Job Shop Scheduling

Job Shop

A work location in which a number of general purpose work stations exist and are used to perform a variety of jobs

Example: Car repair – each operator (mechanic) evaluates plus schedules, gets material, etc. – Traditional machine shop, with similar machine types located together, batch or individual production

Factors to Describe Job Shop Scheduling Problem

1. Arrival Pattern
2. Number of Machines (work stations)
3. Work Sequence
4. Performance Evaluation Criterion

Two Types of Arrival Patterns

- Static - n jobs arrive at an idle shop and must be scheduled for work
- Dynamic – intermittent arrival (often stochastic)

Two Types of Work Sequence

- Fixed, repeated sequence - flow shop
- Random Sequence – All patterns possible

Some Performance Evaluation Criterion

- Makespan – total time to completely process all jobs (Most Common)
- Average Time of jobs in shop
- Lateness
- Average Number of jobs in shop
- Utilization of machines
- Utilization of workers

Gantt Chart

- Simple graphical display technique – suitable for less complex situations
- This does not provide any rules for choosing but simply presents a graphical technique for displaying results (and schedule) and for evaluating results (makespan, idle time, waiting time, machine utilization, etc.)

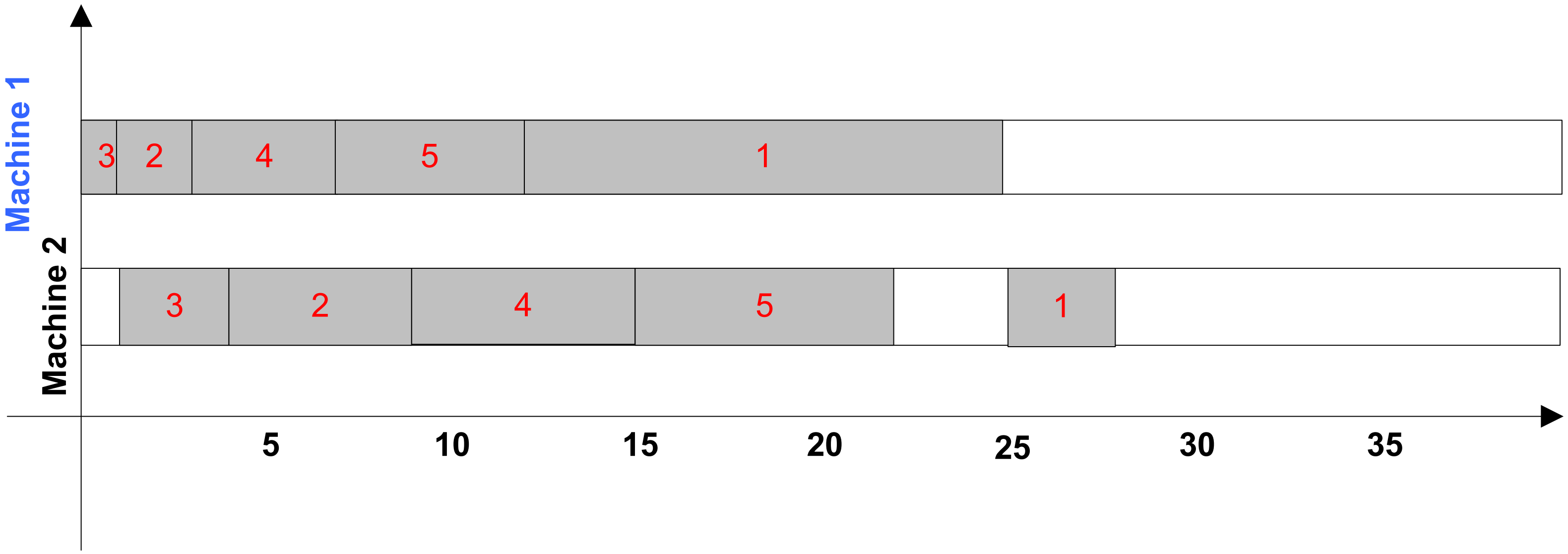
Example of Gantt Chart (I)

5 jobs, 2 machines, each job must first go to machine 1, and then 2 – without changing order. Processing times are:

JOB	Machine 1	Machine 2
1	13	3
2	2	5
3	1	3
4	4	6
5	5	7

Assume order jobs are worked is {3,2,4,5,1}

Example of Gantt Chart (II)



Example of Gantt Chart (III)

Here we assume setup time is included in process time.

- Makespan = 28
- Machine 1 has no idle time except 3 units at end of day
- Machine 2 has 3 units of idle time plus 1 unit at beginning of day.
- Jobs 2, 4 and 5 wait a total of 6 units at machine 2

Scheduling Solutions

- In Order to begin to attempt to develop solution, break the problem in categories:
  1. N jobs, 1 machine
  2. N jobs, 2 machines (flow shop)
  3. N jobs, 2 machines (any order)
  4. N jobs, 3 machines (flow shop)
  5. N jobs, M machines

Scenario 1 – n jobs, 1 machine (I)

- Let P1, P2, … Pn be processing time for each job – (including setup)
- The schedule possibilities are the permutations of n, which is equal to "n!"
- Since the total processing time, or makespan is independent of sequence, this is not a criterion for choice – Consider using minimum mean flow time

Scenario 1 – n jobs, 1 machine (II)

Flow time for job in kth position is:

$$F_{[k]} = \sum_{i=1}^{k} P_{[i]}$$

Mean flow time for n jobs:

$$\overline{F} = \frac{\sum_{k=1}^{n} F_{[k]}}{n} = \frac{\sum_{k=1}^{n} \sum_{i=1}^{k} P_{[i]}}{n}$$

$$\overline{F} = \frac{\sum_{i=1}^{n} (n-i+1) P_i}{n}$$

Scenario 1 – n jobs, 1 machine (III)

It can be proven that $\overline{F}$ is minimized by taking jobs in order of shortest processing time [SPT]

That is order by increasing P, so that

$$P_{[1]} \leq P_{[2]} \leq P_{[3]} \leq \ldots \leq P_{[n]}$$

Scenario 1 – n jobs, 1 machine (IV)

Provide numerical weighting to jobs by priority

(w) – higher w, more important then

$$\overline{F_w} = \frac{\sum_{i=1}^{n} w_i F_{[i]}}{n}$$

and jobs are sequenced by:

$$\frac{P_{[1]}}{w_{[1]}} \leq \frac{P_{[2]}}{w_{[2]}} \leq \frac{P_{[3]}}{w_{[3]}} \leq \ldots \leq \frac{P_{[n]}}{w_{[n]}}$$

Scenario 1 - example

Processing Time:

Job	P	W	P/w
1	10	5	2.0
2	6	10	0.6
3	5	5	1.0
4	4	1	4.0
5	2	3	0.67
6	8	5	1.60

SPT sequence = 5,4,3,2,6,1

SPT / Priority sequence = 2,5,3,6,1,4

Scenario 2 – n jobs, 2 machines, flow shop (I)

These jobs must go to machine 1 first and 2 second – The minimum makespan is determined using **Johnson's Algorithm**

Let P_{ij} = Processing time for job i on machine j

Scenario 2 – n jobs, 2 machines, flow shop (II)

The Algorithm is:

1. Find the job with minimum P_{ij}
2. If j = 1 (machine 1) this job becomes the first job
3. If j = 2 (machine 2) this job becomes the last job
4. Remove assigned job from the list and repeat (break ties at random)

Scenario 2 – n jobs, 2 machines, flow shop (III)

- **Example:** Processing Time as follow

Job	Mach 1	Mach 2
1	4	3
2	1	2
3	5	4
4	2	3
5	5	6

$P_{11} = 4$, $P_{12} = 3$, $P_{41} = 2$, $P_{42} = 3$, … etc.

Using Johnson Rule:

Min Pij = P21 = 1 – now eliminate job 2

Min Pij = P41 = 2 – now job 4

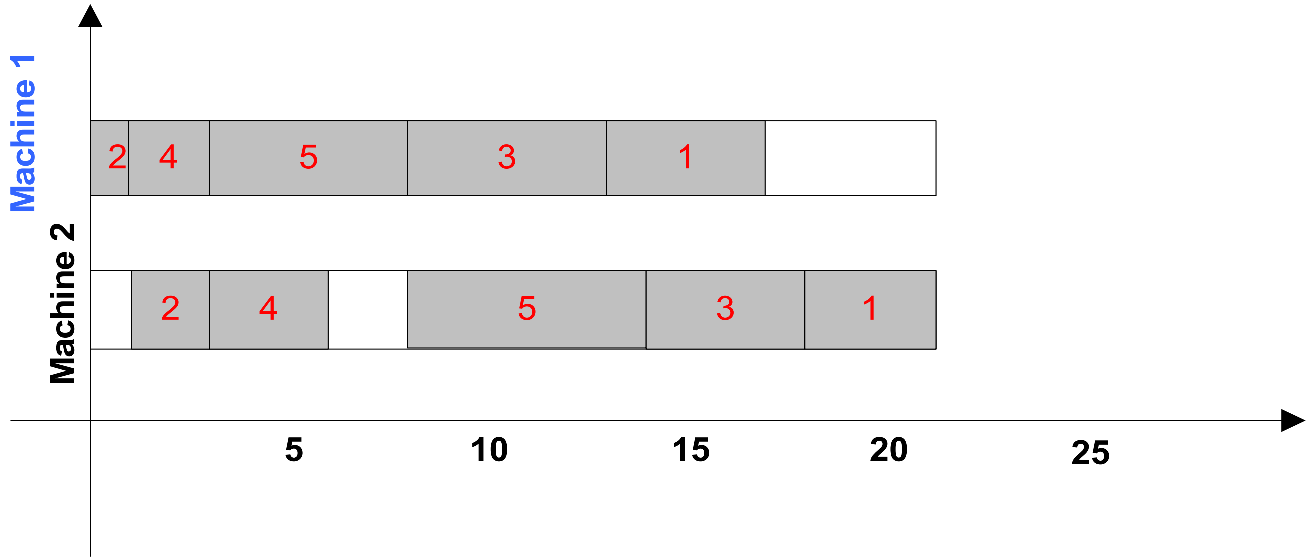
Min Pij = p12 =3 – now job 1 goes to last

Min Pij = p32 …

The Sequence: {2,4,5,3,1}

Scenario 2 – n jobs, 2 machines, flow shop (IV)

- Example con't.



Makespan = 21

Mach 1 = 0 idle plus 4 end of day

Machine 2 = 2 idle + 1 beginning of day

2 wait units (job 3,1)

Scenario 3, n jobs, 2 machines, any order including only 1 machine (I)

- Establish 4 sets:
  - {A} – set of jobs only on machine 1
  - {B} – set of jobs only on machine 2
  - {AB} – set of jobs processing on 1, then 2
  - {BA} – set of jobs processing on 2, then 1
- Sequence jobs in {A,B} by Johnson's Rule
- Sequence jobs in {B,A} by Johnson's Rule
- Sequence jobs in {A} and {B} at random
- Combined as follows without changing order in any set:
  - Machine 1 : Jobs in {A,B} before jobs in {A} before jobs in {B,A}
  - Machine 2 : Jobs in {B,A} before jobs in {B} before jobs in {A,B}

Scenario 3 – example processing time for each machine

JOB	PA	PB	Order
1	4	3	AB
2	1	0	A
3	9	8	AB
4	0	8	B
5	5	1	AB
6	3	7	AB
7	4	6	BA
8	2	1	BA
9	0	6	B
10	4	0	A
11	3	4	BA
12	9	4	BA

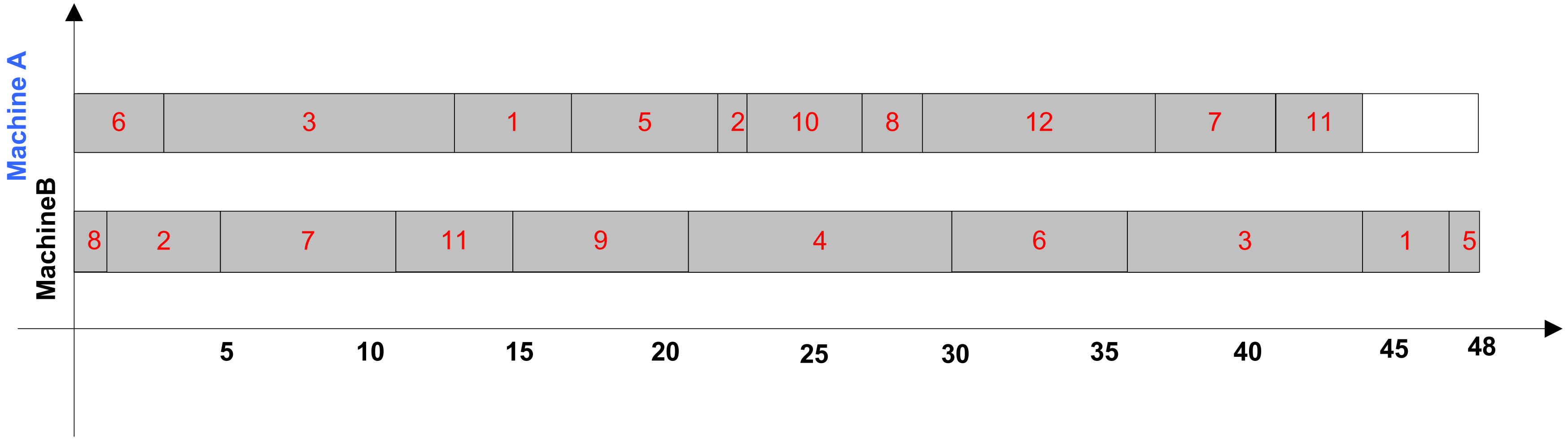
Scenario 3 – Example Con't.

- Set {A, B} – {1,3,5,6)
  Sequence : 6,3,1,5
- Set {B,A} – {7,8,11,12}
  Sequence: 8,12,7,11
- Set {A} – {2,10}
  Sequence: 2,10
- Set {B} – {4,9}
  Sequence: 9,4

Machine A: 6,3,1,5,2,10,8,12,7,11

Machine B: 8,12,7,11,9,4,6,3,1,5

Scenario 3 - Example



2 Jobs, m Machines

Example:

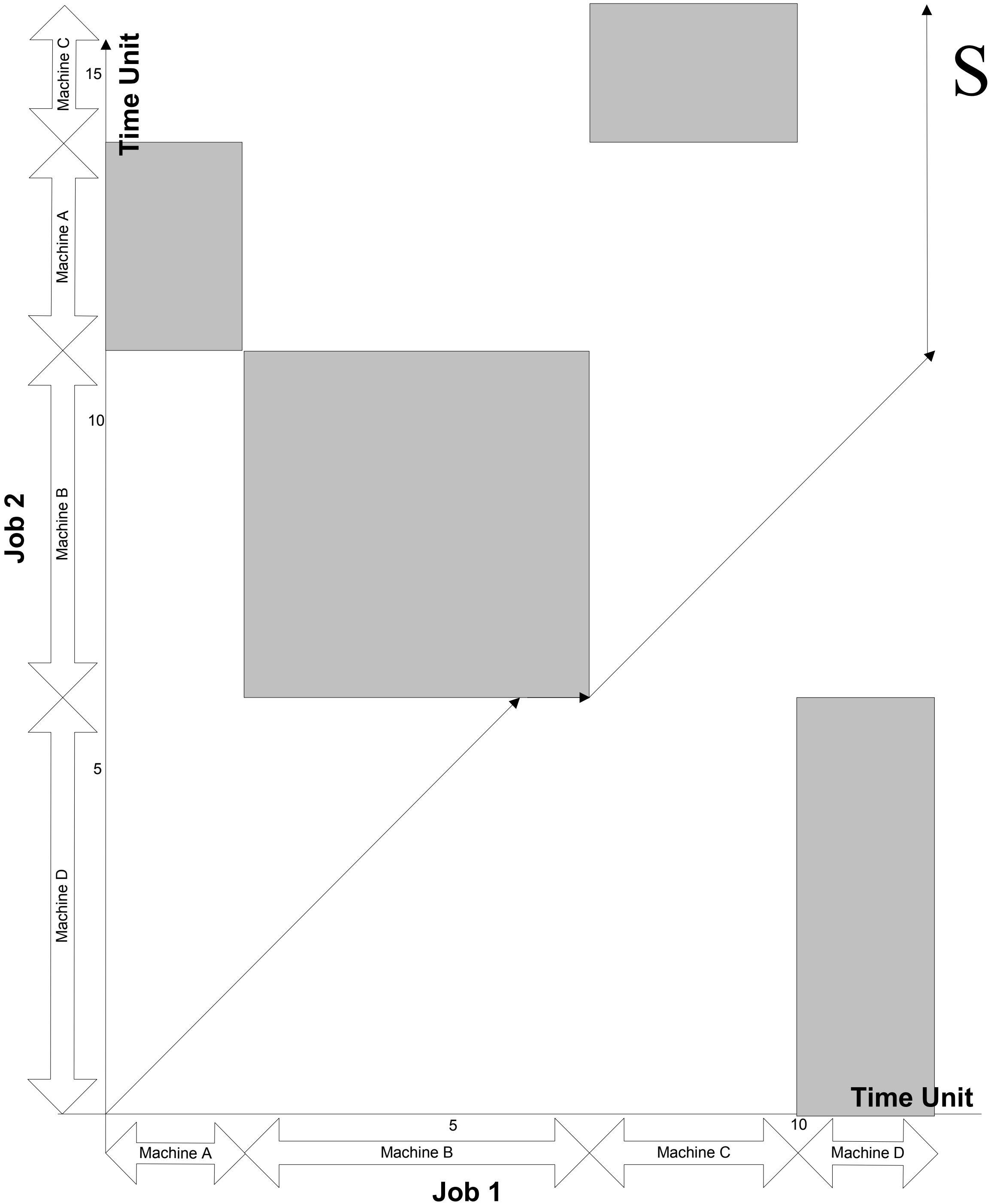
Job 1 Sequence – Machine D, B, A, C

Job 2 Sequence – Machine A, B, C, D

Processing time for each job on each machine

Job	**Machine A**	**Machine B**	**Machine C**	**Machine D**
1	2	5	3	2
2	3	5	2	6

Schedule by graph



N Jobs, M Machines

Number of possible schedules is extremely large, $(n!)^m$

Almost all solved by heuristics which are based on sequencing or dispatching rules.

N Jobs, M Machines

List of Heuristics are as follows:

1. R (Random) – Pick any Job in Queue with equal probability. This rule is often used as benchmark for other rules
2. FCFS (First Come First Serve) – Jobs are processed in the order in which they arrived at the work center (also called earliest release date)
3. SPT (Shortest Processing Time) –This rule tends to reduce both work-in-process inventory, the average job completion (flow) time, and average job lateness.
4. EDD (Earliest Due Date) – Choose Job that has earliest due date
5. CR (Critical Ratio) = Processing Time / Time until due (Due Date – Current Time). Take the highest value.
6. LWR (Least Work Remaining) – This rule is an extension of SPT variant that considers the number of successive operations
7. ST (Slack Time) = Time until job is due - (Sum of processing time remaining). Take the job with the smallest amount of slack time.
8. ST/O (Slack Time per Remaining Operation) = slack time divided by number of operations remaining. Take the job with the smallest amount of slack time per remaining operation

When in Doubt, use SPT. Also, use SPT to break ties.