Incorporating Uncertainty into a Multi-criteria Supplier Selection Problem

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Motivation

- Current business trends
  - On-time delivery of high quality low cost products in shorter time window
  - Global sourcing
  - Reduced supplier base
  - Reduced buffers
  - Shorter lead times
  - More intertwined supply chains
- Potentials to increase risks
- Supplier selection
  - Key component for a successful supply chain
  - Strategic management decision

Plan for Uncertainty

- Critical to plan for uncertainty to mitigate risks
- Two dominant uncertainties in supplier selection
  - Random demand
  - Random supply
- Optimization models that incorporate random events

Impact of Just-In-Time (JIT)

- Reduced supplier base
- Long-term relationships with suppliers

Reduced Supplier Base

- Advantages
  - More leverage on suppliers
  - Better customer services from suppliers
  - Reduced raw material costs
  - Growth of suppliers
  - How to quantify – business volume discounts
- Disadvantages
  - Risks of not having sufficient supplies to meet fluctuating demand
  - Example: Phillips fire in 2000 cost Phillips $40 million, but Ericsson lost $2.34 billion in mobile phone division

Global Sourcing

- Risk associated with global sourcing
  - Low raw material costs offered by overseas suppliers
  - Long lead times and transportation routes

- Strategies to ensure a sufficient supply
  - To build up pipeline inventory - expensive
  - To establish relationships with carefully selected local and overseas suppliers

Handle Risks Proactively

- Another major risk: uncertain demand

  Example: Cisco $2.5 billion inventory in Q2, 2001 due to weakening demand and locked-in supply agreements
- Current focus on supply chain efficiency and lean practice amplifies risks
- Substantial benefits to plan flexibility into supply chain to handle risks proactively

Multiple Criteria for Strategic Supplier Selection

- Quality
- Delivery
- Number of suppliers
- Cost – total cost of ownership
  - Purchasing costs
  - Transportation costs
  - Pipeline inventory costs
- Risk

Strategic Supplier Selection Problem

- Uncertain demand
- Uncertain supplier capacity
- Multiple selection criteria
  - Purchasing prices including business volume discount
- Analyses of tradeoffs between costs and risks

Research Overview

- Develop a multi-objective supplier selection model with consideration of *uncertain demand*, and *uncertain supplier capacities*
- Compare:
  - deterministic mixed integer program
  - stochastic program with recourse
  - chance-constrained program
- Demonstrate the quality of the solutions obtained by probabilistic models
- Characterize the tradeoffs between costs and risks in an analytic form using multiparametric sensitivity analysis

Methodologies Used in Supplier Selection

- Rating models
- Statistical models
- Artificial intelligence models
- Mathematical programming models
  - Outperform other models in terms of the total cost of ownership
  - Quantitative tool with clear assumptions
  - Decisions
    - which suppliers to choose
    - how much to order from the selected suppliers

Deterministic Supplier Selection Problem

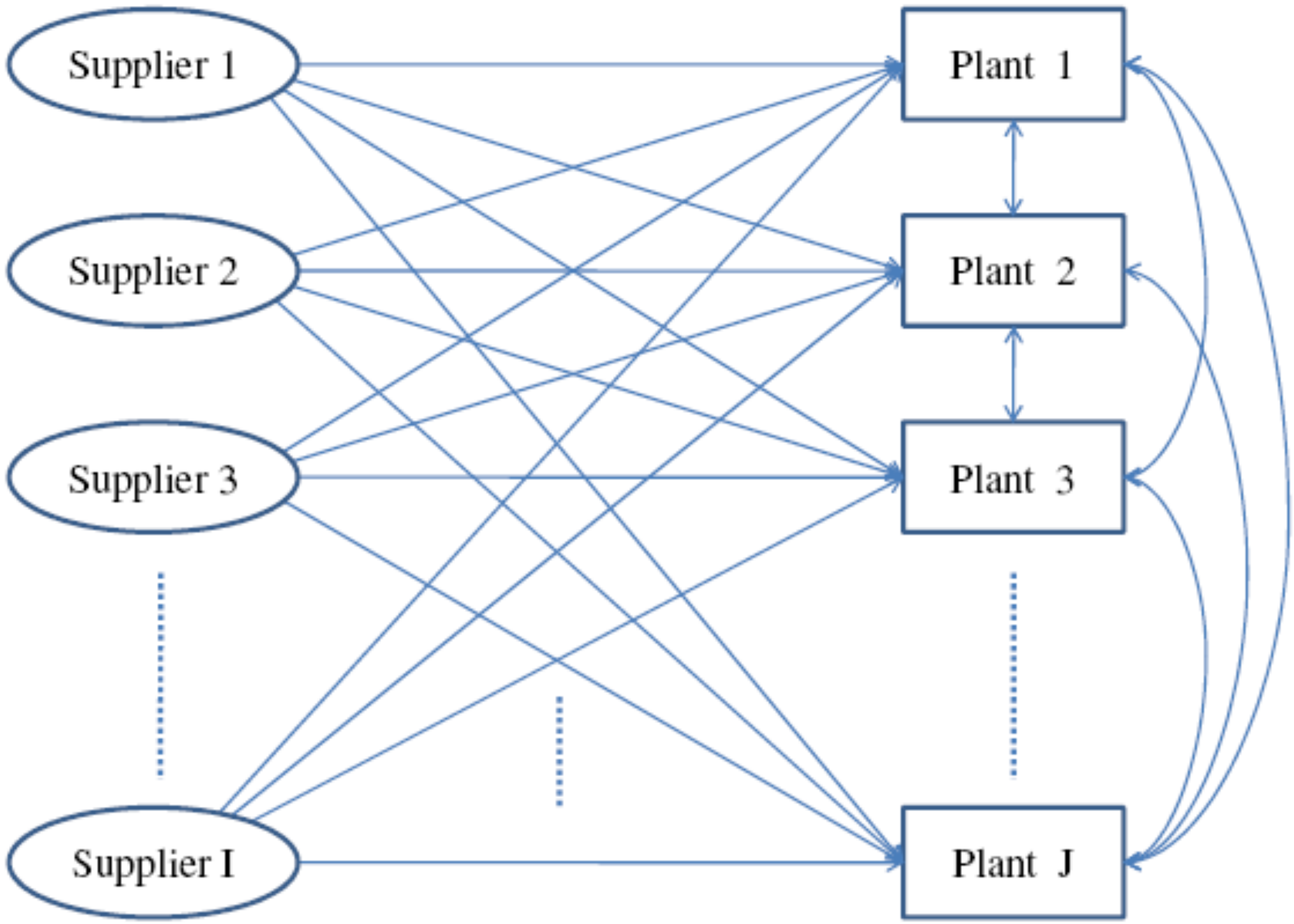
- Linear programming models
- Mixed integer programming models
- Nonlinear programming models
- Goal programming models

- Missing an important factor – uncertainty

Uncertainty in Supplier Selection Decision

- Random supply, random demand in newsvendor problem (Dada et al., 2003; Yang et al., 2005)
- Random demand in multicriteria vendor selection problem (Kasilingam and Lee, 1996)
- Uncertain demand, costs and exchange rates in international sourcing problem (Gutierrez and Kouvelis, 1995; Velarde and Laguna, 2004)
- Supplier failure risk (Berger and Zeng, 2006; Ruiz-Torres and Mahnoodi, 2007)
- Need to address multicriteria supplier selection under random demand and supply

Problem Setting



Approaches

- Mixed integer programming model (deterministic)
- Stochastic programming model with recourse
- Chance-constrained programming model

Probabilistic Models

- To find a minimal set of suppliers
- To incorporate risks originated from
  - Suppliers
  - Customers
- To achieve multiple goals
  - Quality
  - Delivery
  - Cost
  - Probability of satisfying demand
- To include business volume discount

Pareto-optimal Solutions

ε-constraint method

- Easy to implement
- Ability to handle nonconvex solution space
- Difficulty: definition of ε value

Mixed Integer Programming Model

- Deterministic demand
- Deterministic supplier capacity
- Use average demand and supplier capacity from historical data or the distributions

Mixed Integer Programming Model: Decision Variables

Decision Variables	Description
y_i	binary variable, has value of 1 if supplier i is chosen, 0 otherwise, $i \in I$
Y	vector of y_i's
v_{im}	binary variable, has value of 1 if the volume of business awarded to supplier i falls in the m-th interval, 0 otherwise, $i \in I$, $m \in M$
x_{ijk}	amount of item k purchased from supplier i at plant j, $i \in I$,$j \in J$, $k \in K$
$z_{jj'k}$	amount of item k ordered from plant j for plant j', $j, j' \in J$, $j' \neq j$, $k \in K$
b_{im}	business volume in dollars awarded to supplier i falling in the m-th discount interval, $i \in I$, $m \in M$

Mixed Integer Programming Model: Parameters

Sets	Description
I	set of suppliers
J	set of plants
K	set of items
M	set of volume discount intervals

Parameters	Description
d_{jk}	demand for item k at plant j, $j \in J$, $k \in K$
cap_i	aggregate capacity limit of supplier i, $i \in I$
$ucap_{ik}$	unit capacity used by item k for supplier i, $i \in I$, $k \in K$
p_{ijk}	unit price of item k quoted by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
$ship_{ijk}$	unit transportation cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
inv_{ijk}	unit pipeline inventory cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
$c_{jj'k}$	unit coordination cost of item k if plant j orders for plant j', $j, j' \in J, j' \neq j$, $k \in K$
q_{ik}	fraction of poor quality items of type k from supplier i, $i \in I$, $k \in K$
t_{ik}	fraction of late items of type k from supplier i, $i \in I$, $k \in K$
u_{im}	upper cutoff point of discount interval m from supplier i, $i \in I$, $m \in M$
r_{im}	discount rate associated with discount interval m offered by supplier i, $i \in I$, $m \in M$
τ_q	pre-set quality tolerance, equal to $0.05 \sum_{j,k} d_{jk}$
τ_d	pre-set delivery tolerance, equal to $0.05 \sum_{j,k} d_{jk}$

Mixed Integer Programming Model: Objective Functions

$$Min \quad f_s = \sum_{i \in I} y_i$$

Number of suppliers

$$Min \quad f_c = \sum_{i \in I} \sum_{m \in M} (1 - r_{im}) \cdot b_{im}$$

Discounted purchasing costs

$$+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (ship_{ijk} + inv_{ijk}) \cdot x_{ijk}$$

Transportation costs, inventory costs

$$+ \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{k \in K} c_{jj'k} \cdot z_{jj'k}$$

Coordination costs

Mixed Integer Programming Model: Constraints

$$\sum_{i\in I}\sum_{k\in K} q_{ik}\cdot\sum_{j\in J} x_{ijk} \le \tau_q$$

Poor quality items

$$\sum_{i\in I}\sum_{k\in K} t_{ik}\cdot\sum_{j\in J} x_{ijk} \le \tau_d$$

Late delivery

$$\sum_{i\in I} x_{ijk} + \sum_{j'\in J\setminus j} z_{j'jk} - \sum_{j'\in J\setminus j} z_{jj'k} \ge d_{jk} \qquad \forall j\in J, k\in K$$

Demand

$$\sum_{k\in K} ucap_{ik}\cdot\sum_{j\in J} x_{ijk} \le cap_i \qquad \forall i\in I$$

Supplier capacity

$$\sum_{j\in J}\sum_{k\in K} p_{ijk}\cdot x_{ijk} = \sum_{m\in M} b_{im} \qquad \forall i\in I$$

$$b_{im} \le u_{im}\cdot v_{im} \qquad \forall i\in I, m\in M$$

$$b_{i,m+1} \ge u_{im}\cdot v_{i,m+1} \qquad \forall i\in I, m\in\{1,2,...,|M|-1\}$$

$$\sum_{m\in M} v_{im} = 1\cdot y_i \qquad \forall i\in I$$

Business volume discount from selected suppliers

$$y_i\in\{0,1\}, v_{im}\in\{0,1\} \qquad \forall i\in I, m\in M$$

$$x_{ijk}\ge 0, z_{jj'k}\ge 0, b_{im}\ge 0 \qquad \forall i\in I, j\in J, j'\in J, k\in K, m\in M$$

Explore Pareto-optimal Solutions in MIP

- Leave total cost (f_c) as objective function

- Move the objective function on the number of suppliers (f_s) to constraints

$$\sum_{i \in I} y_i \leq \varepsilon_s \qquad 1 \leq \varepsilon_s \leq |I|$$

- Vary ε_s level during solution process

Stochastic Programming Model

- Incorporate uncertainty of demand and supplier capacity into a two-stage stochastic program
- Scenario-based approach: scenarios (ξ) capture random demand & supplier capacity
- First-stage: binary variables for supplier selection, y_i
- Second-stage: continuous variables capture the amount ordered from each supplier, $x_{ijk}(\xi)$, and coordination between plants, $z_{jj'k}(\xi)$, binary and continuous variables are needed to model volume discounts, $v_{im}(\xi)$ and $b_{im}(\xi)$

Stochastic Programming Model: Decision Variables

Decision Variables	Description
y_i	binary variable, has value of 1 if supplier i is chosen, 0 otherwise, $i \in I$
Y	vector of y_i's
$v_{im}(\xi)$	binary variable, has value of 1 if the volume of business awarded to supplier i falls in the m-th interval given scenario ξ, 0 otherwise, $i \in I, m \in M$ and $\xi \in \Xi$
$x_{ijk}(\xi)$	amount of item k purchased from supplier i at plant j given scenario ξ, $i \in I$,$j \in J$, $k \in K$ and $\xi \in \Xi$
$z_{jj'k}(\xi)$	amount of item k ordered from plant j for plant j' given scenario ξ, $j, j' \in J, j' \neq j$, $k \in K$ and $\xi \in \Xi$
$ocap_i(\xi)$	amount of exceeded capacity of supplier i given scenario ξ, $i \in I$ and $\xi \in \Xi$
$b_{im}(\xi)$	business volume in dollars awarded to supplier i falling in the m-th discount interval given scenario ξ, $i \in I, m \in M$ and $\xi \in \Xi$

Stochastic Programming Model: Parameters

Sets	Description
I	set of suppliers
J	set of plants
K	set of items
M	set of volume discount intervals
Ξ	set of scenarios

Parameters	Description
$d_{jk}(\xi)$	demand for item k at plant j given scenario ξ, $j \in J$, $k \in K$ and $\xi \in \Xi$
$cap_i(\xi)$	aggregate capacity limit of supplier i given scenario ξ, $i \in I$ and $\xi \in \Xi$
$ucap_{ik}$	unit capacity used by item k for supplier i, $i \in I$ and $k \in K$
p_{ijk}	unit price of item k quoted by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
$ship_{ijk}$	unit transportation cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
inv_{ijk}	unit pipeline inventory cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
$c_{jj'k}$	unit coordination cost of item k if plant j orders for plant j', $j, j' \in J, j' \neq j$ and $k \in K$
q_{ik}	fraction of poor quality items of type k from supplier i, $i \in I$ and $k \in K$
t_{ik}	fraction of late items of type k from supplier i, $i \in I$ and $k \in K$
u_{im}	upper cutoff point of discount interval m from supplier i, $i \in I$ and $m \in M$
r_{im}	discount rate associated with discount interval m offered by supplier i, $i \in I$ and $m \in M$
e_i	unit penalty of exceeding capacity of supplier i, $i \in I$
$\tau_q(\xi)$	pre-set quality tolerance given scenario ξ, equal to $0.05 \sum_{j,k} d_{jk}(\xi)$
$\tau_d(\xi)$	pre-set delivery tolerance given scenario ξ, equal to $0.05 \sum_{j,k} d_{jk}(\xi)$

Stochastic Programming Model: First-Stage Problem

$$Min \quad f_s = \sum_{i \in I} y_i$$

Number of suppliers

$$Min \quad f_c = E_\xi[C_f(Y, \xi)]$$

Cost of discounted purchasing, transportation and pipeline inventory, coordination and penalty of exceeding capacity

Subject to

$$y_i \in \{0, 1\} \quad \forall i \in I$$

Stochastic Programming Model: Second-Stage Problem (for Given Selected Suppliers Y and Scenario ξ) – Objective Function

$$C_f(Y,\xi) = Min \sum_{i \in I} \sum_{m \in M} (1 - r_{im}) \cdot b_{im}(\xi)$$

Discounted purchasing costs

$$+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (ship_{ijk} + inv_{ijk}) \cdot x_{ijk}(\xi)$$

Transportation costs, inventory costs

$$+ \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{k \in K} c_{jj'k} \cdot z_{jj'k}(\xi)$$

Coordination costs

$$+ \sum_{i \in I} e_i \cdot ocap_i(\xi)$$

Penalty costs of exceeding capacity

Stochastic Programming Model: Second-Stage Problem (for Given Selected Suppliers Y and Scenario ξ) - Constraints

$$\sum_{i\in I}\sum_{k\in K} q_{ik}\cdot\sum_{j\in J} x_{ijk}(\xi) \le \tau_q(\xi)$$

Poor quality items

$$\sum_{i\in I}\sum_{k\in K} t_{ik}\cdot\sum_{j\in J} x_{ijk}(\xi) \le \tau_d(\xi)$$

Late delivery

$$\sum_{i\in I} x_{ijk}(\xi) + \sum_{j'\in J\setminus j} z_{j'jk}(\xi) - \sum_{j'\in J\setminus j} z_{jj'k}(\xi) \ge d_{jk}(\xi) \quad \forall j\in J, k\in K$$

Demand

$$\sum_{k\in K} ucap_{ik}\cdot\sum_{j\in J} x_{ijk}(\xi) - ocap_i(\xi) \le cap_i(\xi) \quad \forall i\in I$$

Supplier capacity

$$\sum_{j\in J}\sum_{k\in K} p_{ijk}\cdot x_{ijk}(\xi) = \sum_{m\in M} b_{im}(\xi) \quad \forall i\in I$$

$$b_{im}(\xi) \le u_{im}\cdot v_{im}(\xi) \quad \forall i\in I, m\in M$$

$$b_{i,m+1}(\xi) \ge u_{im}\cdot v_{i,m+1}(\xi) \quad \forall i\in I, m\in\{1,2,...,|M|-1\}$$

$$\sum_{m\in M} v_{im}(\xi) = 1\cdot y_i \quad \forall i\in I$$

Business volume discount from selected suppliers

$$v_{im}(\xi)\in\{0,1\} \quad \forall i\in I, m\in M$$

$$x_{ijk}(\xi), z_{jj'k}(\xi), b_{im}(\xi), ocap_i(\xi) \ge 0 \quad \forall i\in I, j\in J, j'\in J, k\in K, m\in M$$

Explore Pareto-optimal Solutions in SP

- Leave *expected* total cost (f_c) as objective function
- Move the objective function on the number of suppliers (f_s) to constraints

$$\sum_{i \in I} y_i \leq \varepsilon_s \qquad 1 \leq \varepsilon_s \leq |I|$$

- Vary the number of suppliers, ε_s level
- Vary unit penalty cost for exceeding supplier capacity, e_i

Chance-constrained Programming Model

- Assume probability distributions for demand and supply capacity
- Use probabilistic constraints
- Specify system reliability

Chance-constrained Programming Model: Decision Variables

Decision Variables	Description
y_i	binary variable, has value of 1 if supplier i is chosen, 0 otherwise, $i \in I$
Y	vector of y_i's
v_{im}	binary variable, has value of 1 if the volume of business awarded to supplier i falls in the m-th interval, 0 otherwise, $i \in I$, $m \in M$
x_{ijk}	amount of item k purchased from supplier i at plant j, $i \in I$,$j \in J$, $k \in K$
$z_{jj'k}$	amount of item k ordered from plant j for plant j', $j, j' \in J$, $j' \neq j$, $k \in K$
b_{im}	business volume in dollars awarded to supplier i falling in the m-th discount interval, $i \in I$, $m \in M$

Chance-constrained Programming Model: Parameters

Sets	Description
I	set of suppliers
J	set of plants
K	set of items
M	set of volume discount intervals
Parameters	**Description**
D_{jk}	demand for item k at plant j, $j \in J$ and $k \in K$, random variable with mean $\mu_{D_{jk}}$, standard deviation $\sigma_{D_{jk}}$, and cumulative probability distribution function $F_{D_{jk}}$
CAP_i	aggregate capacity limit of supplier i, $i \in I$, random variable with mean μ_{CAP_i}, standard deviation σ_{CAP_i}, and cumulative probability distribution function F_{CAP_i}
$ucap_{ik}$	unit capacity used by item k for supplier i, $i \in I$, $k \in K$
p_{ijk}	unit price of item k quoted by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
$ship_{ijk}$	unit transportation cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
inv_{ijk}	unit pipeline inventory cost of item k shipped by supplier i to plant j, $i \in I$, $j \in J$, $k \in K$
$c_{jj'k}$	unit shipping cost of item k from plant j to plant j', $j, j' \in J, j' \neq j$, $k \in K$
q_{ik}	fraction of poor quality items of type k from supplier i, $i \in I$, $k \in K$
t_{ik}	fraction of late items of type k from supplier i, $i \in I$, $k \in K$
u_{im}	upper cutoff point of discount interval m from supplier i, $i \in I$, $m \in M$
r_{im}	discount rate associated with discount interval m offered by supplier i, $i \in I$, $m \in M$
τ_q	pre-set quality tolerance, equal to $0.05 \sum_{j,k} \mu_{D_{jk}}$
τ_d	pre-set delivery tolerance, equal to $0.05 \sum_{j,k} \mu_{D_{jk}}$
ε_d	pre-determined satisfaction level of probabilistic demand constraint
ε_c	pre-determined satisfaction level of probabilistic capacity constraint

Chance-constrained Programming Model: Objective Functions

$$Min \quad f_s = \sum_{i \in I} y_i$$

Number of suppliers

$$Min \quad f_c = \sum_{i \in I} \sum_{m \in M} (1 - r_{im}) \cdot b_{im}$$

Discounted purchasing costs

$$+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (ship_{ijk} + inv_{ijk}) \cdot x_{ijk}$$

Transportation costs, inventory costs

$$+ \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{k \in K} c_{jj'k} \cdot z_{jj'k}$$

Coordination costs

Chance-constrained Programming Model: Constraints

$$\sum_{i\in I}\sum_{k\in K} q_{ik}\cdot\sum_{j\in J} x_{ijk} \le \tau_q$$

Poor quality items

$$\sum_{i\in I}\sum_{k\in K} t_{ik}\cdot\sum_{j\in J} x_{ijk} \le \tau_d$$

Late delivery

$$Prob\left(\sum_{i\in I} x_{ijk} + \sum_{j'\in J\setminus j} z_{j'jk} - \sum_{j'\in J\setminus j} z_{jj'k} \ge D_{jk}\right) \ge \varepsilon_d \qquad \forall j\in J, k\in K$$

Probability of meeting demand

$$Prob\left(\sum_{k\in K} ucap_{ik}\cdot\sum_{j\in J} x_{ijk} \le CAP_i\right) \ge \varepsilon_c \qquad \forall i\in I$$

Probability of not exceeding supplier capacity

$$\sum_{j\in J}\sum_{k\in K} p_{ijk}\cdot x_{ijk} = \sum_{m\in M} b_{im} \qquad \forall i\in I$$

$$b_{im} \le u_{im}\cdot v_{im} \qquad \forall i\in I, m\in M$$

$$b_{i,m+1} \ge u_{im}\cdot v_{i,m+1} \qquad \forall i\in I, m\in\{1,2,...,|M|-1\}$$

$$\sum_{m\in M} v_{im} = 1\cdot y_i \qquad \forall i\in I$$

Business volume discount from selected suppliers

$$y_i\in\{0,1\}, v_{im}\in\{0,1\} \qquad \forall i\in I, m\in M$$

$$x_{ijk}, z_{jj'k}, b_{im} \ge 0 \qquad \forall i\in I, j\in J, j'\in J, k\in K, m\in M$$

Chance-constrained Programming Model: Transformation of Probabilistic Constraints

$$Prob\left(\sum_{i\in I} x_{ijk} + \sum_{j'\in J\setminus j} z_{j'jk} - \sum_{j'\in J\setminus j} z_{jj'k} \geq D_{jk}\right) \geq \varepsilon_d \quad \forall j \in J, k \in K$$

$$Prob\left(\sum_{k\in K} ucap_{ik} \cdot \sum_{j\in J} x_{ijk} \leq CAP_i\right) \geq \varepsilon_c \quad \forall i \in I$$

$$\sum_{i\in I} x_{ijk} + \sum_{j'\in J\setminus j} z_{j'jk} - \sum_{j'\in J\setminus j} z_{jj'k} \geq F^{-1}_{D_{jk}}(\varepsilon_d) \quad \forall j \in J, k \in K$$

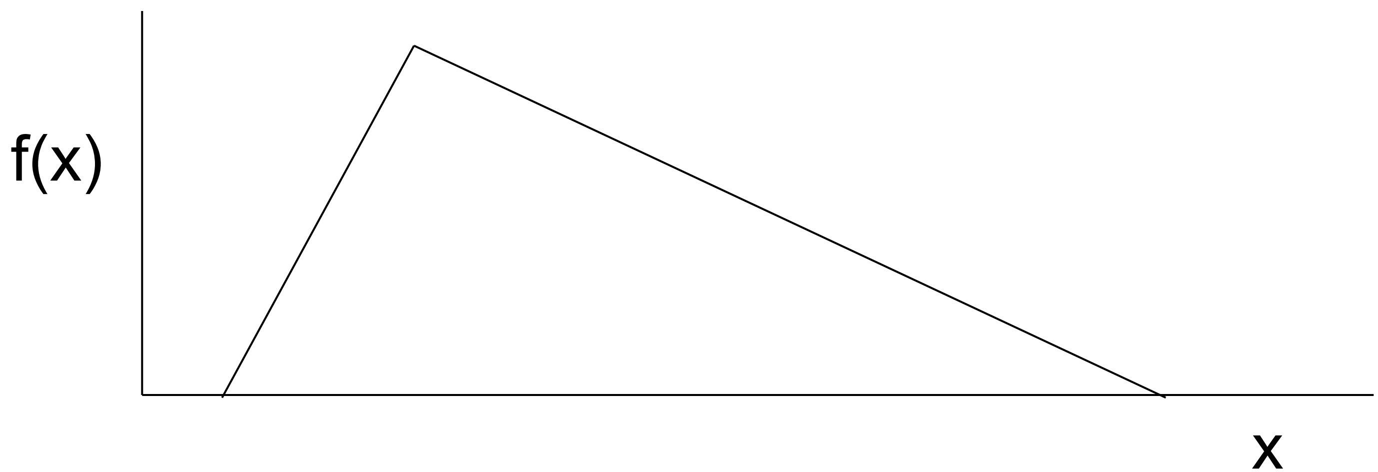
$$\sum_{k\in K} ucap_{ik} \cdot \sum_{j\in J} x_{ijk} \leq F^{-1}_{CAP_i}(1-\varepsilon_c) \quad \forall i \in I$$

Chance-constrained Programming Model: Distributions Used

- Normal distribution
  - Commonly used
  - Described by two parameters: mean and variance
  - Symmetric
- Triangular distribution
  - Limited region
  - Described by three parameters: min, mode, max
  - Skewed or symmetric

Triangle Distribution

- Can be easily described in practice by three parameters
  - Min
  - Mode
  - Max



Explore Pareto-optimal Solutions in CCP

- Leave total cost (f_c) as objective function
- Move the objective function on the number of suppliers (f_s) to constraints

$$\sum_{i \in I} y_i \leq \varepsilon_s \qquad 1 \leq \varepsilon_s \leq |I|$$

- Vary the number of suppliers, ε_s level
- Vary probabilistic levels of meeting demand and not exceeding supplier capacity: ε_d, ε_c ($\varepsilon_d = \varepsilon_c$)

Sample Problem

- 10 suppliers (3 big ones, 4 mid-sized, 3 small ones), 4 plants, 50 components, 3 discount rates from suppliers
- Demand with normal or triangular distribution
- Supplier capacity with normal or triangular distribution
- Big suppliers (possibly overseas) with high capacity levels, lower prices, high transportation costs and pipeline inventory costs, higher poor-quality rate and late-delivery rate, deeper discount rates to large sales
- Lead time differences among suppliers are captured into the transportation and inventory costs

Evaluation of Supplier Selection Decisions

- 5 performance measurements
  - Number of suppliers
  - Probability of having sufficient supplier capacities to meet demands
  - Total costs
  - Quality flexibility
  - Delivery flexibility
- Use CCP and SP models to evaluate solutions under normal and triangular distributions

Quality and Delivery Flexibility

- Quality Flexibility

$$1 - \frac{\text{total poor quality items received}}{\text{pre - set quality tolerance}}$$

$$1 - \frac{\sum_i \sum_k q_{ik} \sum_j x^*_{ijk}}{\tau_q} \quad \text{or} \quad E_\xi\left(1 - \frac{\sum_i \sum_k q_{ik} \sum_j x^*_{ijk}(\xi)}{\tau_q(\xi)}\right)$$

- Delivery Flexibility

$$1 - \frac{\text{total late deliveries received}}{\text{pre - set delivery tolerance}}$$

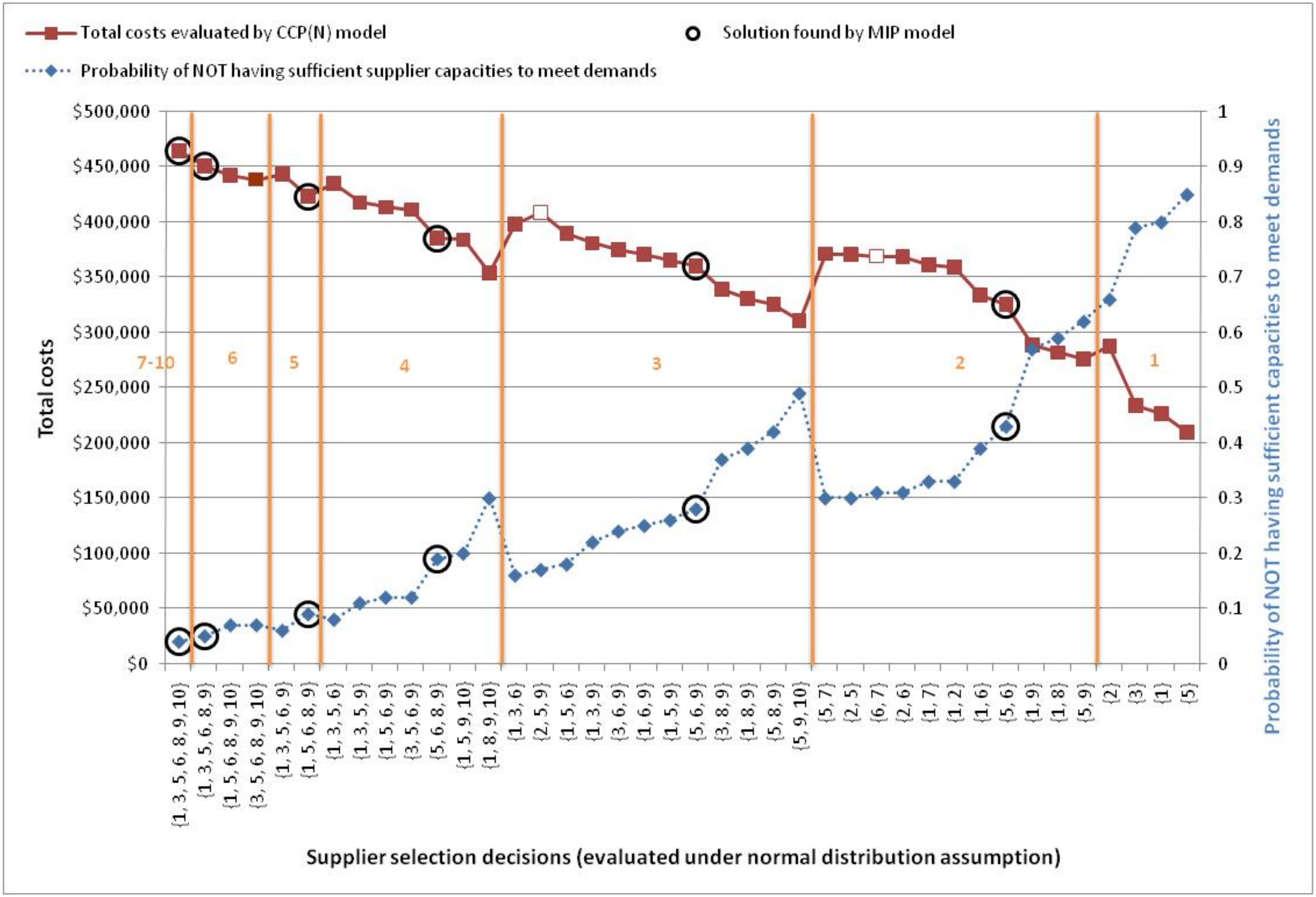
$$1 - \frac{\sum_i \sum_k t_{ik} \sum_j x^*_{ijk}}{\tau_d} \quad \text{or} \quad E_\xi\left(1 - \frac{\sum_i \sum_k t_{ik} \sum_j x^*_{ijk}(\xi)}{\tau_d(\xi)}\right)$$

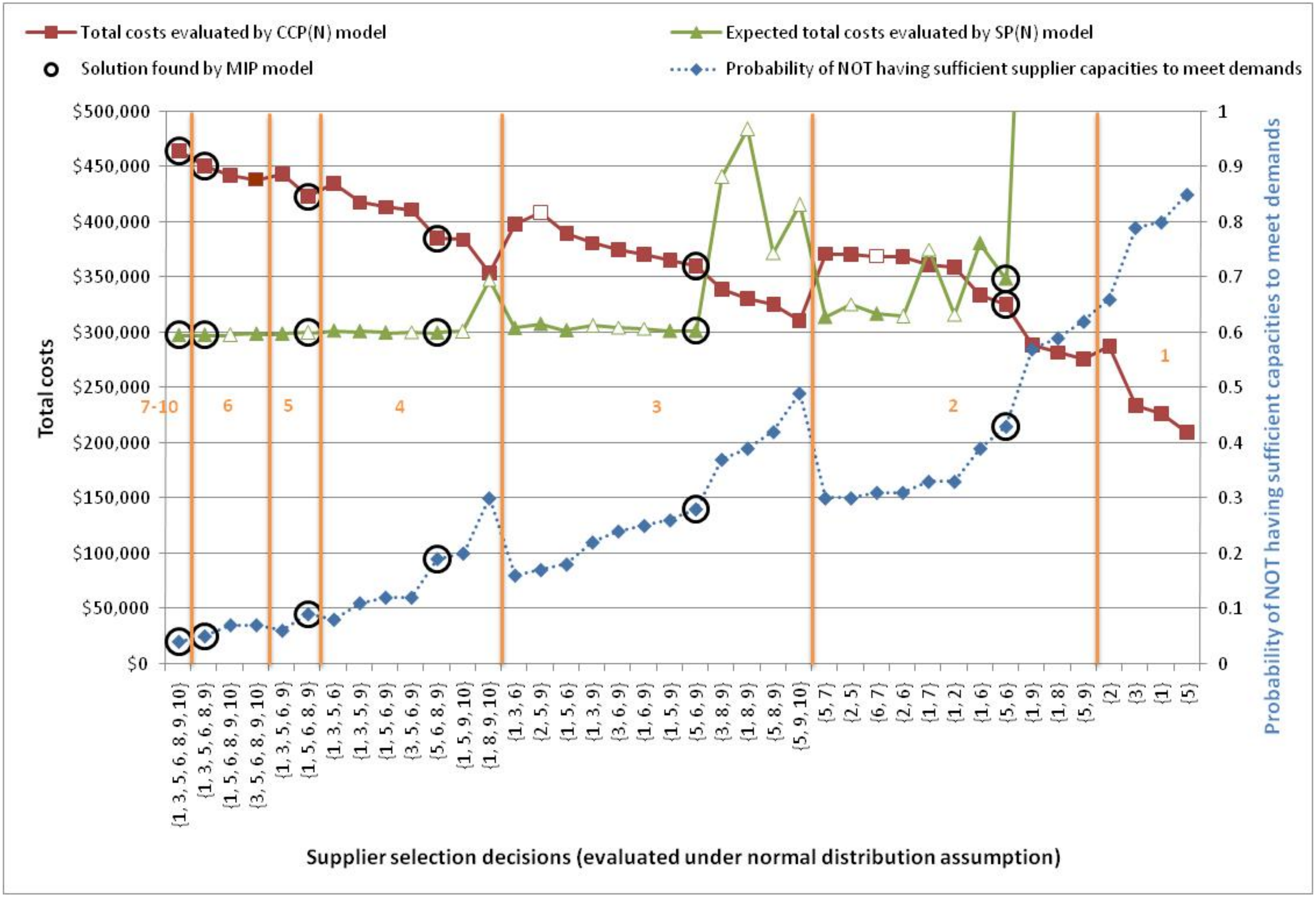
Pareto-optimal Solutions

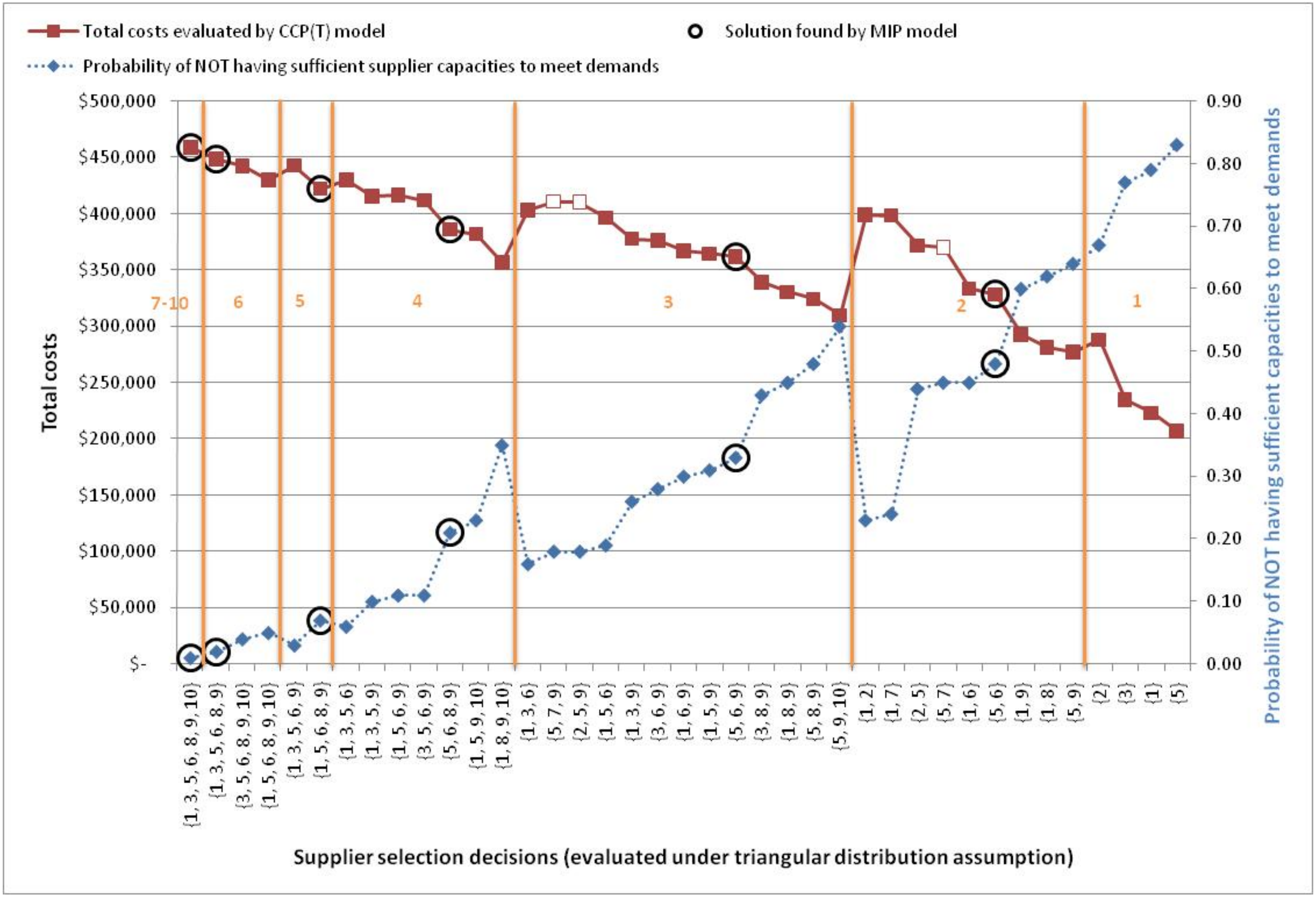
- Efficient solutions obtained by varying
  - Number of suppliers
  - Unit penalty cost of exceeding capacity, e_i (SP model)
  - Probability levels of having sufficient supplier capacity to meet demand, ε (CCP model)

	Efficient Solutions MIP model		Efficient Solutions CCP model		Efficient Solutions SP model	
Evaluation model	Normal	Triangular	Normal	Triangular	Normal	Triangular
eCCP	6	6	26	26	24	22
eSP	5	6	11	15	15	20

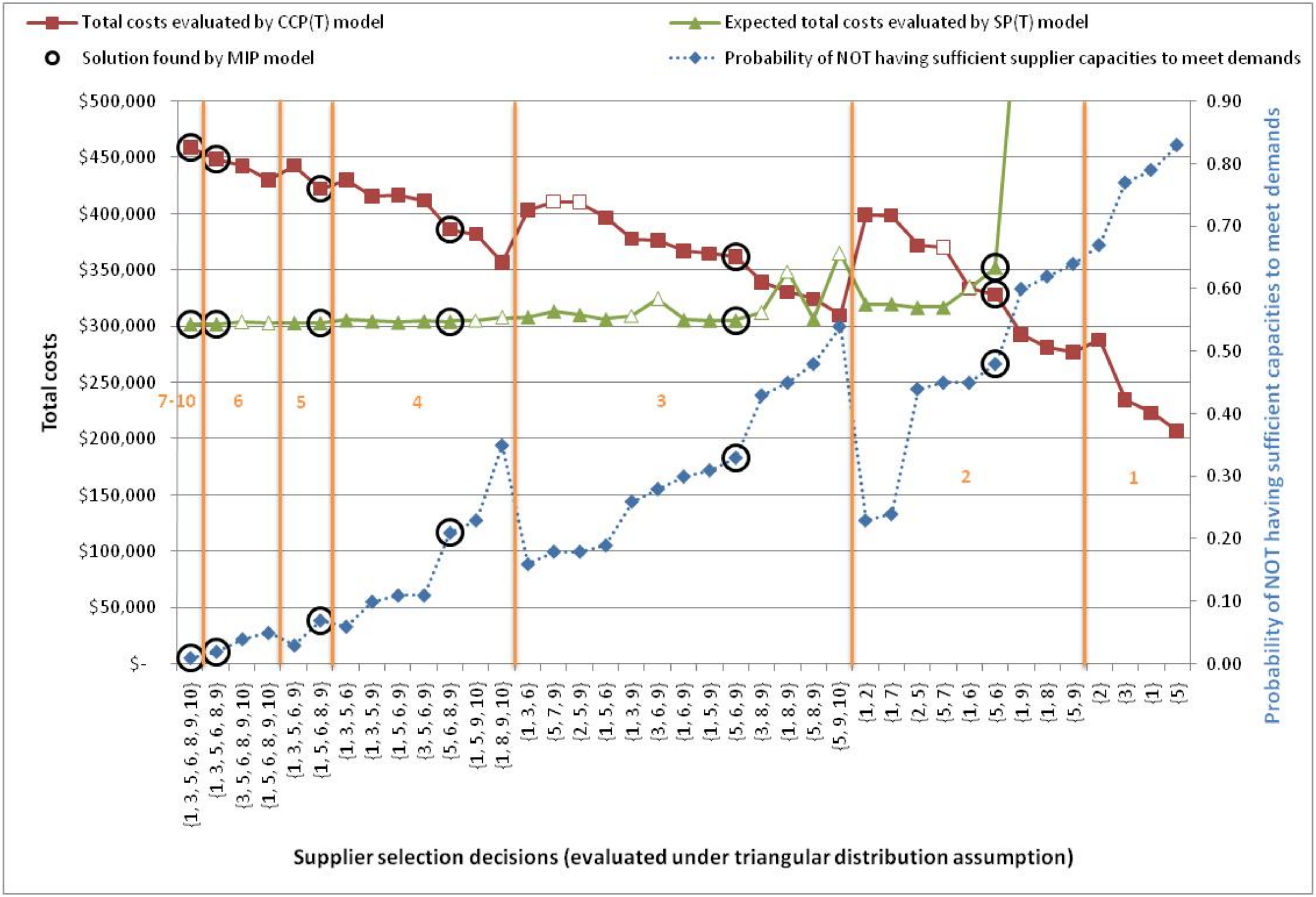
Number of suppliers	Selection decision	Evaluation Model	Probability of having sufficient supplier capacities to meet demands	Total costs	Quality flexibility	Delivery flexibility	Solution model
7	{1,3,5,6,8,9,10}	eCCP(N)	0.96	$ 464,548	46%	10%	MP,CCP(N),CCP(T),SP(N),SP(T)
		eSP(N)	0.96	$ 297,606	63%	38%	
		eCCP(T)	0.99	$ 459,344	46%	10%	
		eSP(T)	0.99	$ 302,354	63%	38%	
6	{1,3,5,6,8,9}	eCCP(N)	0.95	$ 450,694	45%	8%	MP,CCP(N),SP(N),SP(T)
		eSP(N)	0.95	$ 297,775	63%	38%	
		eCCP(T)	0.98	$ 448,770	45%	8%	
		eSP(T)	0.98	$ 302,400	63%	39%	
6	{3,5,6,8,9,10}	eCCP(N)	0.93	$ 438,353	50%	16%	SP(T)
		eSP(N)	0.93	$ 298,853	63%	39%	
		eCCP(T)	0.96	$ 442,640	50%	16%	
		eSP(T)	0.96	$ 303,793	63%	39%	
6	{1,5,6,8,9,10}	eCCP(N)	0.93	$ 442,070	51%	19%	CCP(T),SP(N),SP(T)
		eSP(N)	0.93	$ 298,091	63%	38%	
		eCCP(T)	0.95	$ 429,909	52%	22%	
		eSP(T)	0.95	$ 303,115	63%	38%	
5	{1,3,5,8,9}	eCCP(N)	0.94	$ 443,256	43%	4%	SP(N),SP(T)
		eSP(N)	0.94	$ 298,846	62%	38%	
		eCCP(T)	0.97	$ 442,987	43%	4%	
		eSP(T)	0.97	$ 303,567	63%	38%	
5	{1,5,6,8,9}	eCCP(N)	0.91	$ 422,974	50%	19%	MP,CCP(N),CCP(T),SP(N),SP(T)
		eSP(N)	0.91	$ 300,211	62%	37%	
		eCCP(T)	0.93	$ 422,495	51%	19%	
		eSP(T)	0.93	$ 303,386	63%	38%	
4	{1,3,5,6}	eCCP(N)	0.92	$ 434,893	42%	3%	CCP(N),CCP(T)
		eSP(N)	0.92	$ 301,561	62%	38%	
		eCCP(T)	0.94	$ 430,189	42%	4%	
		eSP(T)	0.94	$ 305,985	62%	37%	
4	{1,3,8,9}	eCCP(N)	0.89	$ 417,647	47%	11%	SP(N)
		eSP(N)	0.89	$ 301,174	62%	37%	
		eCCP(T)	0.90	$ 415,661	47%	11%	
		eSP(T)	0.90	$ 303,371	62%	37%	
4	{3,5,8,9}	eCCP(N)	0.88	$ 411,163	48%	14%	SP(N)
		eSP(N)	0.88	$ 300,257	62%	38%	
		eCCP(T)	0.89	$ 412,013	48%	14%	
		eSP(T)	0.89	$ 306,069	63%	39%	
4	{1,5,8,9}	eCCP(N)	0.88	$ 413,444	40%	15%	CCP(N),CCP(T),SP(N),SP(T)
		eSP(N)	0.88	$ 299,914	63%	38%	
		eCCP(T)	0.89	$ 416,838	40%	15%	
		eSP(T)	0.89	$ 303,811	62%	37%	
4	{5,6,8,9}	eCCP(N)	0.81	$ 380,091	58%	31%	MP,CCP(N)
		eSP(N)	0.81	$ 299,150	63%	39%	
		eCCP(T)	0.79	$ 383,716	58%	31%	
		eSP(T)	0.79	$ 304,700	63%	39%	
4	{1,5,9,10}	eCCP(N)	0.80	$ 383,165	57%	29%	CCP(T)
		eSP(N)	0.80	$ 300,137	63%	38%	
		eCCP(T)	0.77	$ 382,084	58%	30%	
		eSP(T)	0.77	$ 305,413	62%	37%	
4	{1,8,9,10}	eCCP(N)	0.70	$ 355,809	68%	43%	CCP(T)
		eSP(N)	0.70	$ 348,047	63%	38%	
		eCCP(T)	0.65	[illegible]	68%	43%	
		eSP(T)	0.65	$ 307,800	63%	38%	
3	{1,3,9}	eCCP(N)	0.84	$ 388,231	48%	9%	SP(N)
		eSP(N)	0.84	$ 303,180	61%	36%	
		eCCP(T)	0.84	$ 403,800	48%	9%	
		eSP(T)	0.81	$ 302,223	61%	36%	
3	{2,5,9}	eCCP(N)	0.83	$ 408,005	40%	0%	SP(N)
		eSP(N)	0.83	$ 307,710	62%	37%	
		eCCP(T)	0.82	$ 410,168	40%	0%	
		eSP(T)	0.82	$ 310,742	62%	37%	
3	{5,7,9}	eCCP(N)	0.83	$ 408,889	40%	0%	SP(N),SP(T)
		eSP(N)	0.83	$ 312,427	61%	36%	
		eCCP(T)	0.82	$ 410,031	40%	0%	
		eSP(T)	0.82	$ 313,773	64%	40%	
3	{1,5,6}	eCCP(N)	0.82	$ 388,616	48%	16%	CCP(N),CCP(T),SP(N)
		eSP(N)	0.82	$ 303,068	62%	37%	
		eCCP(T)	0.81	[illegible]	48%	16%	
		eSP(T)	0.81	$ 300,731	62%	38%	
3	{1,3,9}	eCCP(N)	0.78	$ 380,751	51%	18%	CCP(T),SP(T)
		eSP(N)	0.78	$ 308,802	60%	34%	
		eCCP(T)	0.74	$ 379,302	51%	18%	
		eSP(T)	0.74	$ 308,890	60%	34%	
3	{3,6,9}	eCCP(N)	0.76	$ 324,242	58%	23%	SP(T)
		eSP(N)	0.76	$ 319,663	61%	35%	
		eCCP(T)	0.72	$ 345,787	54%	22%	
		eSP(T)	0.72	$ 324,063	61%	34%	
3	{1,6,9}	eCCP(N)	0.75	$ 370,765	58%	25%	CCP(N),CCP(T)
		eSP(N)	0.75	$ 303,881	62%	32%	
		eCCP(T)	0.70	$ 357,338	58%	25%	
		eSP(T)	0.70	$ 305,502	61%	35%	
3	{1,5,9}	eCCP(N)	0.74	$ 365,938	58%	27%	CCP(N),SP(N),SP(T)
		eSP(N)	0.74	$ 301,477	62%	37%	
		eCCP(T)	0.69	[illegible]	58%	27%	
		eSP(T)	0.69	[illegible]	62%	32%	
3	{5,8,9}	eCCP(N)	0.72	$ 380,073	58%	31%	MP,CCP(N),SP(T)
		eSP(N)	0.72	$ 301,345	63%	39%	
		eCCP(T)	0.67	$ 381,771	58%	31%	
		eSP(T)	0.67	[illegible]	63%	39%	
3	{3,8,9}	eCCP(N)	0.63	$ 330,004	64%	38%	SP(N)
		eSP(N)	0.63	$ 440,000	58%	30%	
		eCCP(T)	0.57	$ 335,030	64%	39%	
		eSP(T)	0.57	$ 311,943	62%	36%	
3	{1,8,9}	eCCP(N)	0.61	$ 330,882	64%	41%	CCP(T)
		eSP(N)	0.61	$ 409,801	58%	33%	
		eCCP(T)	0.55	$ 330,808	64%	41%	
		eSP(T)	0.55	$ 349,036	63%	38%	
3	{5,8,9}	eCCP(N)	0.58	$ 325,643	69%	48%	CCP(N)
		eSP(N)	0.58	$ 372,343	63%	38%	
		eCCP(T)	0.52	$ 324,307	69%	48%	
		eSP(T)	0.52	$ 308,882	64%	40%	
3	{5,9,10}	eCCP(N)	0.51	$ 310,900	71%	51%	CCP(T)
		eSP(N)	0.51	$ 416,154	60%	34%	
		eCCP(T)	0.46	$ 309,939	71%	51%	
		eSP(T)	0.46	$ 364,711	62%	36%	
2	{2,5}	eCCP(N)	0.70	$ 370,430	40%	0%	CCP(N),SP(T)
		eSP(N)	0.70	$ 325,787	57%	28%	
		eCCP(T)	0.56	$ 371,898	40%	0%	
		eSP(T)	0.56	$ 317,307	60%	34%	
2	{5,7}	eCCP(N)	0.70	$ 371,118	41%	0%	SP(T)
		eSP(N)	0.70	$ 314,119	57%	29%	
		eCCP(T)	0.55	$ 370,383	41%	0%	
		eSP(T)	0.55	$ 317,357	61%	34%	
2	{6,7}	eCCP(N)	0.69	$ 368,871	40%	0%	SP(T)
		eSP(N)	0.69	$ 317,003	59%	32%	
		eCCP(T)	0.39	$ 372,908	41%	0%	
		eSP(T)	0.39	$ 334,196	54%	24%	
2	{2,6}	eCCP(N)	0.69	$ 368,402	40%	0%	SP(N)
		eSP(N)	0.69	$ 314,958	56%	27%	
		eCCP(T)	0.34	$ 374,768	40%	0%	
		eSP(T)	0.34	$ 342,157	52%	21%	
2	{1,2}	eCCP(N)	0.67	$ 369,397	40%	0%	SP(T)
		eSP(N)	0.67	$ 316,325	50%	18%	
		eCCP(T)	0.77	$ 399,404	27%	0%	
		eSP(T)	0.77	$ 320,027	52%	21%	
2	{1,7}	eCCP(N)	0.67	$ 360,989	41%	0%	SP(T)
		eSP(N)	0.67	$ 375,304	53%	22%	
		eCCP(T)	0.76	$ 398,715	28%	0%	
		eSP(T)	0.76	$ 320,376	53%	21%	
2	{1,6}	eCCP(N)	0.61	$ 333,959	56%	28%	CCP(N),CCP(T)
		eSP(N)	0.61	$ 381,001	61%	35%	
		eCCP(T)	0.55	$ 333,681	56%	28%	
		eSP(T)	0.55	$ 334,838	60%	33%	
2	{5,6}	eCCP(N)	0.57	$ 325,460	60%	33%	MP,CCP(N),CCP(T),SP(N)
		eSP(N)	0.57	$ 348,901	61%	35%	
		eCCP(T)	0.52	$ 328,264	59%	32%	
		eSP(T)	0.52	$ 352,858	62%	37%	
2	{1,9}	eCCP(N)	0.43	$ 288,703	66%	43%	CCP(N)
		eSP(N)	0.43	$ 882,115	48%	17%	
		eCCP(T)	0.40	$ 292,818	66%	43%	
		eSP(T)	0.40	$ 629,728	54%	26%	
2	{1,8}	eCCP(N)	0.41	$ 281,657	67%	44%	CCP(N),CCP(T)
		eSP(N)	0.41	$ 830,438	48%	17%	
		eCCP(T)	0.38	$ 281,121	67%	44%	
		eSP(T)	0.38	$ 664,540	50%	12%	
2	{5,9}	eCCP(N)	0.38	$ 276,017	71%	51%	CCP(N)
		eSP(N)	0.38	$ 667,325	56%	26%	
		eCCP(T)	0.36	$ 276,924	71%	51%	
		eSP(T)	0.36	$ 509,151	61%	34%	
1	{2}	eCCP(N)	0.34	$ 287,210	40%	1%	CCP(N),CCP(T)
		eSP(N)	0.34	infeasible			
		eCCP(T)	0.33	$ 288,039	40%	0%	
		eSP(T)	0.33	infeasible			
1	{3}	eCCP(N)	0.21	$ 233,673	67%	45%	SP(N),SP(T)
		eSP(N)	0.21	$ 1,478,687	52%	18%	
		eCCP(T)	0.23	$ 236,122	67%	44%	
		eSP(T)	0.23	$ 1,336,047	49%	12%	
1	{1}	eCCP(N)	0.20	$ 226,235	69%	47%	CCP(N),CCP(T),SP(N),SP(T)
		eSP(N)	0.20	$ 1,498,125	48%	17%	
		eCCP(T)	0.21	$ 222,961	69%	48%	
		eSP(T)	0.21	$ 1,338,709	50%	20%	
1	{5}	eCCP(N)	0.15	$ 209,380	74%	57%	CCP(N),CCP(T)
		eSP(N)	0.15	$ 1,265,621	56%	26%	
		eCCP(T)	0.17	$ 206,779	75%	57%	
		eSP(T)	0.17	$ 1,141,054	61%	34%	

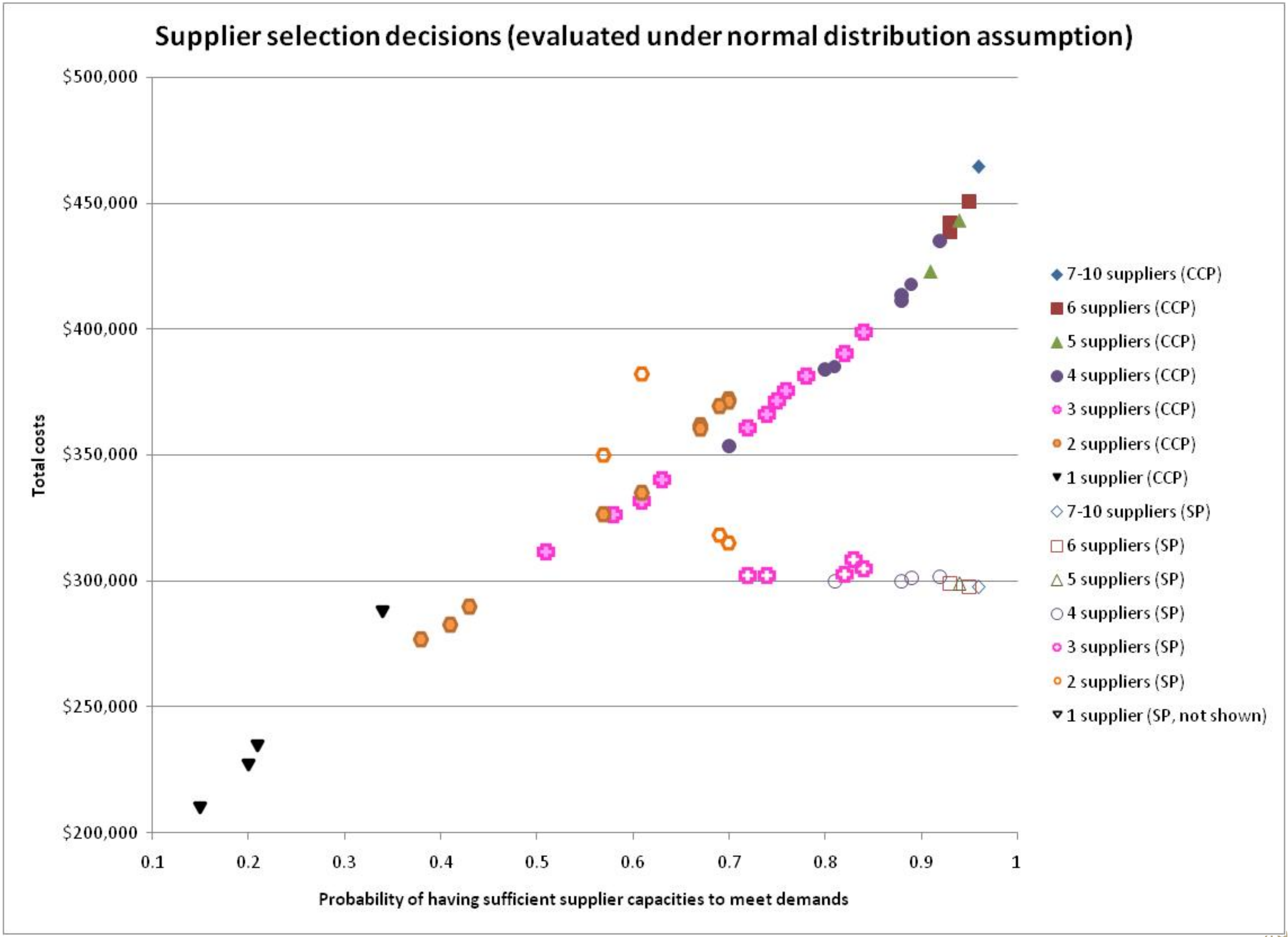

Total costs evaluated by CCP(N) model
Solution found by MIP model
Probability of NOT having sufficient supplier capacities to meet demands
Total costs
Probability of NOT having sufficient capacities to meet demands
$500,000
$450,000
$400,000
$350,000
$300,000
$250,000
$200,000
$150,000
$100,000
$50,000
$0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
7-10
6
5
4
3
2
1
Supplier selection decisions (evaluated under normal distribution assumption)

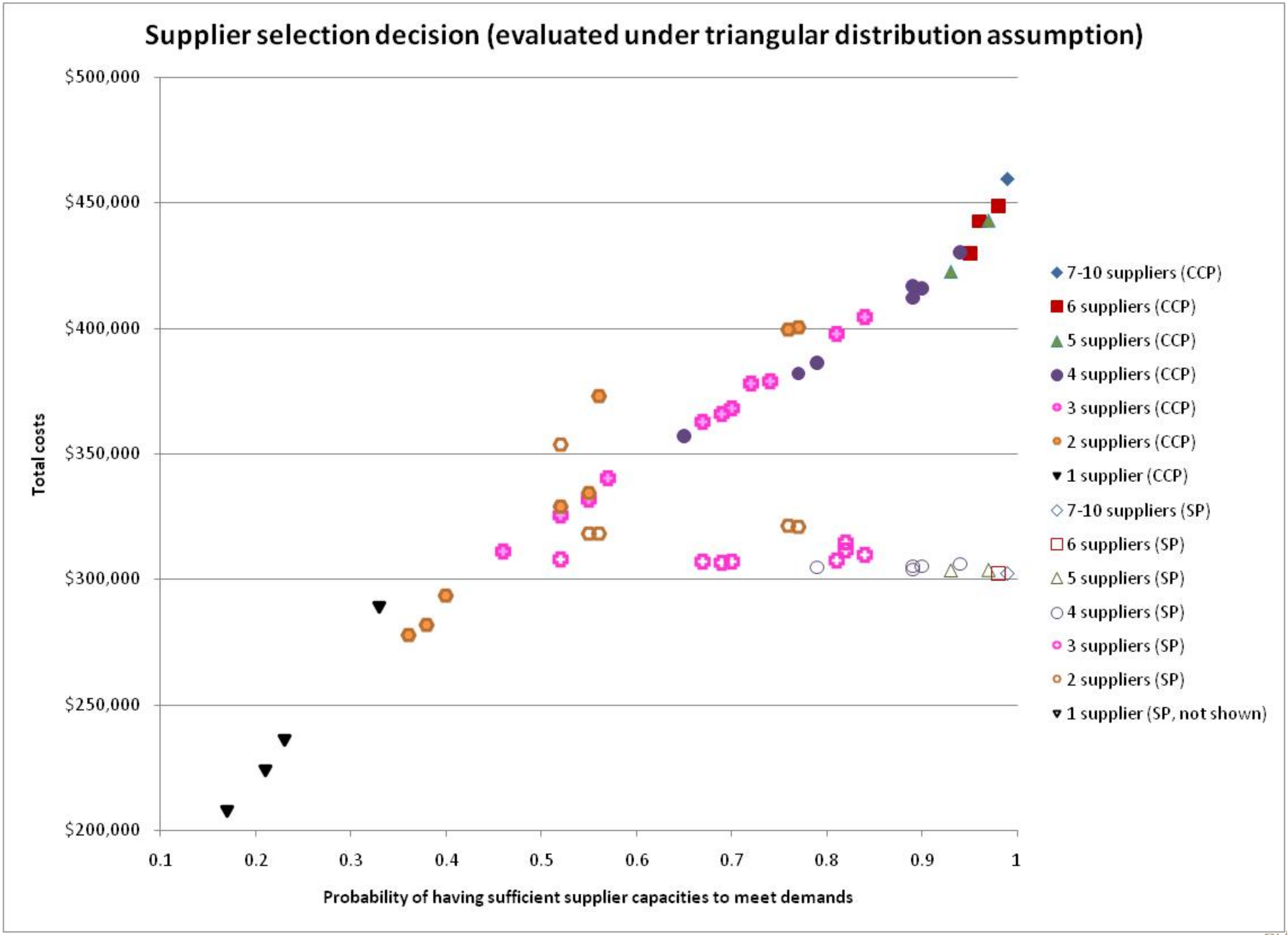

Total costs evaluated by CCP(N) model
Solution found by MIP model
Expected total costs evaluated by SP(N) model
Probability of NOT having sufficient supplier capacities to meet demands
Total costs
$500,000
$450,000
$400,000
$350,000
$300,000
$250,000
$200,000
$150,000
$100,000
$50,000
$0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Probability of NOT having sufficient capacities to meet demands
7-10
6
5
4
3
2
1
{1, 3, 5, 6, 8, 9, 10}
{1, 3, 5, 6, 8, 9}
{1, 5, 6, 8, 9, 10}
{3, 5, 6, 8, 9, 10}
{1, 3, 5, 6, 9}
{1, 5, 6, 8, 9}
{1, 3, 5, 6}
{1, 3, 5, 9}
{1, 5, 6, 9}
{3, 5, 6, 9}
{5, 6, 8, 9}
{1, 5, 9, 10}
{1, 8, 9, 10}
{1, 3, 6}
{2, 5, 9}
{1, 5, 6}
{1, 3, 9}
{3, 6, 9}
{1, 6, 9}
{1, 5, 9}
{5, 6, 9}
{3, 8, 9}
{1, 8, 9}
{5, 8, 9}
{5, 9, 10}
{5, 7}
{2, 5}
{6, 7}
{2, 6}
{1, 7}
{1, 2}
{1, 6}
{5, 6}
{1, 9}
{1, 8}
{5, 9}
{2}
{3}
{1}
{5}
Supplier selection decisions (evaluated under normal distribution assumption)



Total costs evaluated by CCP(T) model
Solution found by MIP model
Probability of NOT having sufficient supplier capacities to meet demands
Total costs
Probability of NOT having sufficient capacities to meet demands
Supplier selection decisions (evaluated under triangular distribution assumption)
7-10
6
5
4
3
2
1


Total costs evaluated by CCP(T) model
Expected total costs evaluated by SP(T) model
Solution found by MIP model
Probability of NOT having sufficient supplier capacities to meet demands
Total costs
Probability of NOT having sufficient capacities to meet demands
7-10
6
5
4
3
2
1
Supplier selection decisions (evaluated under triangular distribution assumption)


Supplier selection decisions (evaluated under normal distribution assumption)
Total costs
$500,000
$450,000
$400,000
$350,000
$300,000
$250,000
$200,000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacities to meet demands
7-10 suppliers (CCP)
6 suppliers (CCP)
5 suppliers (CCP)
4 suppliers (CCP)
3 suppliers (CCP)
2 suppliers (CCP)
1 supplier (CCP)
7-10 suppliers (SP)
6 suppliers (SP)
5 suppliers (SP)
4 suppliers (SP)
3 suppliers (SP)
2 suppliers (SP)
1 supplier (SP, not shown)


Supplier selection decision (evaluated under triangular distribution assumption)
Total costs
$500,000
$450,000
$400,000
$350,000
$300,000
$250,000
$200,000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacities to meet demands
7-10 suppliers (CCP)
6 suppliers (CCP)
5 suppliers (CCP)
4 suppliers (CCP)
3 suppliers (CCP)
2 suppliers (CCP)
1 supplier (CCP)
7-10 suppliers (SP)
6 suppliers (SP)
5 suppliers (SP)
4 suppliers (SP)
3 suppliers (SP)
2 suppliers (SP)
1 supplier (SP, not shown)

Benefits of SP and CCP Models

- Incorporating uncertainty provides insight into the balance between costs and risks
- The stochastic programming model is appropriate for scenario based applications
- The chance-constrained programming model is efficient for known distributions
- The chance-constrained programming model can provide the Pareto-frontier in a *straightforward* manner, and in *less computational time* than the stochastic programming model

Need a More Complete Picture

- Enumerate number of suppliers ε_s and probability level ε_d, ε_c with $\varepsilon_d=\varepsilon_c$
- Arbitrary probability level setting in CCP model (every 0.1)
- Arbitrary weights in SP model
- Need to find analytical relationship between total cost (f_c) and ε_s, ε_d, ε_c

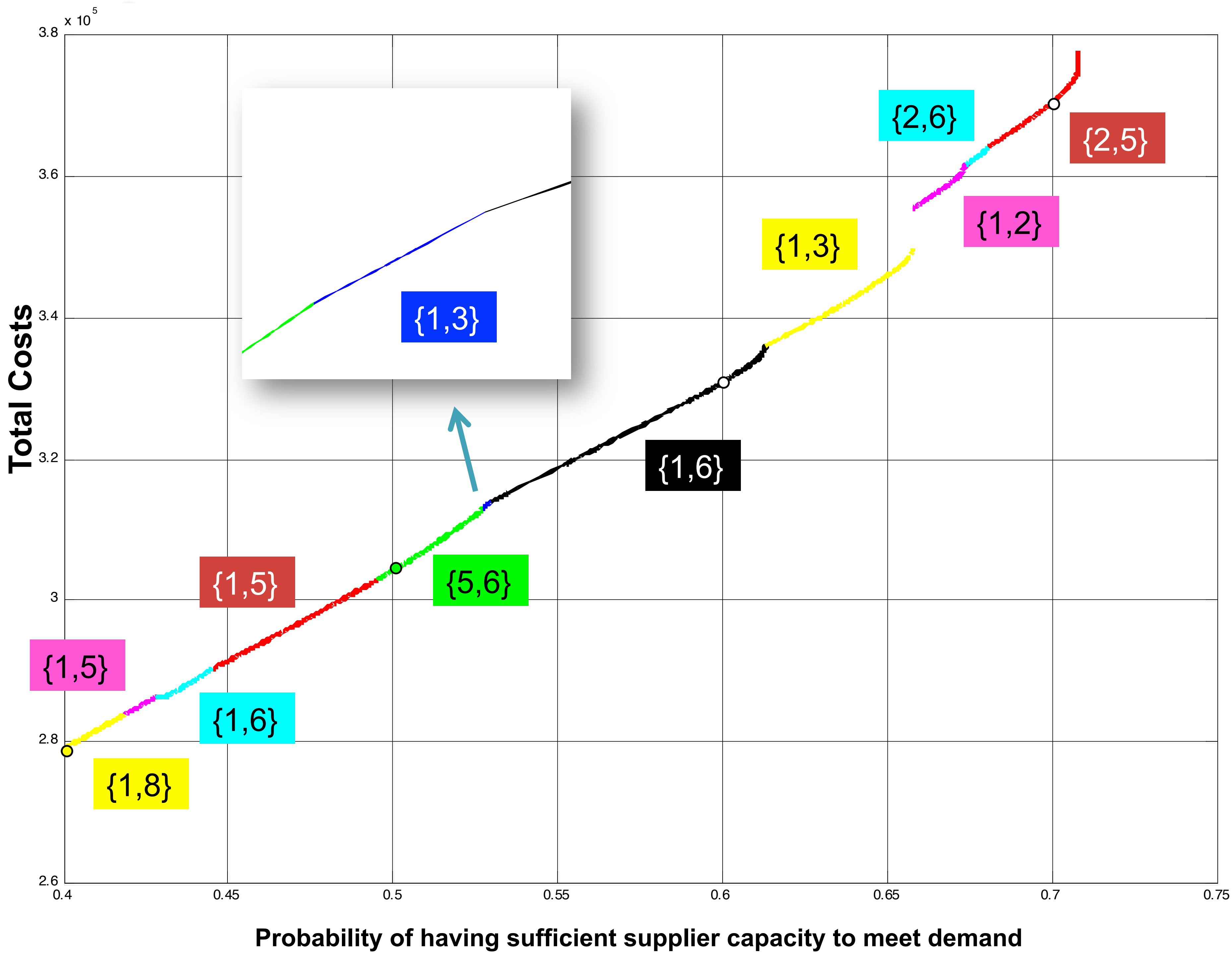
Multiparametric Programming Techniques

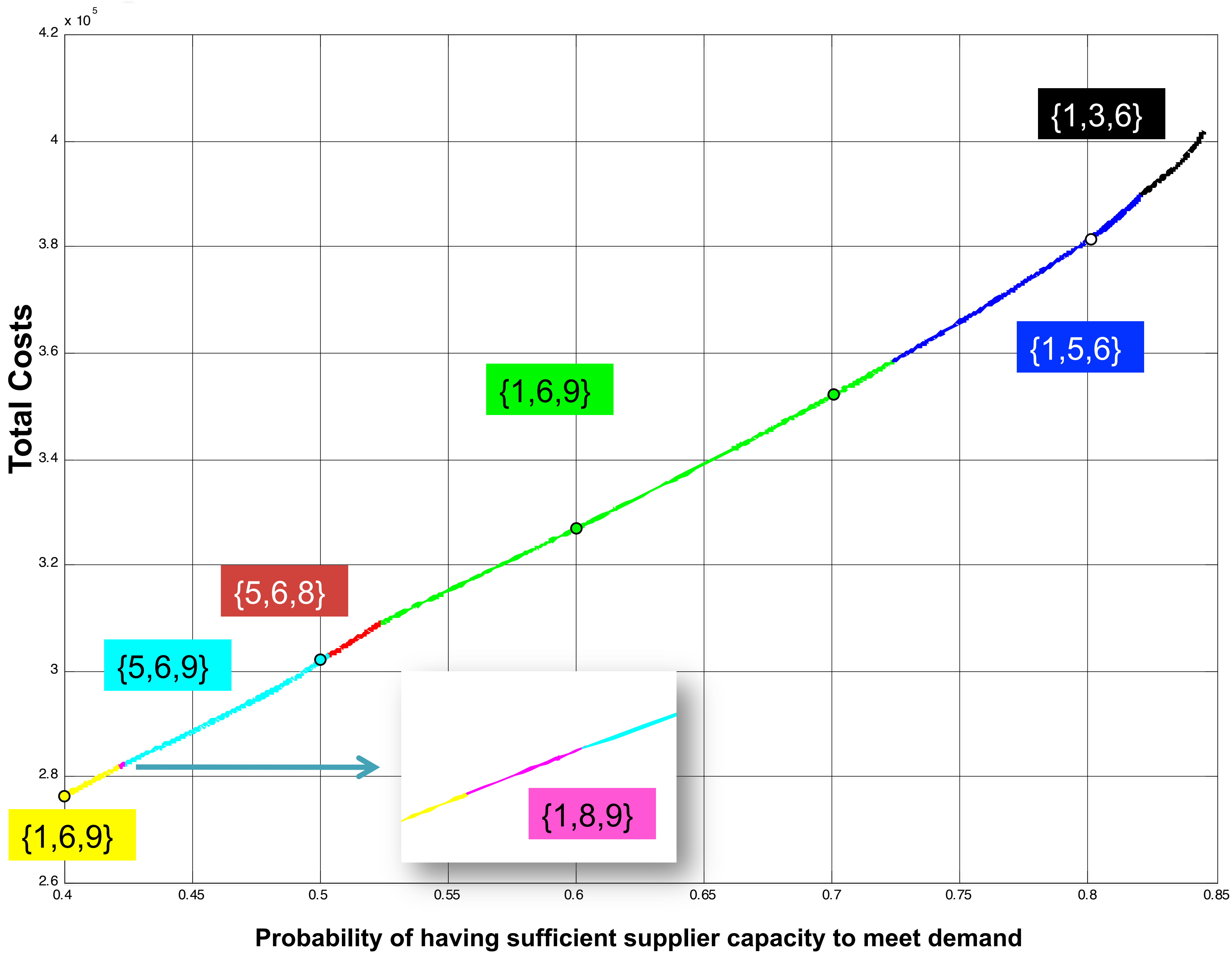
- Posterior analysis of a deterministic model by parametric analysis can *not* plan for uncertainty

- Our CCP model with parametric analysis on ε_d, ε_c *can* plan for uncertainty

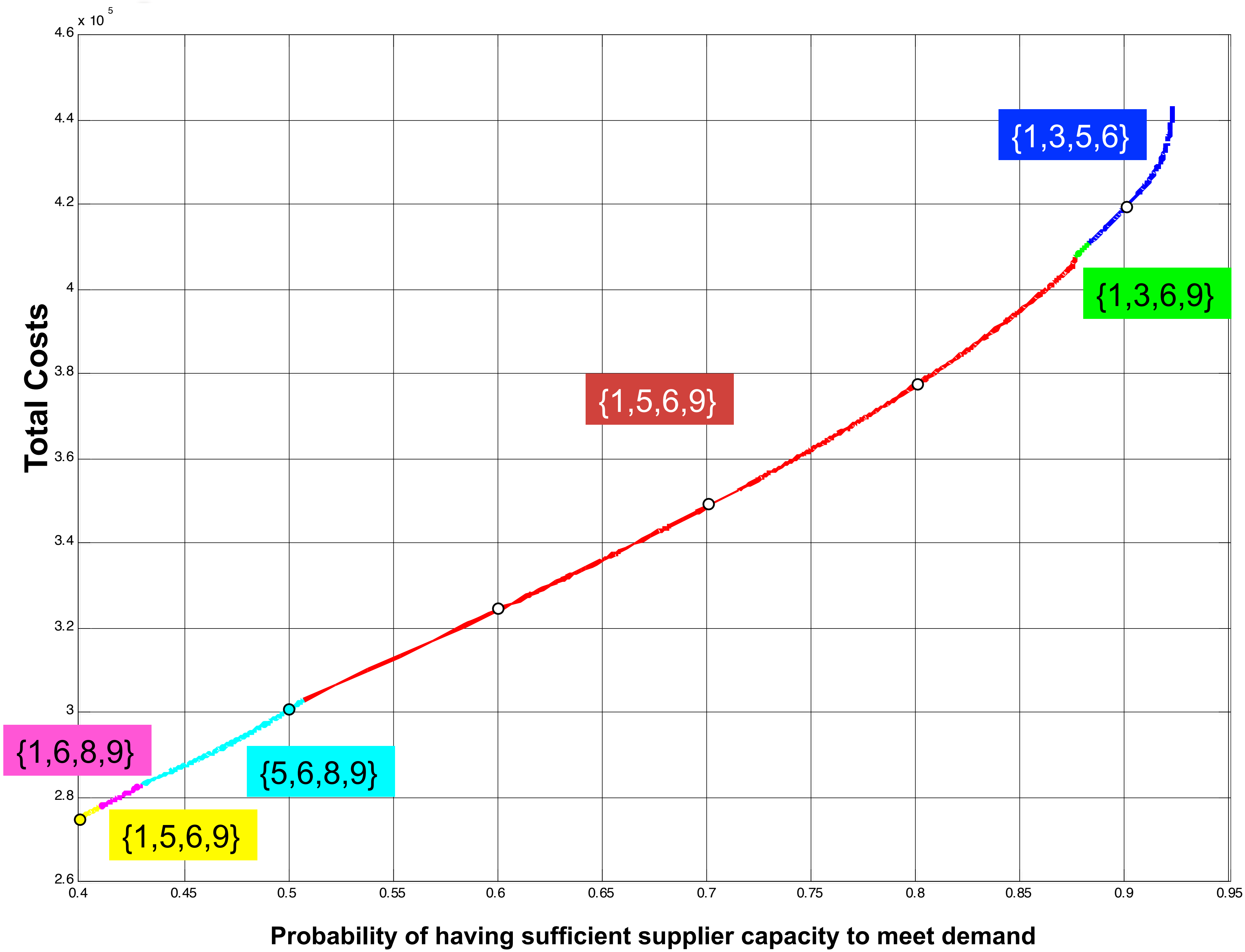
Techniques for Solving Multiparametric MIP with Binary & Continuous Variables

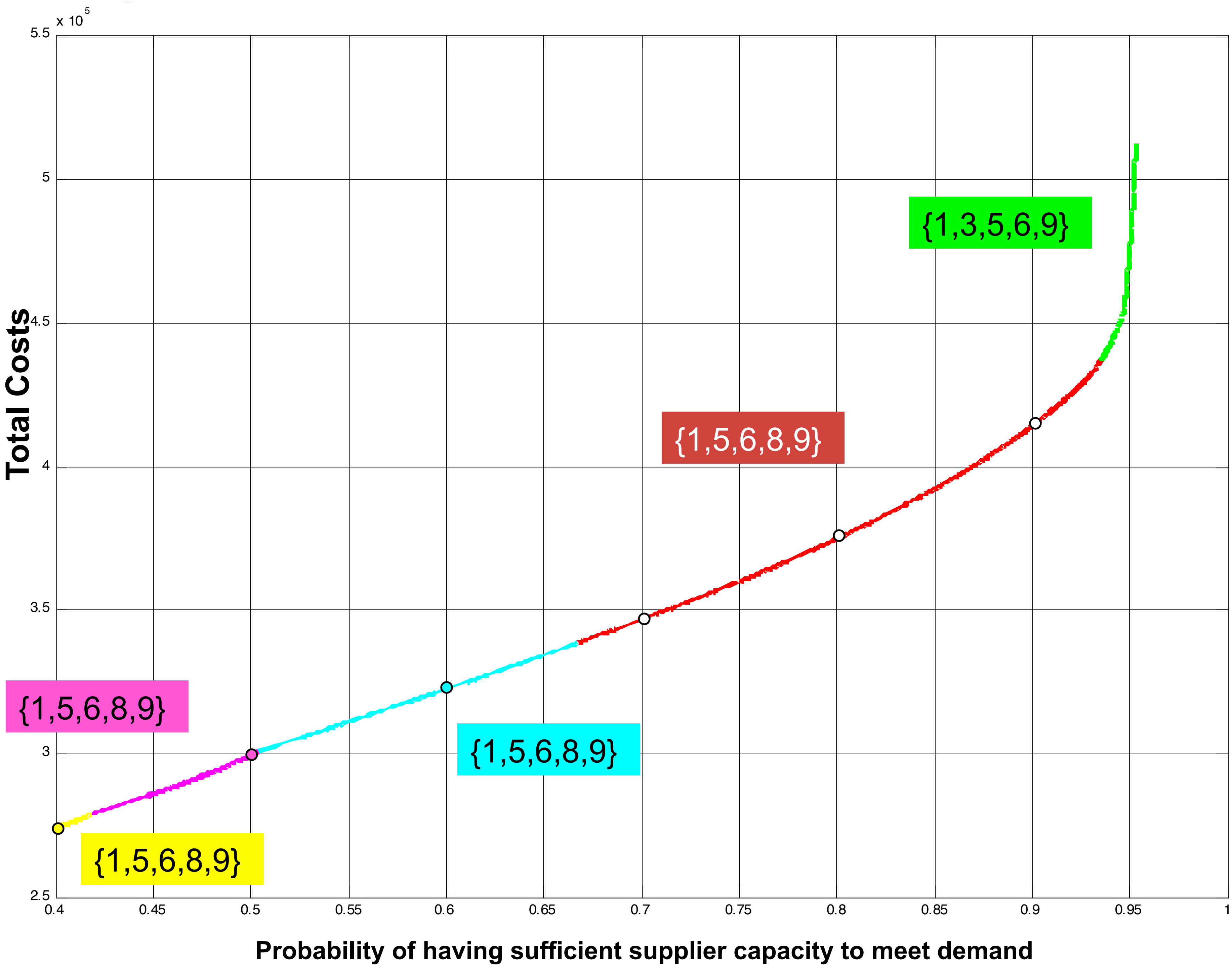
- Initialize: treat parameters as free variables to obtain binary y, v
- Step 1: fix y and v, perform multiparametric LP analysis to obtain critical regions and objective function $f_c(y, v)$
- Step 2: at each critical region, search for the existence of a better integer solution (y_2, v_2)
  - If found, update parameter upper bound at this region, go to step 1 and determine a tighter parametric upper bound
  - If not, y and v are the optimal parametric solutions in this region


x 10^5
3.8
3.6
3.4
3.2
3
2.8
2.6
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
Total Costs
Probability of having sufficient supplier capacity to meet demand
{1,3}
{2,6}
{2,5}
{1,2}
{1,3}
{1,6}
{1,5}
{5,6}
{1,5}
{1,6}
{1,8}

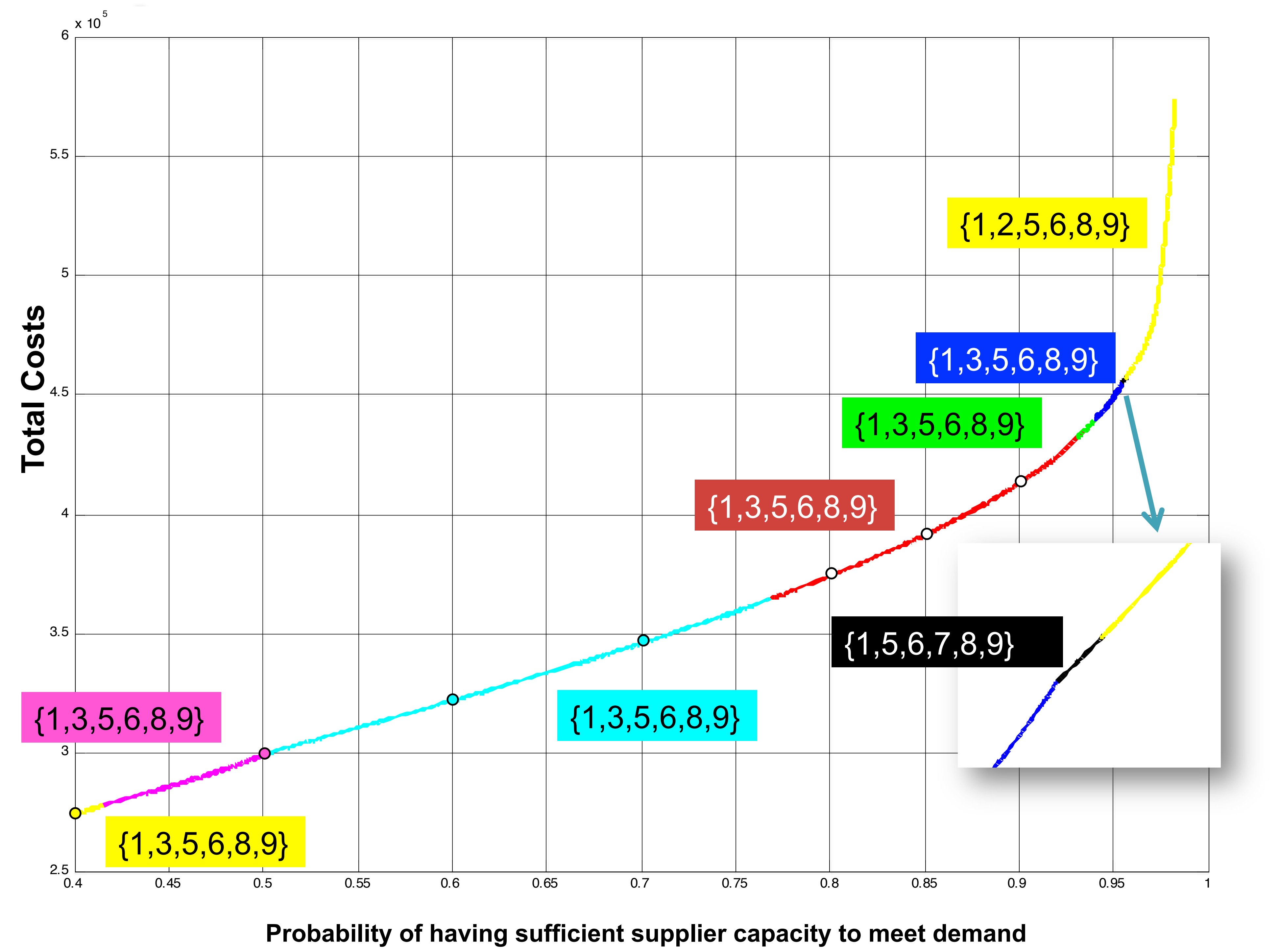


Total Costs
x 10^5
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
Probability of having sufficient supplier capacity to meet demand
{1,3,6}
{1,5,6}
{1,6,9}
{5,6,8}
{5,6,9}
{1,8,9}
{1,6,9}

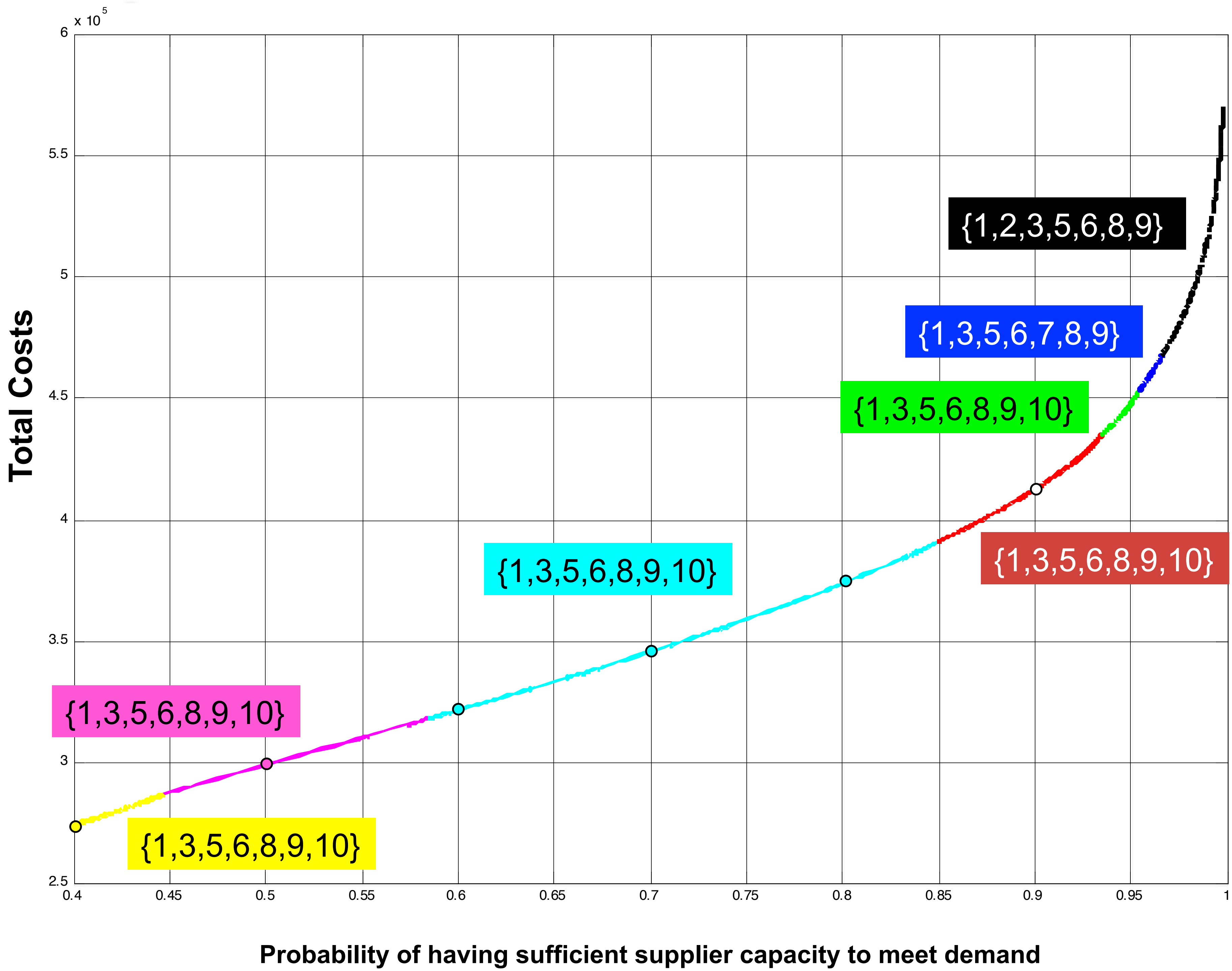

x 10^5
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
Total Costs
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
Probability of having sufficient supplier capacity to meet demand
{1,3,5,6}
{1,3,6,9}
{1,5,6,9}
{1,6,8,9}
{5,6,8,9}
{1,5,6,9}

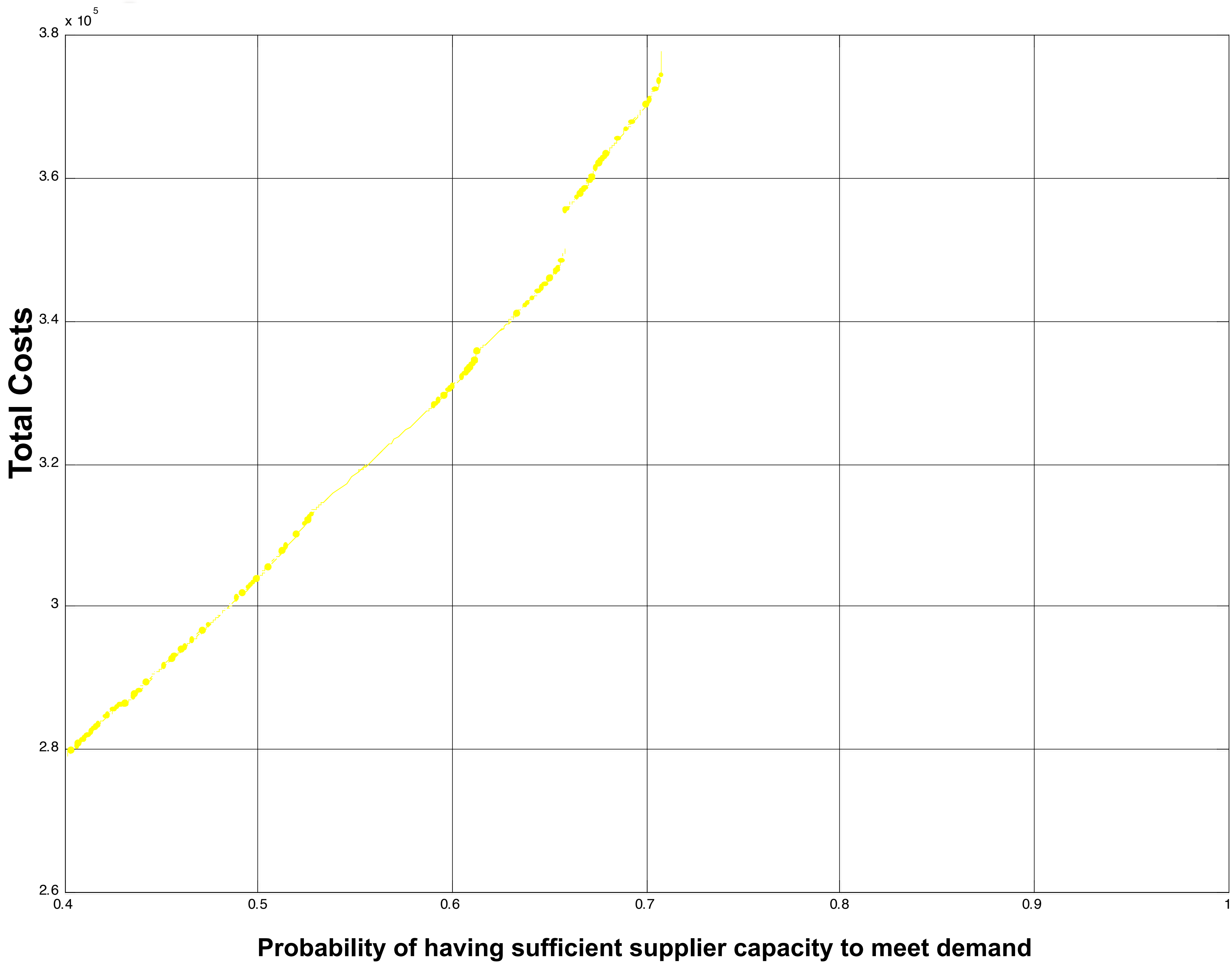


x 10^5
5.5
5
4.5
4
3.5
3
2.5
Total Costs
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Probability of having sufficient supplier capacity to meet demand
{1,3,5,6,9}
{1,5,6,8,9}
{1,5,6,8,9}
{1,5,6,8,9}
{1,5,6,8,9}

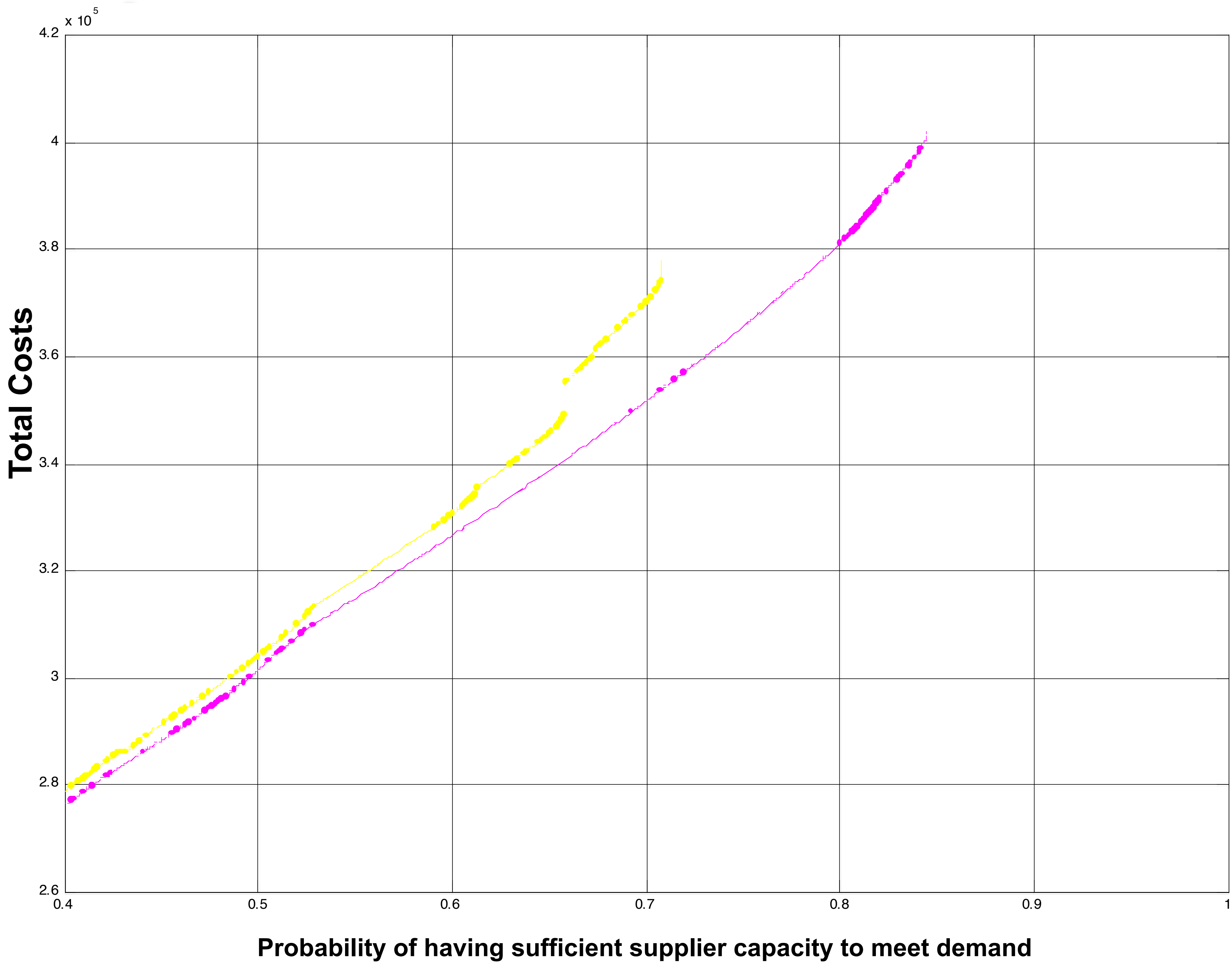


x 10^5
6
5.5
5
4.5
4
3.5
3
2.5
Total Costs
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Probability of having sufficient supplier capacity to meet demand
{1,2,5,6,8,9}
{1,3,5,6,8,9}
{1,3,5,6,8,9}
{1,3,5,6,8,9}
{1,5,6,7,8,9}
{1,3,5,6,8,9}
{1,3,5,6,8,9}
{1,3,5,6,8,9}

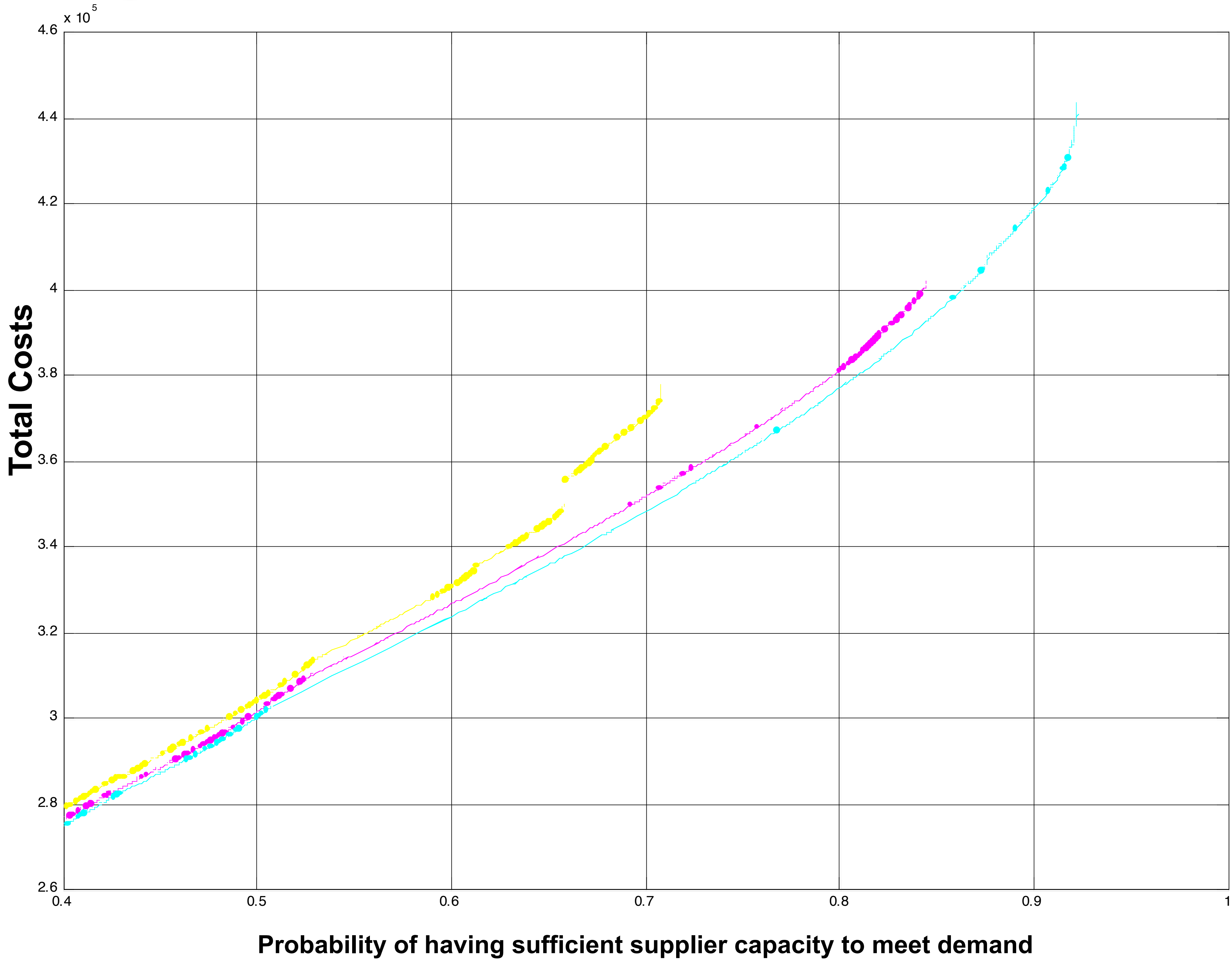

x 10^5
6
5.5
5
4.5
4
3.5
3
2.5
Total Costs
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
Probability of having sufficient supplier capacity to meet demand
{1,2,3,5,6,8,9}
{1,3,5,6,7,8,9}
{1,3,5,6,8,9,10}
{1,3,5,6,8,9,10}
{1,3,5,6,8,9,10}
{1,3,5,6,8,9,10}
{1,3,5,6,8,9,10}



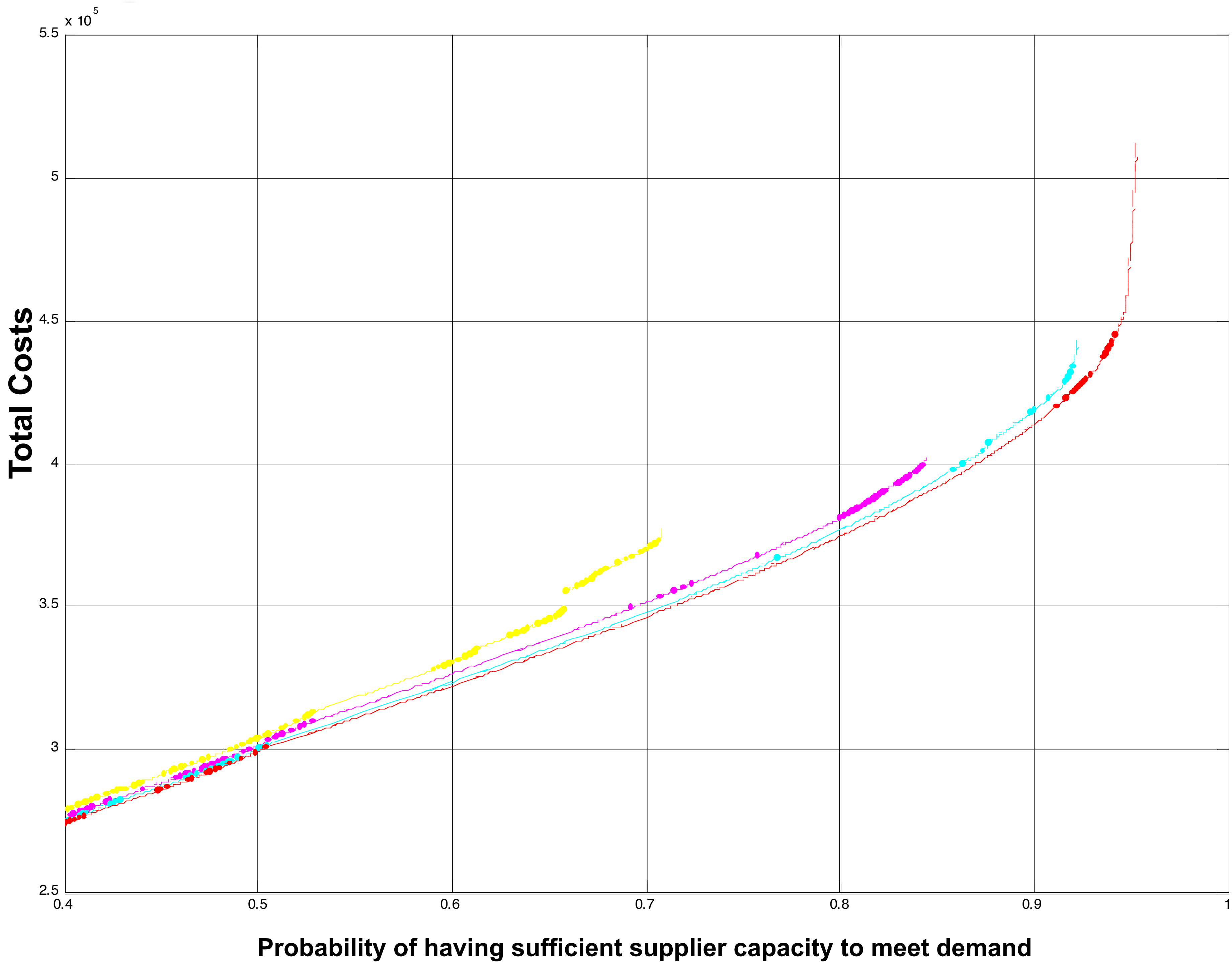
x 10^5
3.8
3.6
3.4
3.2
3
2.8
2.6
Total Costs
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacity to meet demand

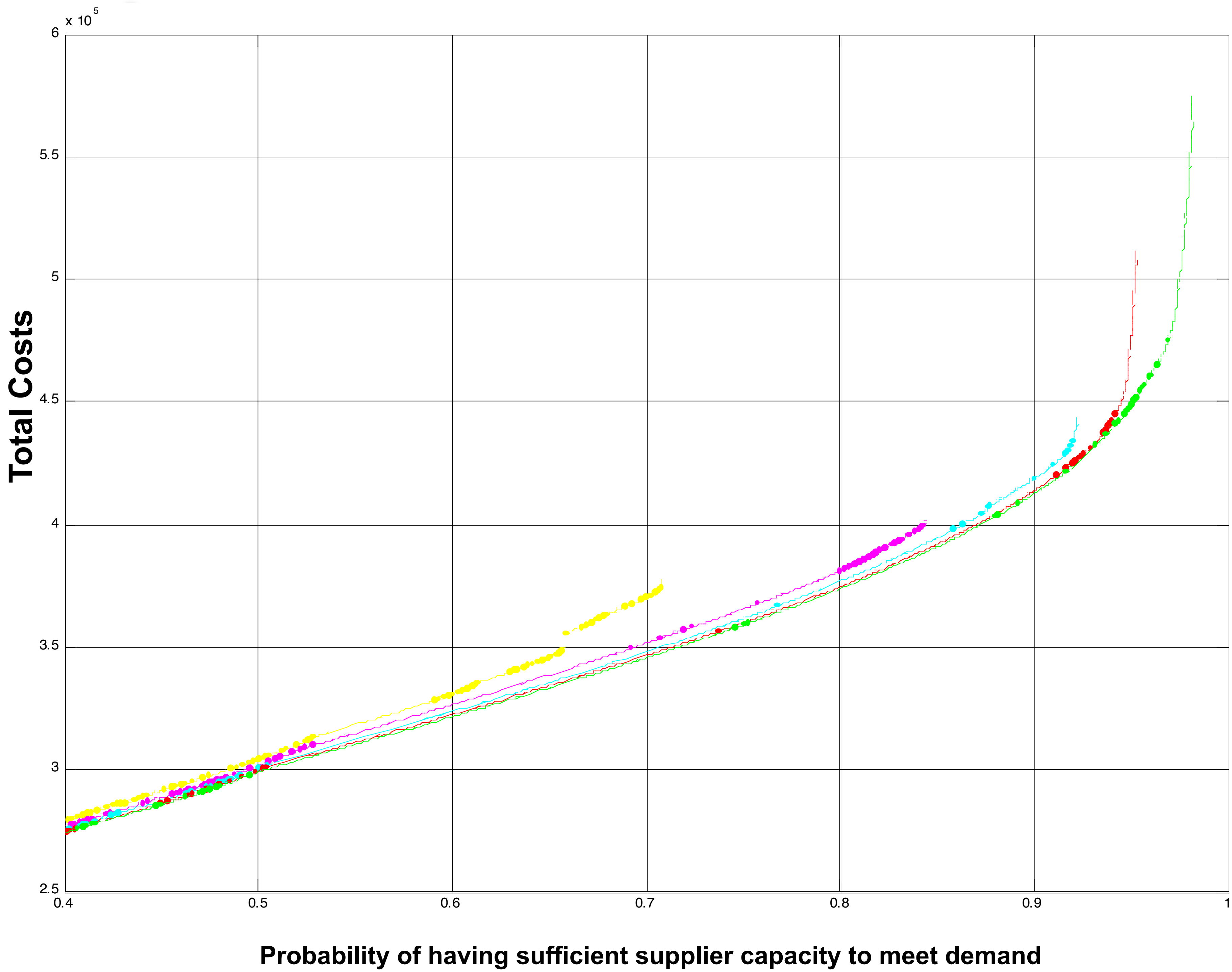


x 10^5
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
Total Costs
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacity to meet demand

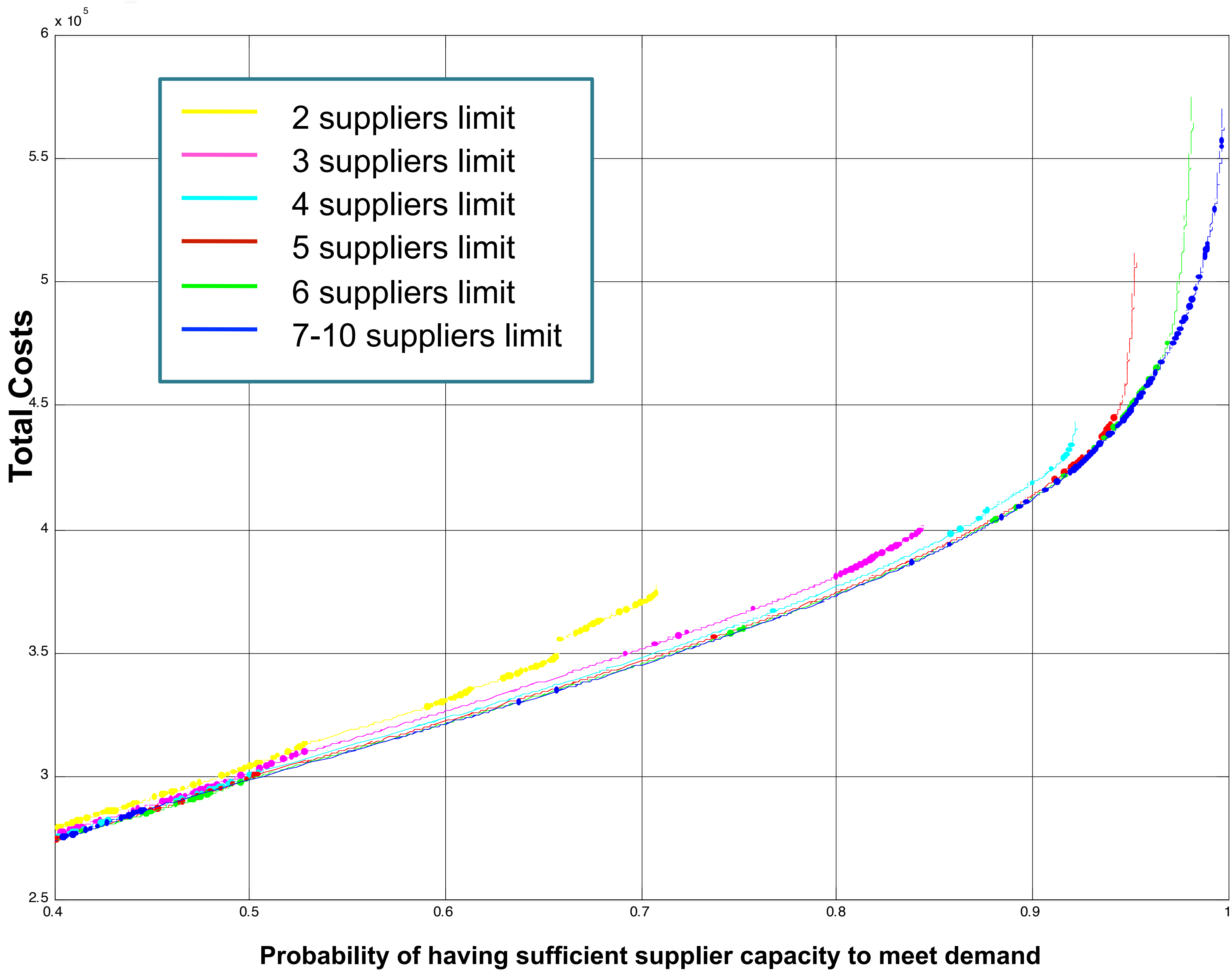


x 10^5
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
Total Costs
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacity to meet demand


x 10^5
5.5
5
4.5
4
3.5
3
2.5
Total Costs
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacity to meet demand



x 10^5
6
5.5
5
4.5
4
3.5
3
2.5
Total Costs
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of having sufficient supplier capacity to meet demand



x 10^5
6
5.5
5
4.5
4
3.5
3
2.5
0.4
0.5
0.6
0.7
0.8
0.9
1
2 suppliers limit
3 suppliers limit
4 suppliers limit
5 suppliers limit
6 suppliers limit
7-10 suppliers limit
Total Costs
Probability of having sufficient supplier capacity to meet demand

Two Suppliers

ε range	Supplier Decision	Note
40% ~ 41.86%	{1, 8}	
41.86% ~ 42.80%	{1, 5}	New selection
42.80% ~ 44.49%	{1, 6}	
44.49% ~ 49.57%	{1, 5}	New selection
49.57% ~ 52.71%	{5, 6}	
52.71% ~ 52.97%	{1, 3}	New selection
52.97% ~ 61.30%	{1, 6}	
61.30% ~ 65.77%	{1, 3}	New selection
65.77% ~ 67.36%	{1, 2}	New selection
67.36% ~ 68.06%	{2, 6}	New selection
68.06% ~ 70.76%	{2, 5}	
70.76% up		Infeasible

Three Suppliers

ε range	Supplier Decision	Note
40% ~ 42.14%	{1, 6, 9}	
42.14% ~ 42.37%	{1, 8, 9}	New selection
42.37% ~ 50.33%	{5, 6, 9}	
50.33% ~ 52.36%	{5, 6, 8}	New selection
52.36% ~ 72.30%	{1, 6, 9}	
72.30% ~ 82.03%	{1, 5, 6}	
82.03% ~ 84.47%	{1, 3, 6}	New selection
84.47% up		Infeasible

Four Suppliers

ε range	Supplier Decision	Note
40% ~ 41.04%	{1, 5, 6, 9}	
41.04% ~ 43.00%	{1, 6, 8, 9}	New selection
43.00% ~ 50.71%	{5, 6, 8, 9}	
50.71% ~ 87.60%	{1, 5, 6, 9}	
87.60% ~ 88.28%	{1, 3, 6, 9}	New selection
88.28% ~ 92.22%	{1, 3, 5, 6}	
92.22% up		Infeasible

Five Suppliers

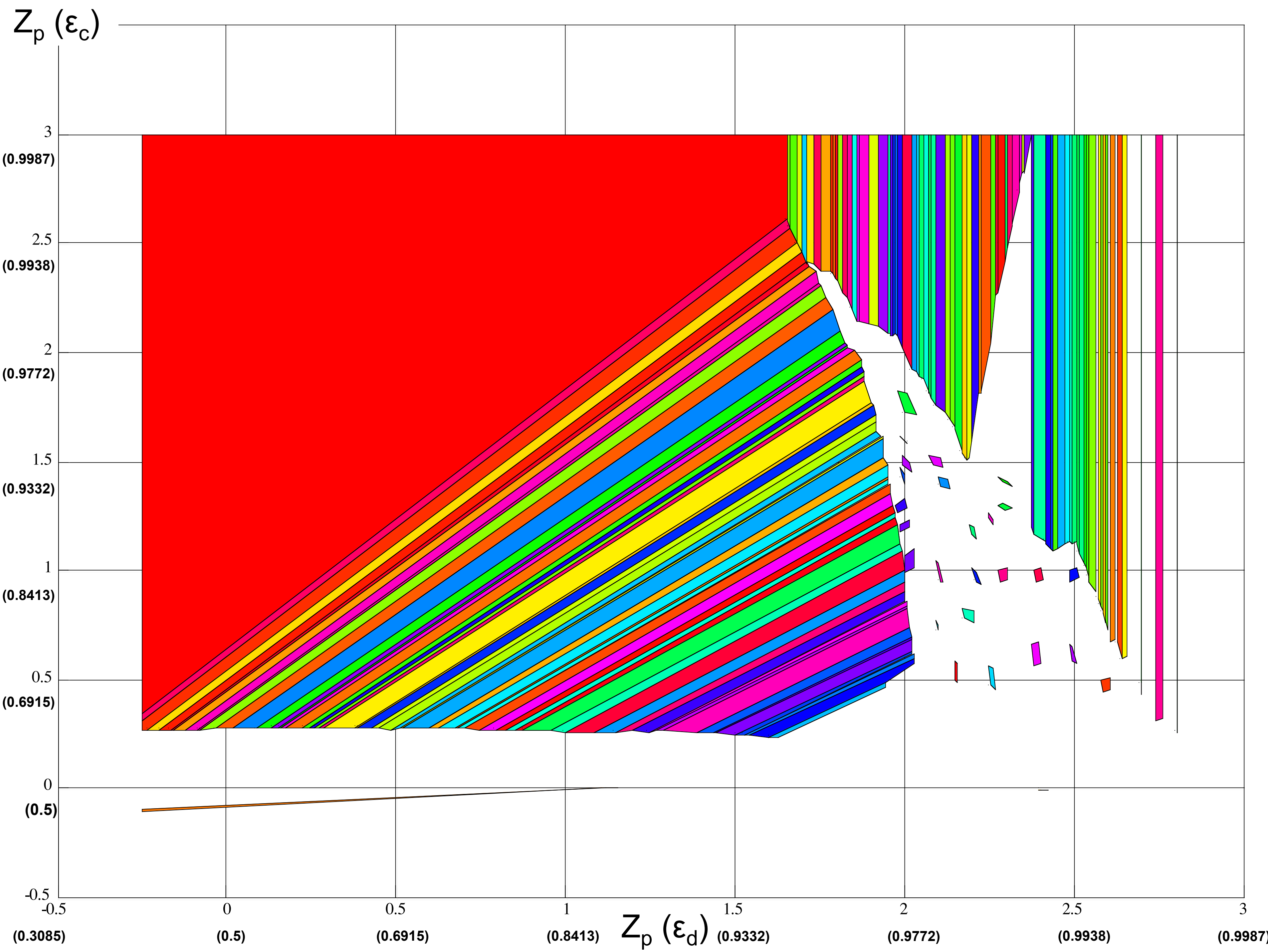
ε range	Supplier Decision	Note
40% ~ 41.82%	{1, 5, 6, 8, 9}	
41.82% ~ 50.39%	{1, 5, 6, 8, 9}	
50.39% ~ 66.71%	{1, 5, 6, 8, 9}	
66.71% ~ 93.53%	{1, 5, 6, 8, 9}	
93.53% ~ 95.23%	{1, 3, 5, 6, 9}	New selection
95.23% up		Infeasible

Six Suppliers

ε range	Supplier Decision	Note
40% ~ 41.54%	{1, 3, 5, 6, 8, 9}	
41.54% ~ 50.26%	{1, 3, 5, 6, 8, 9}	
50.26% ~ 76.85%	{1, 3, 5, 6, 8, 9}	
76.85% ~ 93.13%	{1, 3, 5, 6, 8, 9}	
93.13% ~ 93.97%	{1, 3, 5, 6, 8, 9}	
93.97% ~ 95.46%	{1, 3, 5, 6, 8, 9}	
95.46% ~ 95.64%	{1, 5, 6, 7, 8, 9}	New selection
95.64% ~ 98.18%	{1, 2, 5, 6, 8, 9}	New selection
98.18% up		Infeasible

Seven to Ten Suppliers

ε range	Supplier Decision	Note
40% ~ 44.71%	{1, 3, 5, 6, 8, 9, 10}	
44.71% ~ 58.40%	{1, 3, 5, 6, 8, 9, 10}	
58.40% ~ 84.89%	{1, 3, 5, 6, 8, 9, 10}	
84.89% ~ 93.46%	{1, 3, 5, 6, 8, 9, 10}	
93.46% ~ 95.35%	{1, 3, 5, 6, 8, 9, 10}	
95.35% ~ 96.61%	{1, 3, 5, 6, 7, 8, 9}	New selection
96.61% ~ 99.72%	{1, 2, 3, 5, 6, 8, 9}	New selection
99.72% up		Infeasible



Zp (εc)
3
(0.9987)
2.5
(0.9938)
2
(0.9772)
1.5
(0.9332)
1
(0.8413)
0.5
(0.6915)
0
(0.5)
-0.5
-0.5
(0.3085)
0
(0.5)
0.5
(0.6915)
1
(0.8413)
1.5
(0.9332)
2
(0.9772)
2.5
(0.9938)
3
(0.9987)
Zp (εd)

Summary

- Developed multiobjective SP and CCP models to incorporate uncertain demand and uncertain supply
- Characterized tradeoffs between cost and risk under alternative Pareto-optimal supplier selection solutions
- Future research

  To add industry-specific details to models and explore other probabilistic distributions