**FIGURE 10.7**
The complete project network showing ES and LS (in parentheses above the node) and EF and LF (in parentheses below the node) for each activity of the Reliable Construction Co. project. The darker arrows show the critical path through the project network.

S=(ES, LS)
F=(EF, LF)

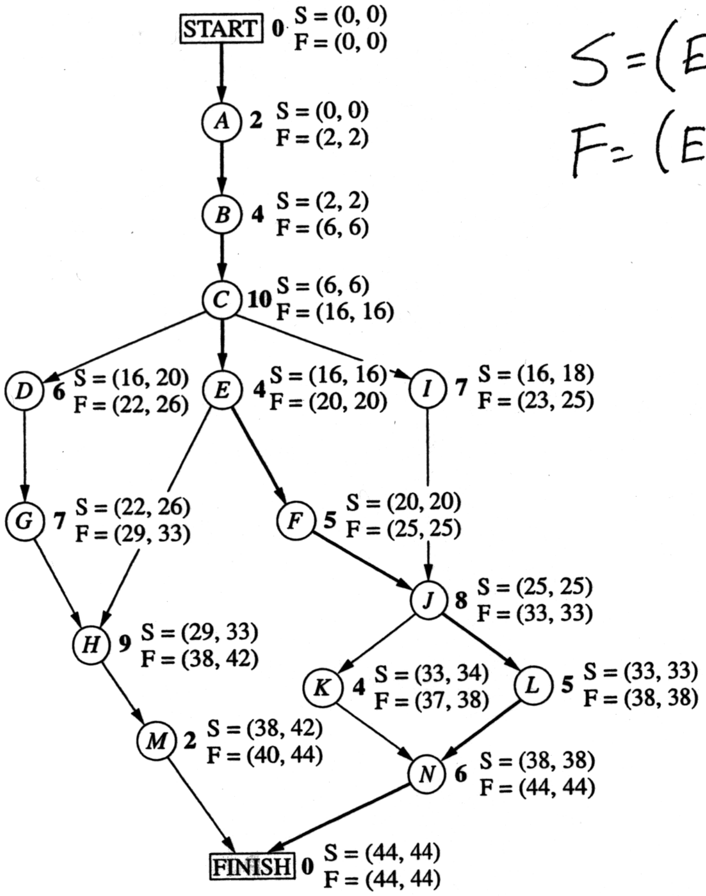



**TABLE 10.3** Slack for Reliable's activities

| Activity | Slack (LF − EF) | On Critical Path? |
|---|---|---|
| *A* | 0 | Yes |
| *B* | 0 | Yes |
| *C* | 0 | Yes |
| *D* | 4 | No |
| *E* | 0 | Yes |
| *F* | 0 | Yes |
| *G* | 4 | No |
| *H* | 4 | No |
| *I* | 2 | No |
| *J* | 0 | Yes |
| *K* | 1 | No |
| *L* | 0 | Yes |
| *M* | 4 | No |
| *N* | 0 | Yes |

## Review

Now let us review Mr. Perty's questions at the beginning of the section and see how all of them have been answered by the PERT/CPM scheduling procedure.

*Question 2:* What is the total time required to complete the project if no delays occur? This is the earliest finish time at the FINISH node (EF = 44 weeks), as given at the bottom of Figs. 10.5 and 10.7.

*Question 3:* When do the individual activities need to start and finish (at the latest) to meet this project completion time? These times are the latest start times (LS) and latest finish times (LF) given in Figs. 10.6 and 10.7. These times provide a "last chance schedule" to complete the project in 44 weeks if no further delays occur.

*Question 4:* When can the individual activities start and finish (at the earliest) if no delays occur? These times are the earliest start times (ES) and earliest finish times (EF) given in Figs. 10.5 and 10.7. These times usually are used to establish the initial schedule for the project. (Subsequent delays may force later adjustments in the schedule.)

*Question 5:* Which are the critical bottleneck activities where any delays must be avoided to prevent delaying project completion? These are the activities on the critical path shown by the darker arrows in Fig. 10.7. Mr. Perty needs to focus most of his attention on keeping these particular activities on schedule in striving to keep the overall project on schedule.

*Question 6:* For the other activities, how much delay can be tolerated without delaying project completion? These tolerable delays are the positive slacks given in the middle column of Table 10.3.

# Incorporate Uncertain Activity Duration Times (Probabilistic)

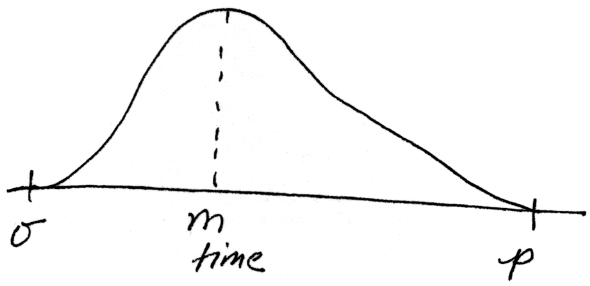
## PERT Three Estimate Approach

$m$ most likely estimate of activity duration time

$o$ optimistic estimate

$p$ = pessimistic estimate

Assume Beta distribution of activity time




Approximately $\mu$ $\dfrac{o + 4m + p}{6}$

$\sigma$ $\left(\dfrac{p-o}{6}\right)^2$

**TABLE 10.4** Expected value and variance of the duration of each activity for Reliable's project

| Activity | Optimistic Estimate $o$ | Most Likely Estimate $m$ | Pessimistic Estimate $p$ | Mean $\mu = \frac{o + 4m + p}{6}$ | Variance $\sigma^2 =$ [illegible] |
|---|---|---|---|---|---|
| $A$ | 1 | 2 | 3 | 2 | [illegible] |
| $B$ | 2 | $3\frac{1}{2}$ | 8 | 4 | [illegible] |
| $C$ | 6 | 9 | 18 | 10 | [illegible] |
| $D$ | 4 | $5\frac{1}{2}$ | 10 | 6 | [illegible] |
| $E$ | 1 | $4\frac{1}{2}$ | 5 | 4 | [illegible] |
| $F$ | 4 | 4 | 10 | 5 | [illegible] |
| $G$ | 5 | $6\frac{1}{2}$ | 11 | 7 | [illegible] |
| $H$ | 5 | 8 | 17 | 9 | [illegible] |
| $I$ | 3 | $7\frac{1}{2}$ | 9 | 7 | [illegible] |
| $J$ | 3 | 9 | 9 | 8 | [illegible] |
| $K$ | 4 | 4 | 4 | 4 | [illegible] |
| $L$ | 1 | $5\frac{1}{2}$ | 7 | 5 | [illegible] |
| $M$ | 1 | 2 | 3 | 2 | [illegible] |
| $N$ | 5 | $5\frac{1}{2}$ | 9 | 6 | [illegible] |

**TABLE 10.5** The paths and path lengths through Reliable's project network when the duration of each activity equals its pessimistic estimate

| Path | Length |
|---|---|
| START→$A$→$B$→$C$→$D$→$G$→$H$→$M$→FINISH | 3 + 8 + 18 + 10 + 11 + 17 + 3 = [illegible] |
| START→$A$→$B$→$C$→$E$→$H$→$M$→FINISH | 3 + 8 + 18 + 5 + 17 + 3 = [illegible] |
| START→$A$→$B$→$C$→$E$→$F$→$J$→$K$→$N$→FINISH | 3 + 8 + 18 + 5 + 10 + 9 + 4 + 9 = [illegible] |
| START→$A$→$B$→$C$→$E$→$F$→$J$→$L$→$N$→FINISH | 3 + 8 + 18 + 5 + 10 + 9 + 7 + 9 = [illegible] |
| START→$A$→$B$→$C$→$I$→$J$→$K$→$N$→FINISH | 3 + 8 + 18 + 9 + 9 + 4 + 9 = [illegible] |
| START→$A$→$B$→$C$→$I$→$J$→$L$→$N$→FINISH | 3 + 8 + 18 + 9 + 9 + 7 + 9 = [illegible] |

For a path (typically the critical path),

find the mean length (time) $\mu_p$

and the variance

p

Mean length of path $\mu_p$ = sum of mean activity times on the path

(because $E[X+Y] = E[X] + E[Y]$)

Variance of length of path $\sigma_p^2$ = sum of variances of activity times on path

(because we assume independence... $Var[X+Y] = Var[X] + Var[Y]$ **if** X, Y indep)

Ex: critical path Start → A → B → C → E → F → J → L → N → Finish

$\mu_p = 2+4+10+4+5+8+5+6 = 44$

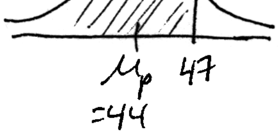
$\sigma_p^2 = \frac{1}{9} + 1 + 4 + \frac{4}{9} + 1 + 1 + 1 + \frac{4}{9} = 9$

Find the probability the project is completed in 47 weeks

$P(\text{total project time, critical path time} \leq 47)$ using an assumption of a Normal distribution

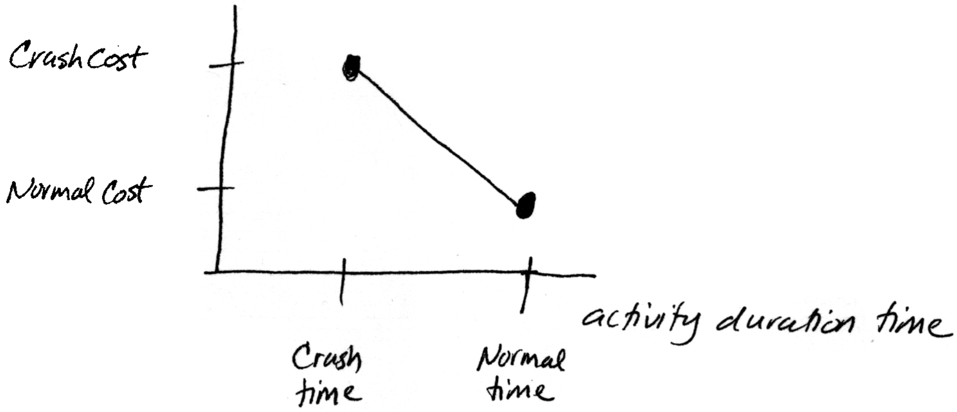
$= P\left(Z \leq \frac{47-\mu_p}{\sigma_p}\right)$

$= P\left(Z \leq \frac{47-4}{3}\right) = P(Z \leq 1)$ (11) $\approx 0.84$



# Time - Cost Trade-offs

For an activity, could pay extra to reduce time (crash)




How much would it cost to reduce total project duration from 44 weeks to 40 weeks? Which activities should be "crashed"? Use LP!!

**TABLE 10.7** Time-cost trade-off data for the activities of Reliable's project

| Activity | Time | | Cost | | Maximum Reduction in Time | Crash Co[illegible] per We[illegible] Saved |
|---|---|---|---|---|---|---|
| | Normal | Crash | Normal | Crash | | |
| A | 2 weeks | 1 week | $180,000 | $ 280,000 | 1 week | $100,00[illegible] |
| B | 4 weeks | 2 weeks | $320,000 | $ 420,000 | 2 weeks | $ 50,00[illegible] |
| C | 10 weeks | 7 weeks | $620,000 | $ 860,000 | 3 weeks | $ 80,00[illegible] |
| D | 6 weeks | 4 weeks | $260,000 | $ 340,000 | 2 weeks | $ 40,00[illegible] |
| E | 4 weeks | 3 weeks | $410,000 | $ 570,000 | 1 week | $160,00[illegible] |
| F | 5 weeks | 3 weeks | $180,000 | $ 260,000 | 2 weeks | $ 40,00[illegible] |
| G | 7 weeks | 4 weeks | $900,000 | $1,020,000 | 3 weeks | $ 40,00[illegible] |
| H | 9 weeks | 6 weeks | $200,000 | $ 380,000 | 3 weeks | $ 60,00[illegible] |
| I | 7 weeks | 5 weeks | $210,000 | $ 270,000 | 2 weeks | $ 30,00[illegible] |
| J | 8 weeks | 6 weeks | $430,000 | $ 490,000 | 2 weeks | $ 30,00[illegible] |
| K | 4 weeks | 3 weeks | $160,000 | $ 200,000 | 1 week | $ 40,00[illegible] |
| L | 5 weeks | 3 weeks | $250,000 | $ 350,000 | 2 weeks | $ 50,00[illegible] |
| M | 2 weeks | 1 week | $100,000 | $ 200,000 | 1 week | $100,00[illegible] |
| N | 6 weeks | 3 weeks | $330,000 | $ 510,000 | 3 weeks | $ 60,00[illegible] |

# Linear Programming to Make Crashing Decisions

$x_j$ = reduction in the duration of activity $j$ due to crashing
$j = A, B, C, \ldots, N$

$y_{FINISH}$ = project duration, time at which FINISH node is reached

minimize crash costs $= Z = \sum_{j=A}^{N} (\text{slope of Crashcost}_j)(x_j)$

$= 100{,}000\, x_A + 50{,}000\, x_B + \cdots + 60{,}000\, x_N$

subject to

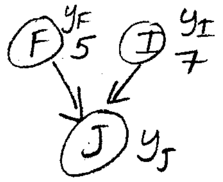
- $y_{FINISH} \leq 40$ weeks (total project duration)
- $0 \leq x_j \leq$ maximum reduction in time, normal time − crash time

  $0 \leq x_A \leq 1$
  $0 \leq x_B \leq 2$
  $\vdots$
  $0 \leq x_N \leq 3$
  (max. time reduction)

- non-negativity constraints
- let $y_j$ = start time of activity $j$ (start times)

  $\geq y_i + \text{normal time}_i - x_i$

  $i$ is an immediate predecessor of $j$




$y_J \geq y_F + 5 - x_F$
$y_J \geq y_I + 7 \quad x_I$

- non-negativity constraints: $x_j \geq 0$
  $y_j \geq 0$
  $y_{FINISH} \geq 0$

**TABLE 10.9** **The final table for performing marginal cost analysis on Reliable's project**

| Activity to Crash | Crash Cost | Length of Path | | | | | |
|---|---|---|---|---|---|---|---|
| | | ABCDGHM | ABCEHM | ABCEFJKN | ABCEFJLN | ABCIJKN | ABCIJLN |
| | | 40 | 31 | 43 | 44 | 41 | 42 |
| J | $30,000 | 40 | 31 | 42 | 43 | 40 | 41 |
| J | $30,000 | 40 | 31 | 41 | 42 | 39 | 40 |
| F | $40,000 | 40 | 31 | 40 | 41 | 39 | 40 |
| F | $40,000 | 40 | 31 | 39 | 40 | 39 | 40 |

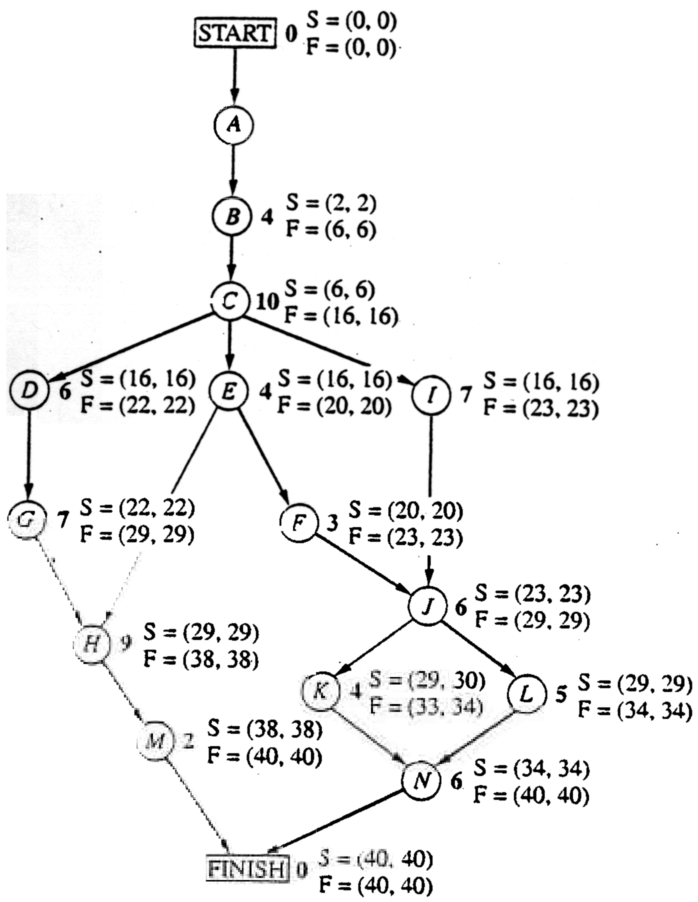



Figure 10.12
network if
after J crashed
J and F are fully
all other
mal) for
ject. The darker
the various
through the
ork.

**10.17**
...edule of cumulative ...costs when all ...begin at their ...start times (the top ...urve) or at their latest ...mes (the bottom cost