>
<td>Corporate/C</td>
<td>8.0%</td>
<td>10.0%</td>
<td>11.5%</td>
<td>13.0%</td>
<td>14.5%</td>
</tr>
<tr>
<td>Yields</td>
<td>US Gvt.</td>
<td>3.1%</td>
<td>3.5%</td>
<td>3.9%</td>
<td>3.9%</td>
<td>4.4%</td>
</tr>
<tr>
<td>Term</td>
<td>Bank 1</td>
<td>6.4%</td>
<td>8.1%</td>
<td>8.0%</td>
<td>9.1%</td>
<td>9.7%</td>
</tr>
<tr>
<td>Loans</td>
<td>Bank 2</td>
<td>6.5%</td>
<td>8.1%</td>
<td>8.3%</td>
<td>9.5%</td>
<td>9.7%</td>
</tr>
<tr>
<td>(APR)</td>
<td>Bank 3</td>
<td>6.3%</td>
<td>7.7%</td>
<td>8.2%</td>
<td>9.2%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>

**Solution:**

a) [30] For each separate cash flow you have to determine which opportunities are comparable (in terms of riskiness) and among the comparable alternatives, which one constitutes the best alternative opportunity.

Year 1: Comparable Opportunities – issue a B rated corporate bond at 6.5% interest, or take out a 1 year term loan at 6.4%, 6.5%, or 6.3% interest. *The best available opportunity is borrowing $100,000 from Bank 3 at 6.3% interest.*

Year 2: Comparable Opportunities – Purchase a 2 year US zero-coupon bond that pays 6.1% interest.

Year 3: Comparable Opportunities – Purchase a 3 year C-grade zero-coupon corporate bond that pays 11.5% interest.

Year 4: Comparable Opportunities – issue a B rated corporate bond at 9.0% interest or take out a 4 year term loan at 9.1%, 9.5%, or 9.2% interest. *The best available opportunity is issuing a Grade B corporate bond paying 9.0% interest.*

\[ PV = -\frac{100}{1.063} + \frac{100}{(1.061)^2} + \frac{100}{(1.115)^3} - \frac{50}{(1.09)^4} = 31,477 \]

b) [10] The risk premiums are the interest rates used above subtract the US bond rate for the same term. They are 0.8%, 0%, 5%, and 2.2%, for years 1, 2, 3, and 4, respectively. The **boldface** premiums are those that you pay as a class B firm. In year 2 you do not earn a risk premium since the cash flow is without risk; whereas, in year 3 you earn a hefty 5% premium since you’re dealing with a client that is likely to default.
2. [30] You are looking for debt only financing of a potentially lucrative R&D project within your company. Your company prefers a debt mixture of approximately 80% term debt and 20% bond issues (Grade A with annual coupon). For this project the bond maturities and loan term will each be 5 years.

For 5 year term loans your commercial bank charges 8% nominal interest, compounded quarterly, plus a 3% handling fee deducted directly from the initial loan amount. The loan will require annual payments beginning 1 year from today. To issue a Bond you will pay the investment bank a 7% floating charge.

Suppose you issue 22 par bonds, each with a face value of $1,000 and the spot rates and your risk premiums given at the right:

<table>
<thead>
<tr>
<th>US Bond Spot Rates</th>
<th>A Grade Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 yr</td>
<td>2 yrs</td>
</tr>
<tr>
<td>5.3%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

a) How much will you raise from the bond issue?
b) What coupon rate must you pay on the bond?
c) If you need $100,000 to finance the project, how much will you need to borrow with a term loan?
d) What is your APR on the term loan?
e) Calculate your annual payment on the term loan.
f) Draw a table and/or cash flow diagram describing your overall financing. What is the present value of this cash flow and describe, in words, what it tells you?

Solution:

a) [5] Par bonds implies you sell them each for the face value of $1,000. Taking out the floatation charge you are left with $22,000*(1-0.07) = $20,460.

b) [5] To make it a par bond, \( C \) must satisfy:

\[
1,000 = 1,000\left(\frac{1}{1.06} + \frac{1}{1.07^2} + \frac{1}{1.07^5} + \frac{1}{1.08^4} + \frac{1}{1.085^7}\right) + \frac{1,000}{1.085^5}
\]

\[
\Rightarrow C = \left(1 - \frac{1}{1.085^7}\right)/4.02 = 8.33\%
\]

c) [5] After fees have been deducted you will need $100,000 - $20,460 = $79,540. So you will need to find \( X \) satisfying \( X(1 - 0.03) = 79,540 \), implying that \( X = 82,000 \).

d) [5] Ignoring the loan fees, the effective annual rate of interest is \( (1 + 0.08/4)^4 - 1 = 8.24\% \), but we have to account for the 3% fee in the APR. Instead of $1 in debt today growing to $1.0824 in a year, we have $0.97 growing to $1.0824, so the rate of growth in our debt is actually \( r = 1.0824/0.97 = 8.5\% \).

e) [5] \( A = rP/\left[1 - (1+r)^{-5}\right] = (0.085)(82,000)/\left[1 - (1.085)^{-5}\right] = 20,807.80 \)

f) [5] In $1,000:

\[
\begin{array}{cccccc}
& 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{t} & \text{100} & -22.6 & -22.6 & -22.6 & -22.6 & -44.6 \\
\end{array}
\]
Since the cash flows are risk free, to find the NPV we discount using the spot rates:

\[
NPV = 22,640 \left[ \frac{1}{1.053} + \frac{1}{1.059^2} + \frac{1}{1.063^3} + \frac{1}{1.068^4} + \frac{1}{1.075^5} \right] + \frac{22,000}{1.07^5} - 100,000 = 9,766
\]

This is the equivalent price you would have to pay today to finance your project (in addition to the $100,000).
3. [30] Consider a modification of the pizza delivery capital budgeting problem. You will run the business out of the UW residence halls for the next four years, use a 10% MARR, and pay a tax rate of 30%. You are considering the purchase of at most one of the following four alternatives. As long as it has value, you will salvage any vehicle after year 4. The Moped and Vespa each lose $200/yr in resale value.

1) A new 50cc Moped has a cost basis of $1,000 and a 5 year useful life.

2) A used 50cc Vespa has a cost basis of just $600, but only a 3 year useful life. If you purchase a Vespa, after 3 years you will replace it with another Vespa at the same price.

3) A 150cc Wildfire has a cost basis of $1,500 and a useful life of 4 years, but provides $200/yr more in after tax profits than a Moped.

4) A used 800cc Harley Davidson has a cost basis of $7,000 and a useful life of 4 years, but provides $5,000/yr in after tax profits.

You are also considering an expansion of your business by purchasing a new mega-toaster oven for your dorm room. If you expand, you will be forced to purchase either the Wildfire or the Harley to be able to deliver the extra pizzas on time. You have estimated the pre-tax cash flow of the oven in the table below (data is in $100 and assumes you’re able to deliver all of the pizzas). The oven is in the 3 year property class and has the following MACRS schedule, which switches from 1.5DB to SL depreciation in year 3.

<table>
<thead>
<tr>
<th>Time</th>
<th>Pre-tax cash flow</th>
<th>3 year MACRS</th>
<th>Depreciation</th>
<th>Book Value</th>
<th>Taxable Income</th>
<th>Tax</th>
<th>After tax cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-21</td>
<td>0</td>
<td>0</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>-21</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>33.33%</td>
<td>7.00</td>
<td>14</td>
<td>10</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>13.14</td>
<td>44.45%</td>
<td>9.34</td>
<td>4.66</td>
<td>3.8</td>
<td>1.14</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>14.81%</td>
<td>3.12</td>
<td>1.54</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>9.34</td>
<td>7.41%</td>
<td>1.54</td>
<td>0</td>
<td>7.8</td>
<td>2.34</td>
<td>7</td>
</tr>
</tbody>
</table>

a) Using the assumptions of MACRS, fill in the details of the above table and determine the NPV of business expansion. (Round the depreciation values up to two decimal places, as in yr 2 above.)

b) The Moped and Vespa alternatives will bring in $3,300 per year in after tax profits and you have a $9,000 budget. Giving specific values for each parameter, formulate an optimization problem to find the best mix of alternatives. (Hint: consider how salvage values will affect your budget.)

c) By inspection of (b), what choices do you think are optimal? Assume the oven has no salvage value.

Solution: a) [10]

b) [15] Let $x_i = 1$ if we invest in transportation alternative $i$, and $x_i = 0$ if we do not invest in this alternative. Let $x_5$ denote the binary decision of whether or not to invest in the business expansion. We formulate the optimization problem as follows:
There are two challenging parts of this problem. One is defining the “expand only if” constraint and the other is that salvage costs reduce your costs and may make otherwise infeasible alternatives feasible. Note we could have written the constraint as:

\[
\sum_{i=1}^{s} c_i x_i \leq \$9,000
\]

Max NPV

\[
\max_{x_i} \sum_{i=1}^{5} p_i x_i
\]

Budget

\[
\sum_{i=1}^{s} x_i \leq 1
\]

Mutual Exclusion

\[
x_5 \leq x_3 + x_4
\]

Expand only if buy Harley or Wildfire

Yes/No Decisions

\[
x_i = 0 \text{ or } 1 \text{ for } i = 1 \text{ to } 5
\]

The parameter values are as follows:

\[
p_i = \text{PV(benefits)} + \text{PV(salvage costs)} = \text{PV(benefits)} + c_i \text{ for option } i \text{ (in $1,000)}
\]

\[
p_1 = (3.3/0.1)[1 - (1.1)^{-4}] + [0.2(1.1)^{-4} - 1] = \$9,280
\]

\[
p_2 = (3.3/0.1)[1 - (1.1)^{-4}] + [0.4(1.1)^{-4} - (0.6 + 0.6(1.1)^{-3})] = \$9,366
\]

\[
p_3 = (3.5/0.1)[1 - (1.1)^{-4}] + [0 - 1.5] = \$9,278
\]

\[
p_4 = (5/0.1)[1 - (1.1)^{-4}] + [0 - 7] = \$8,849
\]

so the parameter values are:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(c_i)</td>
<td>(p_i)</td>
</tr>
<tr>
<td>1</td>
<td>$863</td>
<td>$9,280</td>
</tr>
<tr>
<td>2</td>
<td>$778</td>
<td>$9,366</td>
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<tr>
<td>3</td>
<td>$1,500</td>
<td>$9,278</td>
</tr>
<tr>
<td>4</td>
<td>$6,317</td>
<td>$9,532</td>
</tr>
<tr>
<td>5</td>
<td>$2,100</td>
<td>$8,680</td>
</tr>
</tbody>
</table>

c) [5] **Buying a Harley and expanding** will give you the maximum NPV = $18,212. Looking strictly at initial costs, this alternative appears infeasible ($7,000 + $2,100 > $9,000), but recovering the salvage value reduces the effective cost of the Harley enough to make this option feasible.