Ling 566
Oct 2, 2025

Feature Structures
Headed Rules, Trees

Overview

- Review: problems with CFG, modeling
- Feature structures, unification (pizza)
- Features for linguistic description
- Reformulate grammar rules
- Notion of head/headedness
- Licensing of trees
- Reading questions

Our Goals

- Descriptive, generative grammar
  - Describing English (in this case)
  - Generating all possible well-formed sentences (and no ill-formed ones)
    - Assigning appropriate structures
- Design/discover an appropriate *type* of model (through incremental improvement)
- Create a particular model (grammar fragment) for English

Problems with Context-Free Grammar (atomic node labels)

- Potentially arbitrary rules
- Gets clunky quickly with cross-cutting properties
- Not quite powerful enough for natural languages

Solution: Replace atomic node labels with feature structures.

Cross-cutting Grammatical Properties

	3rd singular subject	plural subject
direct object NP	*denies*	*deny*
no direct object NP	*appears*	*appear*

Two Kinds of Language Models

- Model of: Speakers' internalized knowledge (their grammar)
- Model of: Set of sentences in the language
- (Both distinct from model of: Probability distribution over strings)

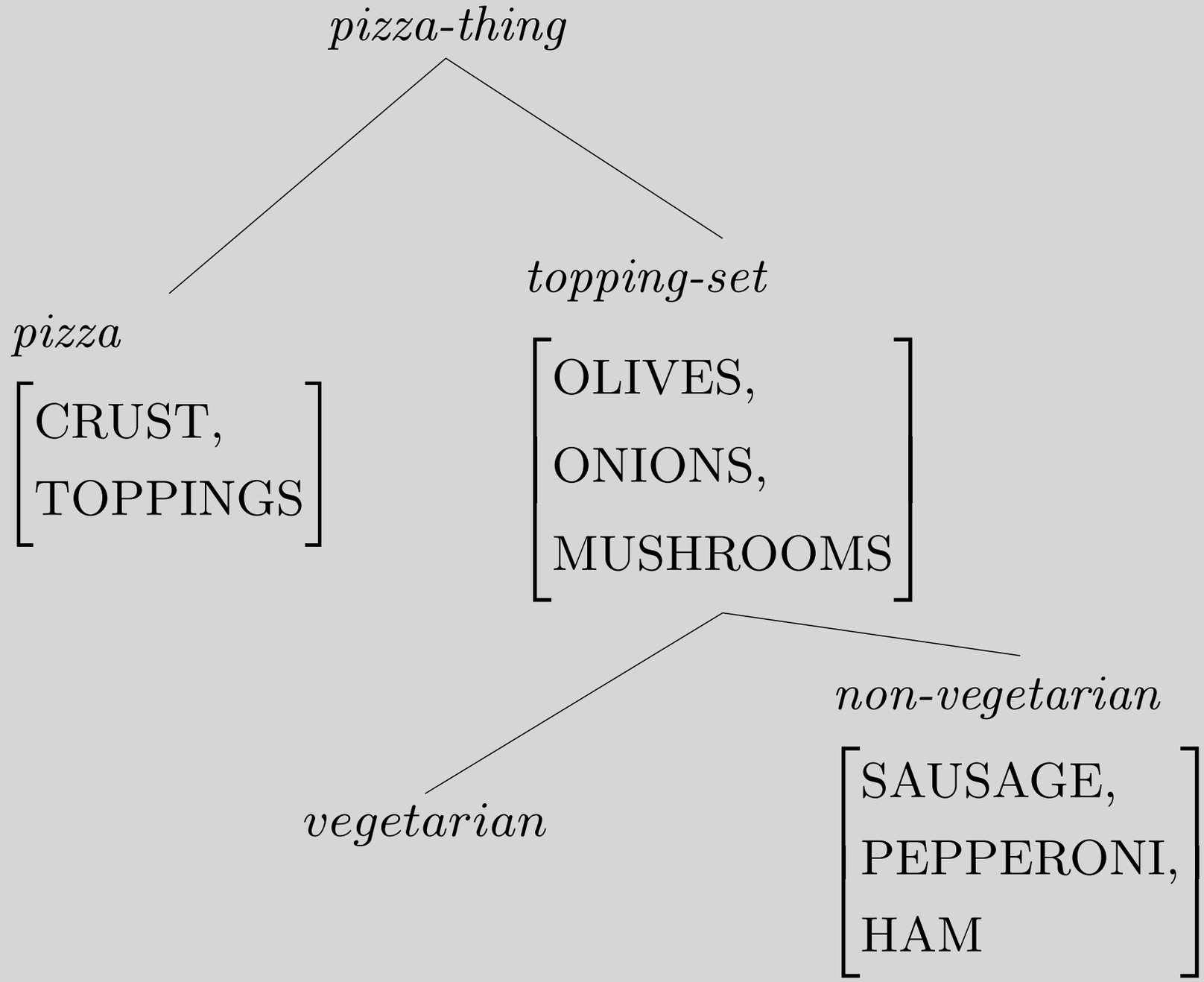
Things Involved in Modeling Language

- Real world entities (utterance types)
- Models (here: fully specified trees)
- Descriptions of the models (here: rules, principles, lexical entries)

Feature Structure Descriptions

$$\begin{bmatrix} \text{FEATURE}_1 & \text{VALUE}_1 \\ \text{FEATURE}_2 & \text{VALUE}_2 \\ \ldots & \\ \text{FEATURE}_n & \text{VALUE}_n \end{bmatrix}$$

A Pizza Type Hierarchy



TYPE	FEATURES/VALUES	IST
pizza-thing		
pizza	$\begin{bmatrix} \text{CRUST} & \{\text{thick, thin, stuffed}\} \\ \text{TOPPINGS} & \textit{topping-set} \end{bmatrix}$	*pizza-thing*
topping-set	$\begin{bmatrix} \text{OLIVES} & \{+, -\} \\ \text{ONIONS} & \{+, -\} \\ \text{MUSHROOMS} & \{+, -\} \end{bmatrix}$	*pizza-thing*
vegetarian		*topping-set*
non-vegetarian	$\begin{bmatrix} \text{SAUSAGE} & \{+, -\} \\ \text{PEPPERONI} & \{+, -\} \\ \text{HAM} & \{+, -\} \end{bmatrix}$	*topping-set*

Type Hierarchies

A type hierarchy....

- ... states what kinds of objects we claim exist (the types)
- ... organizes the objects hierarchically into classes with shared properties (the type hierarchy)
- ... states what general properties each kind of object has (the feature and feature value declarations).

Pizza Descriptions and Pizza Models

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \textit{vegetarian} & \\ \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

How many pizza models (by definition, fully resolved) satisfy this description?

Answer: 2

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \textit{vegetarian} & \\ \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

{<CRUST , thick> , <TOPPINGS , { <OLIVES , + > , <ONIONS, +> , <MUSHROOMS, –>}>}

{<CRUST , thick> , <TOPPINGS , { <OLIVES , + > , <ONIONS, +> , <MUSHROOMS, +>}>}

Pizza Descriptions and Pizza Models

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \textit{vegetarian} & \\ \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

How many pizzas-in-the-world do the pizza models correspond to?

Answer: A large, constantly-changing number.

Pizza Descriptions and Pizza Models

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \textit{vegetarian} & \\ \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

‘type’/‘token’ distinction
applies to sentences as well

Combining Constraints

$$\begin{bmatrix} pizza \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{HAM} & - \end{bmatrix} \end{bmatrix} \;\&\; \begin{bmatrix} pizza \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

Combining Constraints

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{ONIONS} & + \\ \text{HAM} & - \end{bmatrix} \end{bmatrix}$$

Combining Constraints

$$\begin{bmatrix} pizza \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{HAM} & - \end{bmatrix} \end{bmatrix} \;\&\; \begin{bmatrix} pizza \\ \text{CRUST} & \text{thin} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{ONIONS} & + \end{bmatrix} \end{bmatrix}$$

$$= \phi$$

Combining Constraints

$$\begin{bmatrix} pizza \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{HAM} & + \end{bmatrix} \end{bmatrix} \;\&\; \begin{bmatrix} pizza \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & vegetarian \end{bmatrix}$$

$$= \phi$$

Combining Constraints

$$\begin{bmatrix} \textit{pizza} \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \begin{bmatrix} \text{OLIVES} & + \\ \text{HAM} & - \end{bmatrix} \end{bmatrix} \;\&\; \begin{bmatrix} \textit{pizza} \\ \text{CRUST} & \text{thick} \\ \text{TOPPINGS} & \textit{vegetarian} \end{bmatrix}$$

$$= \phi$$

A New Theory of Pizzas

$$pizza: \begin{bmatrix} \text{CRUST} & \{\text{thick , thin , stuffed}\} \\ \text{ONE-HALF} & \textit{topping-set} \\ \text{OTHER-HALF} & \textit{topping-set} \end{bmatrix}$$

Combining Constraints

$$\begin{bmatrix} pizza \\ \text{ONE-HALF} & \begin{bmatrix} \text{ONIONS} & + \\ \text{OLIVES} & - \end{bmatrix} \end{bmatrix} \;\&\; \begin{bmatrix} pizza \\ \text{OTHER-HALF} & \begin{bmatrix} \text{ONIONS} & - \\ \text{OLIVES} & + \end{bmatrix} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} pizza \\ \text{ONE-HALF} & \begin{bmatrix} \text{ONIONS} & + \\ \text{OLIVES} & - \end{bmatrix} \\ \text{OTHER-HALF} & \begin{bmatrix} \text{ONIONS} & - \\ \text{OLIVES} & + \end{bmatrix} \end{bmatrix}$$

Identity Constraints (tags)

$$\begin{bmatrix} \textit{pizza} & \\ \text{CRUST} & \text{thin} \\ \text{ONE-HALF} & \begin{bmatrix} \text{OLIVES} & \boxed{1} \\ \text{ONIONS} & \boxed{2} \end{bmatrix} \\ \text{OTHER-HALF} & \begin{bmatrix} \text{OLIVES} & \boxed{1} \\ \text{ONIONS} & \boxed{2} \end{bmatrix} \end{bmatrix}$$

Combining Constraints

$$\begin{bmatrix} pizza \\ \text{ONE-HALF} & \boxed{1}\begin{bmatrix}\text{ONIONS} & + \\ \text{OLIVES} & -\end{bmatrix} \\ \text{OTHER-HALF} & \boxed{1} \end{bmatrix} \;\&\; \begin{bmatrix} pizza \\ \text{OTHER-HALF} & \begin{bmatrix}\text{MUSHROOMS} & - \\ \text{OLIVES} & -\end{bmatrix} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} pizza \\ & \begin{bmatrix}\text{ONIONS} & + \\ \text{OLIVES} & - \\ \text{MUSHROOMS} & -\end{bmatrix} \\ \text{ONE-HALF} & \boxed{1} \\ \text{OTHER-HALF} & \boxed{1} \end{bmatrix}$$

Note

$$\begin{bmatrix} \textit{pizza} & \\ \text{ONE-HALF} & \boxed{1}\begin{bmatrix} \text{ONIONS} & + \\ \text{OLIVES} & - \\ \text{MUSHROOMS} & - \end{bmatrix} \\ \text{OTHER-HALF} & \boxed{1} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \textit{pizza} & \\ \text{ONE-HALF} & \boxed{1} \\ \text{OTHER-HALF} & \boxed{1}\begin{bmatrix} \text{ONIONS} & + \\ \text{OLIVES} & - \\ \text{MUSHROOMS} & - \end{bmatrix} \end{bmatrix}$$

Combining Constraints

$$\begin{bmatrix} pizza & \\ \text{ONE-HALF} & \boxed{1}\begin{bmatrix} \text{ONIONS} & + \\ \text{OLIVES} & + \end{bmatrix} \\ \text{OTHER-HALF} & \boxed{1}\ vegetarian \end{bmatrix} \;\&\; \begin{bmatrix} pizza & \\ \text{ONE-HALF} & \begin{bmatrix} \text{SAUSAGE} & + \\ \text{HAM} & - \end{bmatrix} \end{bmatrix}$$

$$= \phi$$

How badly do you want pizza now?

0

PIZZA! NOW!

0

I can wait until dinner

0

Meh

0

I've been eating pizza this whole time

0

Why combine constraints?

- The pizza example illustrates how unification can be used to combine information from different sources.
- In our grammar, information will come from lexical entries, grammar rules, and general principles.

Linguistic Application of Feature Structures: Making the Mnemonic Meaningful

What do these CFG categories have in common?

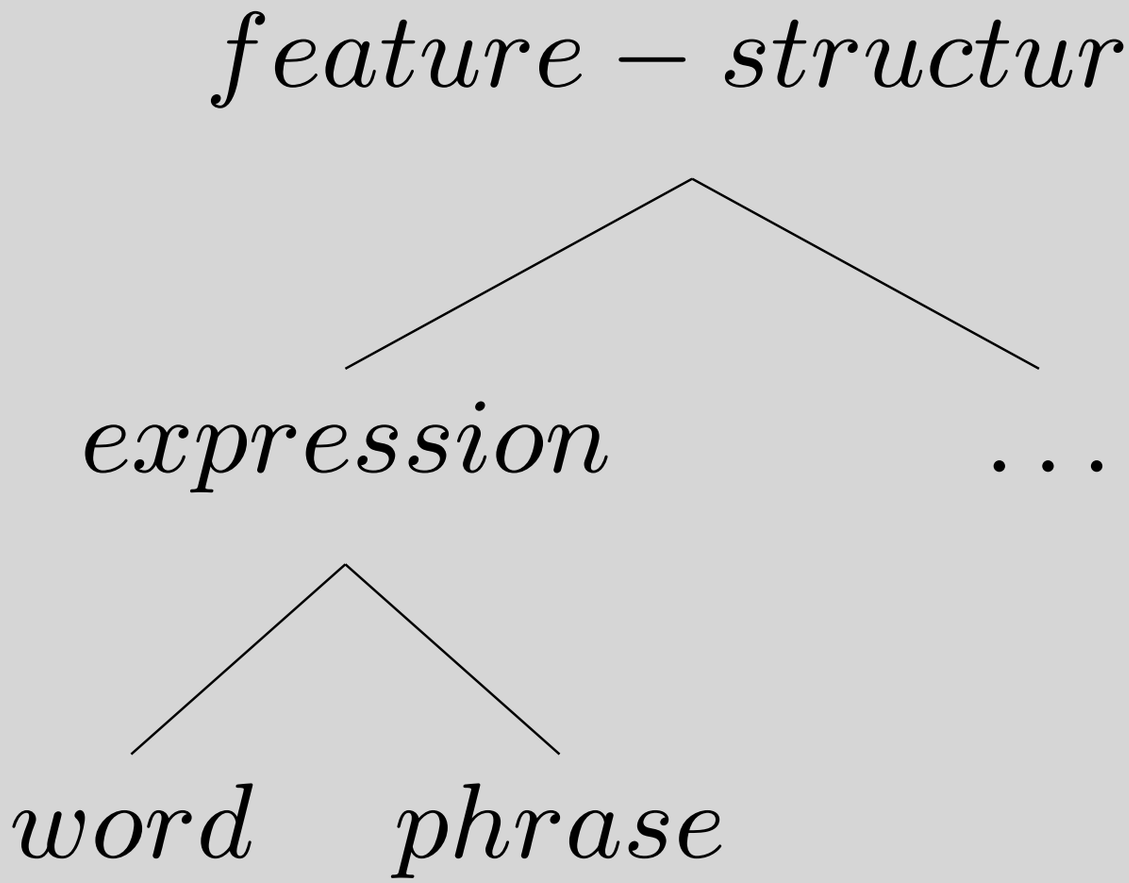
NP & VP: are both phrases

N & V: are both words

NP & N: are both ‘nouny’

VP & V: are both ‘verby’

The Beginnings of Our Type Hierarchy

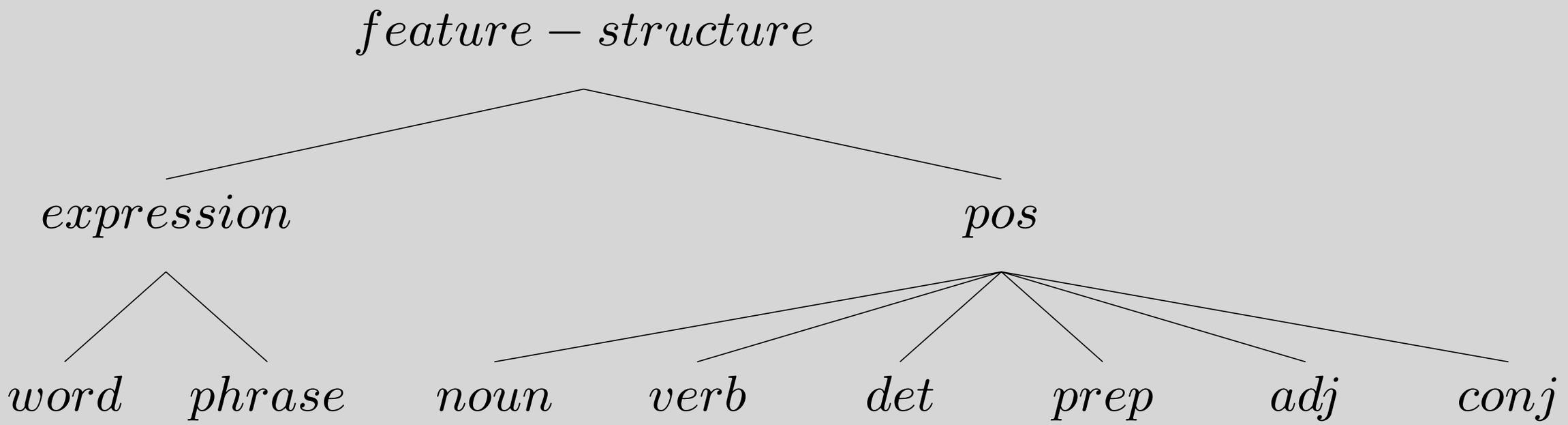


A Feature for Part of Speech

$$\text{NP} = \begin{bmatrix} \textit{phrase} & \\ \text{HEAD} & \textit{noun} \end{bmatrix}$$

$$\left\langle \text{bird} \ , \ \begin{bmatrix} \textit{word} & \\ \text{HEAD} & \textit{noun} \end{bmatrix} \right\rangle$$

Type Hierarchy for Parts of Speech I



Having 'expression' and 'pos' at the same level of that hierarchy

0

Bugs me

0

Didn’t really stand out to me at all

0

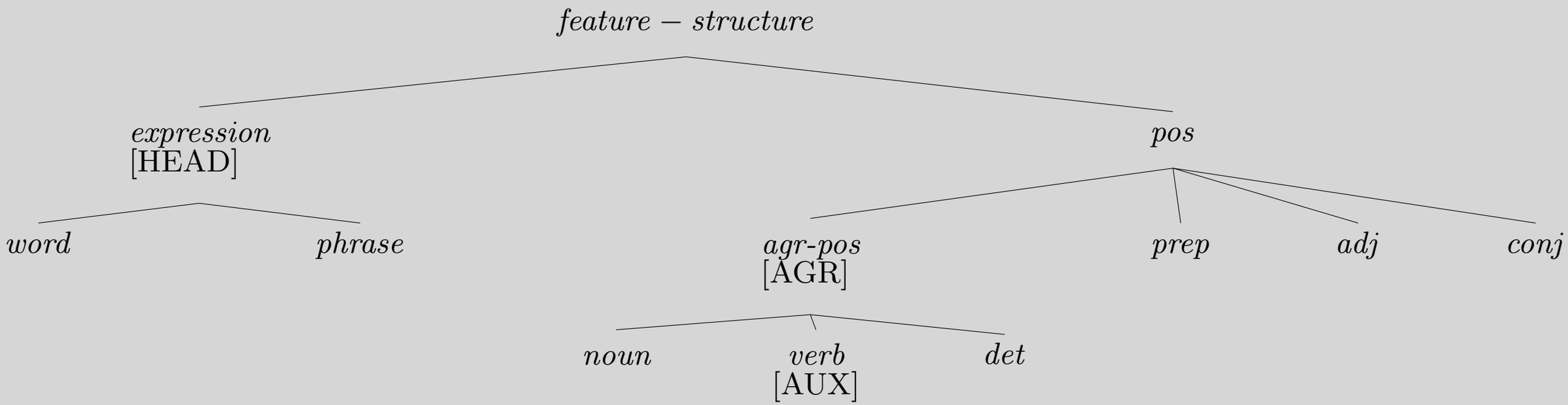
Makes sense

0

None of the above

0

Type Hierarchy for Parts of Speech II



A Feature for Valence

$$\text{IV} = \begin{bmatrix} \textit{word} & \\ \text{HEAD} & \textit{verb} \\ \text{VAL} & [\text{COMPS} \quad \text{itr}] \end{bmatrix}$$

$$\text{TV} = \begin{bmatrix} \textit{word} & \\ \text{HEAD} & \textit{verb} \\ \text{VAL} & [\text{COMPS} \quad \text{str}] \end{bmatrix}$$

$$\text{DTV} = \begin{bmatrix} \textit{word} & \\ \text{HEAD} & \textit{verb} \\ \text{VAL} & [\text{COMPS} \quad \text{dtr}] \end{bmatrix}$$

Underspecification

$$\mathrm{V} = \begin{bmatrix} word & \\ \mathrm{HEAD} & verb \end{bmatrix}$$

$$\mathrm{VP} = \begin{bmatrix} phrase & \\ \mathrm{HEAD} & verb \end{bmatrix}$$

$$[\mathrm{HEAD} \quad verb\ ]$$

Another Valence Feature

$$\text{NP} = \begin{bmatrix} \textit{phrase} & \\ \text{HEAD} & \textit{noun} \\ \text{VAL} & \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & + \end{bmatrix} \end{bmatrix}$$

$$\text{NOM} = \begin{bmatrix} \textit{phrase} & \\ \text{HEAD} & \textit{noun} \\ \text{VAL} & \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix}$$

SPR and Verbs

$$\mathrm{S} = \begin{bmatrix} \textit{phrase} & \\ \mathrm{HEAD} & \textit{verb} \\ \mathrm{VAL} & \begin{bmatrix} \mathrm{COMPS} & \mathrm{itr} \\ \mathrm{SPR} & + \end{bmatrix} \end{bmatrix}$$

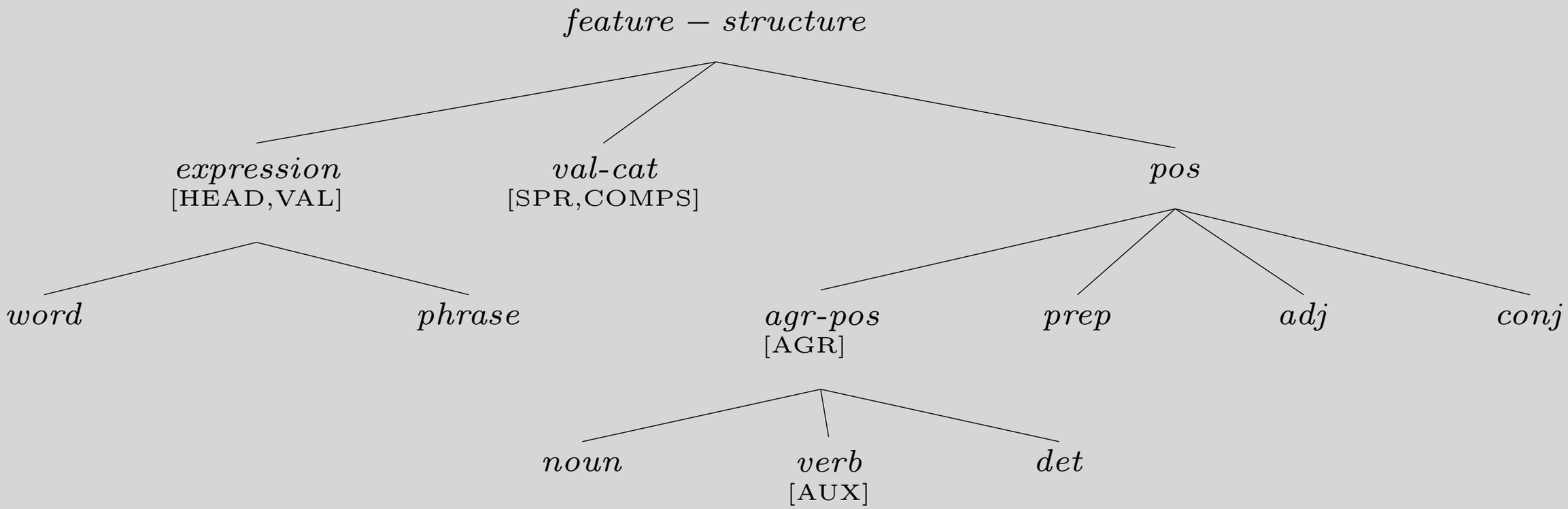
$$\mathrm{VP} = \begin{bmatrix} \textit{phrase} & \\ \mathrm{HEAD} & \textit{verb} \\ \mathrm{VAL} & \begin{bmatrix} \mathrm{COMPS} & \mathrm{itr} \\ \mathrm{SPR} & - \end{bmatrix} \end{bmatrix}$$

S and NP

$$\begin{bmatrix} \text{VAL} & \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & + \end{bmatrix} \end{bmatrix}$$

- We created a monster
- our creation of a monster

Type Hierarchy So Far



Reformulating the Grammar Rules I
Which Ch 2 rules do these correspond to?

Head-Complement Rule 1:

$$\begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \rightarrow \mathbf{H} \begin{bmatrix} \textit{word} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix}$$

Head Complement Rule 2:

$$\begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \rightarrow \mathbf{H} \begin{bmatrix} \textit{word} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{str} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \text{ NP}$$

Head Complement Rule 3:

$$\begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \rightarrow \mathbf{H} \begin{bmatrix} \textit{word} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{dtr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \text{ NP NP}$$

Reformulating the Grammar Rules II

Head-Specifier Rule 1:

$$\begin{bmatrix} phrase \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & + \end{bmatrix} \end{bmatrix} \rightarrow \overset{\text{NP}}{\begin{bmatrix} \text{HEAD} \quad \begin{bmatrix} \text{AGR} \quad \boxed{1} \end{bmatrix} \end{bmatrix}} \quad \mathbf{H} \begin{bmatrix} phrase \\ \text{HEAD} \quad \begin{bmatrix} verb \\ \text{AGR} \quad \boxed{1} \end{bmatrix} \\ \text{VAL} \quad \begin{bmatrix} \text{SPR} & - \end{bmatrix} \end{bmatrix}$$

Head-Specifier Rule 2:

$$\begin{bmatrix} phrase \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & + \end{bmatrix} \end{bmatrix} \rightarrow \text{D} \quad \mathbf{H} \begin{bmatrix} phrase \\ \text{HEAD} & noun \\ \text{VAL} & \begin{bmatrix} \text{SPR} & - \end{bmatrix} \end{bmatrix}$$

Reformulating the Grammar Rules III

Non-Branching NP Rule

$$\begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & + \end{bmatrix} \end{bmatrix} \rightarrow \mathbf{H} \begin{bmatrix} \textit{word} \\ \text{HEAD} \quad \textit{noun} \\ \text{VAL} \quad \begin{bmatrix} \text{SPR} & + \end{bmatrix} \end{bmatrix}$$

Head-Modifier Rule

$$\begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \rightarrow \mathbf{H} \begin{bmatrix} \textit{phrase} \\ \text{VAL} \quad \begin{bmatrix} \text{SPR} & - \end{bmatrix} \end{bmatrix} \text{PP}$$

Coordination Rule

$$\boxed{1} \rightarrow \boxed{1}^{+} \begin{bmatrix} \textit{word} \\ \text{HEAD} \quad \textit{conj} \end{bmatrix} \boxed{1}$$

Advantages of the New Formulation

- Subject-verb agreement is stipulated only once (where?)
- Common properties of verbs with different valences are expressed by common features (for example?)
- Parallelisms across phrase types are captured (for example?)

Disadvantages of the New Formulation

- We still have three head complement rules
- We still have two head specifier rules
- We only deal with three verb valences (Which ones? What are some others?)
- The non-branching rule doesn't really do any empirical work
- Others?

Heads

- Intuitive idea: A phrase typically contains a word that determines its most essential properties, including
  - where it occurs in larger phrases, and
  - what its internal structure is
- This is called the head
- The term “head” is used both for the head word in a phrase and for all the intermediate phrases containing that word
- NB: Not all phrases have heads

Formalizing the Notion of Head

- Expressions have a feature HEAD
- HEAD's values are of type *pos*
- For HEAD values of type *agr-pos*, HEAD's value also includes the feature AGR
- Well-formed trees are subject to the Head Feature Principle

The Head Feature Principle

- Intuitive idea: Key properties of phrases are shared with their heads
- The HFP: In any headed phrase, the HEAD value of the mother and the head daughter must be identical.
- Sometimes described in terms of properties "percolating up" or "filtering down", but this is just metaphorical talk

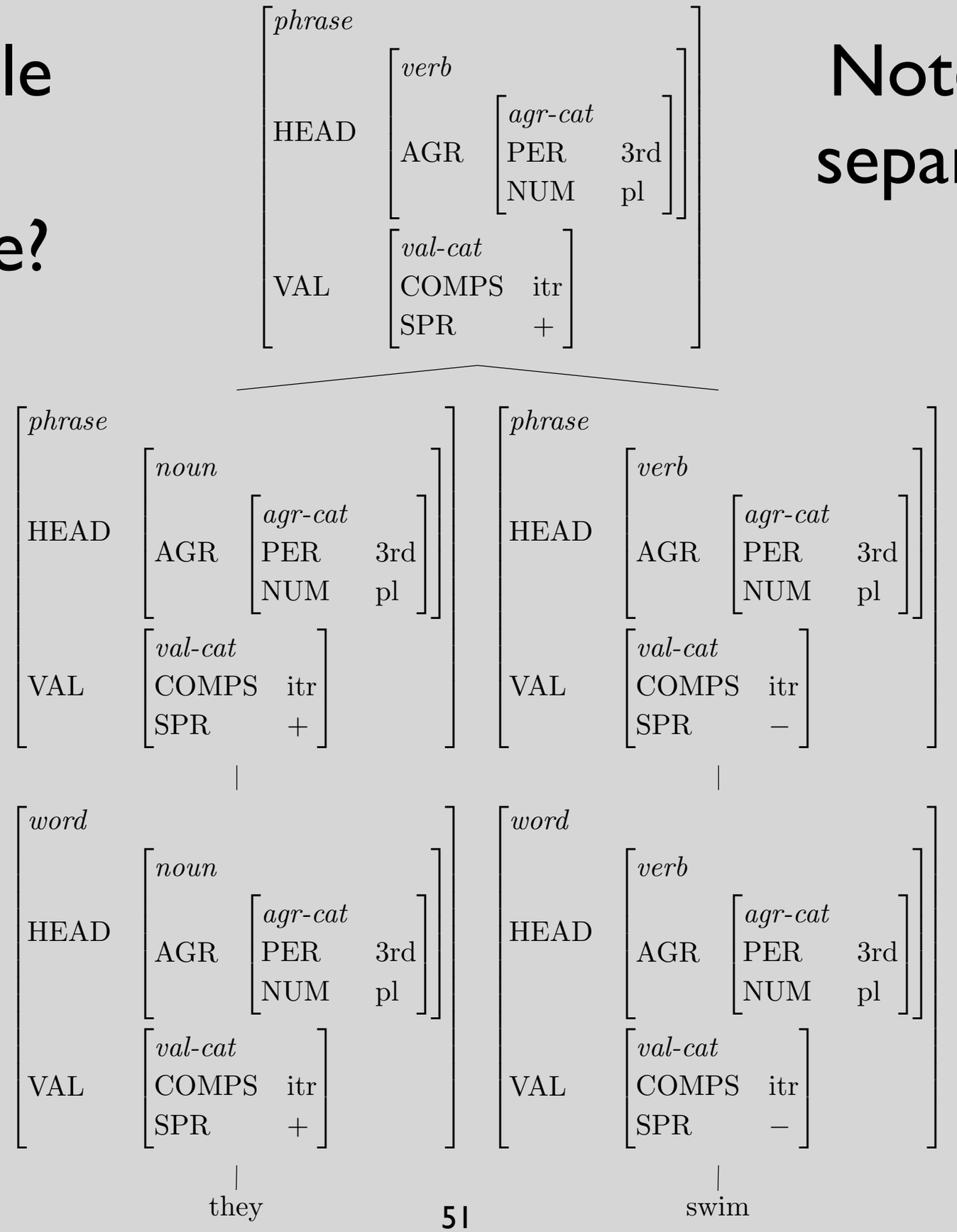
A Tree is Well-Formed if …

- It and each subtree are licensed by a grammar rule or lexical entry
- All general principles (like the HFP) are satisfied.
- NB: Trees are part of our model of the language, so all their features have values (even though we will often be lazy and leave out the values irrelevant to our current point).

Question:

Do phrases that are not headed have HEAD features?

Which rule licenses each node?



Note the three separate uses of DAGs

A Question:

Since the lexical entry for swim below has only [NUM pl] as the value of AGR, how did the tree on the previous slide get [PER 3rd] in the AGR of swim?

$$\left\langle \text{swim} \ , \begin{bmatrix} word & & \\ \text{HEAD} & \begin{bmatrix} verb & \\ \text{AGR} & \begin{bmatrix} \text{NUM} & \text{pl} \end{bmatrix} \end{bmatrix} \\ \text{VAL} & \begin{bmatrix} \text{COMPS} & \text{itr} \\ \text{SPR} & - \end{bmatrix} \end{bmatrix} \right\rangle$$

Overview

- Review: problems with CFG
- Modeling
- Feature structures, unification (pizza)
- Features for linguistic description
- Reformulate grammar rules
- Notion of head/headedness
- Licensing of trees
- Next time: Valence and agreement

RQs: Unification

- Why aren't the two constraints in Ex 1, item C compatible?

C. $\begin{bmatrix} \text{PRESIDENT} & \boxed{1} \\ \text{FOUNDERS} & \langle\, \boxed{1} \,\rangle \end{bmatrix} \;\&\; \begin{bmatrix} \textit{individual} & \\ \text{NAME} & \text{John Hennessy} \end{bmatrix}$

RQs: word v. phrase

- Are "words" entities that come from the lexicon and are the leaf nodes of the tree, while "phrases" are containers that can "house" words as leaf nodes within their trees?

RQs: Valence

- Why is that one called "strict transitive" (str)? Do we handle optionally transitive verbs as well?
- Why call nouns [COMPS itr]?
- Since valence originated as a way to capture the combinatoric behavior of verbs, what is the theoretical motivation for extending it to categories like nouns or full phrases? Does this generalization reflect a deeper linguistic principle about all heads, or does it blur distinctions that are better kept separate?

RQs: Complexity

- I understand the concept of valuence as introduced by this chapter, but how does this relate to the general principle of parsimony we want to establish in our analysis -- the valence diagrams seem to get complicated fast as stacks within stacks of valences are shown, how do we use this new notation in the "simplest" way necessary for a thorough analysis?

RQs: Complexity

- How does this tackle the problem of not wanting our grammar to be overly specific/ complex? It seems that there is still just as much complexity that children would have to internalize, but we just reformatted it so that it's easier for us to read.

RQs: OOP

- Is HPSG related to OOP?

RQs: Theory building

- (3.2, p52) In introducing the motivation for types, the text notes: "As we develop our theory of grammatical types, we will in fact be developing a theory of what kinds of linguistic entities there are, and what kinds of generalizations hold of those entities”

RQs: Theory building

- If building a type hierarchy is a way of saying what kinds of linguistic entities exist, is the point mainly to capture language-specific patterns (like the structure of English verbs) so we don't have to spell them out in the lexicon? Or is the hierarchy supposed to represent more universal constraints, like the basic properties of parts of speech that might hold across all languages?