Ling 566
Nov 1, 2016

Lexical Rules

Overview

- How lexical rules fit in
- Three types of lexical rules, constraints
- Example: Plural noun lexical rule
- Advice on writing lexical rules
- Constant lexemes
- ARG-ST & ARP
- The feature FORM

Lexical Types & Lexical Rules

- Lexemes capture the similarities among *run*, *runs*, *running*, and *ran*
- The lexical type hierarchy captures the similarities among *run*, *sleep*, and *laugh*, among those and other verbs like *devour* and *hand*, and among those and other words like *book*.
- Lexical rules capture the similarities among *runs*, *sleeps*, *devours*, *hands*, ...

Parsimony & Plausibility

- Lexical rules capture **productive** generalizations.
- There may be some ‘precompiling’ going on as well.

Three Kinds of Lexical Rules

- Inflectional: *lexeme* to *word*

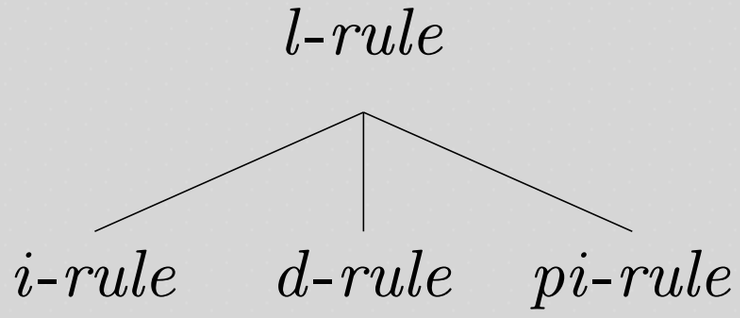
Examples?

- Derivational: *lexeme* to *lexeme*

Examples?

- Post-Inflectional: *word* to *word*
  (Chapters 11, 13, 14)

Three Subtypes of *l-rule*



$$\textit{l-rule}: \begin{bmatrix} \text{INPUT} & \textit{l-sequence}\left\langle \text{X} , [\ \text{SEM} \quad / \boxed{2}\ ] \right\rangle \\ \text{OUTPUT} & \textit{l-sequence}\left\langle \text{Y} , [\ \text{SEM} \quad / \boxed{2}\ ] \right\rangle \end{bmatrix}$$

$$\textit{i-rule}: \begin{bmatrix} \text{INPUT} & \left\langle \text{X} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad \boxed{3} \\ \text{ARG-ST} \quad \boxed{\text{A}} \end{bmatrix} \right\rangle \\ \text{OUTPUT} & \left\langle \text{Y} , \begin{bmatrix} \textit{word} \\ \text{SYN} \quad \boxed{3} \\ \text{ARG-ST} \quad \boxed{\text{A}} \end{bmatrix} \right\rangle \end{bmatrix}$$

$$\textit{d-rule}: \begin{bmatrix} \text{INPUT} & \left\langle \text{X} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad / \boxed{3} \end{bmatrix} \right\rangle \\ \text{OUTPUT} & \left\langle \text{Y} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad / \boxed{3} \end{bmatrix} \right\rangle \end{bmatrix}$$

Plural Noun LR

$$\left[\begin{array}{l} \textit{i-rule} \\ \text{INPUT} \quad \left\langle \boxed{1}\ ,\ \textit{cntn-lxm} \right\rangle \\ \text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1})\ ,\ \left[\begin{array}{l} \textit{word} \\ \text{SYN}\ \left[\text{HEAD}\ \left[\text{AGR}\ \left[\text{NUM}\ \ \text{pl}\right]\right]\right] \end{array}\right] \right\rangle \end{array}\right]$$

Plural Noun LR with Inherited Constraints

$$\left[\textit{i-rule}\right.$$

$$\text{INPUT} \quad \left\langle \boxed{1}, \left[\textit{cntn-lxm}\right] \right\rangle$$

$$\left.\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\textit{word} \quad \left[\text{HEAD} \quad [\text{AGR} \quad [\text{NUM} \quad \text{pl}]]\right]\right]\right\rangle\right]$$

Plural Noun LR with Inherited Constraints

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}, \left[\begin{array}{l} \textit{cntn-lxm} \\ \\ \text{SEM} \quad \boxed{2} \\ \\ \end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\begin{array}{l} \textit{word} \\ \\ \left[\text{HEAD} \quad [\text{AGR} \quad [\text{NUM} \quad \text{pl}]]\right] \\ \\ \text{SEM} \quad \boxed{2} \\ \\ \end{array}\right] \right\rangle
\end{array}\right]
$$

Plural Noun LR with Inherited Constraints

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}, \left[\begin{array}{ll} \textit{cntn-lxm} & \\ & \\ & \\ \text{SEM} & \boxed{2} \\ \text{ARG-ST} & \boxed{\text{B}} \oplus \boxed{\text{C}} \end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\begin{array}{ll} \textit{word} & \\ & \\ & \left[\text{HEAD} \quad [\text{AGR} \quad [\text{NUM} \quad \text{pl}]]\right] \\ & \\ \text{SEM} & \boxed{2} \\ \text{ARG-ST} & \boxed{\text{B}} \oplus \boxed{\text{C}} \end{array}\right] \right\rangle
\end{array}\right]
$$

Plural Noun LR with Inherited Constraints

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}, \left[\begin{array}{ll}
\textit{cntn-lxm} & \\
 & \\
 & \\
\text{SYN} & \boxed{3} \\
 & \\
 & \\
\text{SEM} & \boxed{2} \\
\text{ARG-ST} & \boxed{B} \oplus \boxed{C}
\end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\begin{array}{ll}
\textit{word} & \\
 & \\
\text{SYN} & \boxed{3}\left[\text{HEAD} \quad [\text{AGR} \quad [\text{NUM} \quad \text{pl}]]\right] \\
 & \\
\text{SEM} & \boxed{2} \\
\text{ARG-ST} & \boxed{B} \oplus \boxed{C}
\end{array}\right] \right\rangle
\end{array}\right]
$$

Plural Noun LR with Inherited Constraints

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}, \left[\begin{array}{ll}
\textit{cntn-lxm} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{ll}
\text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \boxed{4}[\text{PER} \quad \text{3rd}] \end{array}\right] \\
\\
\text{VAL} & \left[\text{SPR} \left\langle \left[\begin{array}{ll} & \text{DP} \\ \text{COUNT} & + \\ \text{AGR} & \boxed{4} \end{array}\right] \right\rangle\right]
\end{array}\right] \\
\\
\text{SEM} & \boxed{2}[\text{MODE} \ / \ \text{ref}] \\
\text{ARG-ST} & \boxed{B} \oplus \boxed{C}
\end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\begin{array}{ll}
\textit{word} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{llll} \text{HEAD} & [\text{AGR} & [\text{NUM} & \text{pl}]] \end{array}\right] \\
\\
\text{SEM} & \boxed{2} \\
\text{ARG-ST} & \boxed{B} \oplus \boxed{C}
\end{array}\right] \right\rangle
\end{array}\right]
$$

Plural Noun LR with Inherited Constraints

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}, \left[\begin{array}{ll}
\textit{cntn-lxm} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{ll}
\text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \boxed{4}[\text{PER} \quad \text{3rd}] \end{array}\right] \\
\\
\text{VAL} & \left[\text{SPR} \left\langle \left[\begin{array}{ll} & \text{DP} \\ \text{COUNT} & + \\ \text{AGR} & \boxed{4} \end{array}\right] \right\rangle\right]
\end{array}\right] \\
\\
\text{SEM} & \boxed{2}[\text{MODE} \ / \ \text{ref}] \\
\text{ARG-ST} & \boxed{\text{B}} \oplus \boxed{\text{C}}
\end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}), \left[\begin{array}{ll}
\textit{word} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{ll}
\text{HEAD} & [\text{AGR} \quad [\text{NUM} \quad \text{pl}]] \\
\\
\text{VAL} & \left[\begin{array}{ll} \text{SPR} & \boxed{\text{B}} \\ \text{COMPS} & \boxed{\text{C}} \end{array}\right]
\end{array}\right] \\
\\
\text{SEM} & \boxed{2} \\
\text{ARG-ST} & \boxed{\text{B}} \oplus \boxed{\text{C}}
\end{array}\right] \right\rangle
\end{array}\right]
$$

Practicalities - Applying Lexical Rules

- INPUT is a family of lexical sequences.
- OUTPUT is another family of lexical sequences.
  - ...usually a smaller family
  - ...usually a disjoint one
- The only differences between the families are those stipulated in the rule (or the rule's type).
- Similarities are handled by the constraints on *l-rule* and its subtypes.
- If we've written the LRs correctly, nothing is left underconstrained.

Example: Lexical Entry for *cat*

$$\left\langle \text{cat} \;,\; \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SEM} \quad \left[\begin{array}{ll} \text{INDEX} & k \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array}\right] \right\rangle \end{array}\right] \end{array}\right] \right\rangle$$

Example: *cat*, with inheritance

$$\left\langle \text{cat} , \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SYN} \left[\begin{array}{l} \text{VAL} \left[\begin{array}{l} \\ \\ \end{array} \right] \end{array} \right] \\ \text{SEM} \left[\begin{array}{ll} \text{INDEX} & k \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array} \right] \right\rangle \end{array} \right] \end{array} \right] \right\rangle$$

Example: *cat*, with inheritance

$$\left\langle \text{cat} , \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SYN} \quad \left[\begin{array}{l} \text{VAL} \quad \left[\begin{array}{l} \text{SPR} \quad \langle \left[\text{COUNT} \quad + \right] \rangle \end{array} \right] \end{array} \right] \\ \text{SEM} \quad \left[\begin{array}{l} \text{INDEX} \quad k \\ \text{RESTR} \quad \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array} \right] \right\rangle \end{array} \right] \end{array} \right] \right\rangle$$

Example: *cat*, with inheritance

$$\left\langle \text{cat} , \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SYN} \left[\begin{array}{ll} \text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & [\ \text{PER} \quad \text{3rd}\ ] \end{array}\right] \\ \text{VAL} & \left[\begin{array}{ll} \text{SPR} & \left\langle \left[\begin{array}{ll} \text{DP} & \\ \text{COUNT} & + \end{array}\right] \right\rangle \\ & \end{array}\right] \end{array}\right] \\ \text{SEM} \left[\begin{array}{ll} \text{MODE} & \text{ref} \\ \text{INDEX} & k \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array}\right] \right\rangle \end{array}\right] \\ \text{ARG-ST} \quad \langle \text{X} \rangle \end{array}\right] \right\rangle$$

Example: *cat*, with inheritance

$$\left\langle \text{cat} \;,\; \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SYN} \left[\begin{array}{ll} \text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \boxed{7}\,[\ \text{PER} \quad \text{3rd}\ ] \end{array}\right] \\ \text{VAL} & \left[\begin{array}{ll} \text{SPR} & \left\langle \left[\begin{array}{ll} \text{DP} & \\ \text{COUNT} & + \\ \text{AGR} & \boxed{7} \end{array}\right] \right\rangle \\ & \end{array}\right] \end{array}\right] \\ \text{SEM} \left[\begin{array}{ll} \text{MODE} & \text{ref} \\ \text{INDEX} & k \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array}\right] \right\rangle \end{array}\right] \\ \text{ARG-ST} \quad \langle \text{X} \rangle \end{array}\right] \right\rangle$$

Plural Noun LR

$$\left[\begin{array}{l} \textit{i-rule} \\ \text{INPUT} \quad \left\langle \boxed{1}\ ,\ \textit{cntn-lxm} \right\rangle \\ \text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1})\ ,\ \left[\begin{array}{l} \textit{word} \\ \text{SYN}\ \left[\text{HEAD}\ \left[\text{AGR}\ \left[\text{NUM}\ \ \text{pl}\right]\right]\right] \end{array}\right] \right\rangle \end{array}\right]$$

Licensing *cats*

$$
\left[\begin{array}{l}
\textit{i-rule} \\
\\
\text{INPUT} \quad \left\langle \boxed{1}\text{cat} \;,\; \left[\begin{array}{ll}
\textit{cntn-lxm} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{ll}
\text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \boxed{7}[\ \text{PER} \quad \text{3rd}\ ] \end{array}\right] \\
\\
\text{VAL} & \left[\begin{array}{ll} \text{SPR} & \left\langle \left[\begin{array}{ll} & \text{DP} \\ \text{COUNT} & + \\ \text{AGR} & \boxed{7} \end{array}\right] \right\rangle \end{array}\right]
\end{array}\right] \\
\\
\text{SEM} & \boxed{2}\left[\begin{array}{ll} \text{MODE} & \text{ref} \\ \text{INDEX} & k \\ \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array}\right] \right\rangle \end{array}\right] \\
\\
\text{ARG-ST} & \boxed{B}\langle\ \text{X}\ \rangle \oplus \boxed{C}\langle\ \rangle
\end{array}\right] \right\rangle \\
\\
\text{OUTPUT} \quad \left\langle \text{F}_{NPL}(\boxed{1}) \;,\; \left[\begin{array}{ll}
\textit{word} & \\
\\
\text{SYN} & \boxed{3}\left[\begin{array}{lll} \text{HEAD} & [\text{AGR} & [\text{NUM} \quad \text{pl}]] \\ \\ \text{VAL} & \left[\begin{array}{ll} \text{SPR} & \boxed{B} \\ \text{COMPS} & \boxed{C} \end{array}\right] & \end{array}\right] \\
\\
\text{SEM} & \boxed{2} \\
\text{ARG-ST} & \boxed{B} \oplus \boxed{C}
\end{array}\right] \right\rangle
\end{array}\right]
$$

cats: The Lexical Sequence

$$\left\langle \text{cats} \;,\; \left[\begin{array}{ll} \textit{word} & \\ \text{SYN} & \left[\begin{array}{ll} \text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \textit{3pl} \end{array}\right] \\ \text{VAL} & \left[\begin{array}{ll} \text{SPR} & \boxed{\text{B}}\left\langle \left[\begin{array}{ll} & \text{DP} \\ \text{COUNT} & + \\ \text{AGR} & \boxed{7} \end{array}\right] \right\rangle \\ \text{COMPS} & \langle\,\rangle \end{array}\right] \end{array}\right] \\ \text{SEM} & \left[\begin{array}{ll} \text{MODE} & \text{ref} \\ \text{INDEX} & k \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \text{cat} \\ \text{INST} & k \end{array}\right] \right\rangle \end{array}\right] \\ \text{ARG-ST} & \boxed{\text{B}} \end{array}\right] \right\rangle$$

Practicalities -- Writing Lexical Rules

- Determine the type of the LR.
- Determine the class of possible inputs.
- Determine what should change.
  - If INPUT and OUTPUT values are identified (by default or otherwise) and only OUTPUT value is mentioned, then...
    information is added.

    (Lexical sequences incompatible with that value are not possible inputs)
  - If INPUT and OUTPUT values are identified by default, but different values are given on the INPUT and OUTPUT of the rule, then...
    information is changed.
  - If INPUT and OUTPUT values are identified by an inviolable constraint, but different values are given on the INPUT and OUTPUT of the rule, then...
    there is no well-formed output

Constant lexemes

- What kinds of words are constant lexemes in our grammar?
- Why do we need a rule for these words?
- What would be an alternative analysis?

Constant Lexeme LR

$$\left[\begin{array}{ll} \textit{i-rule} & \\ \text{INPUT} & \langle\ \boxed{1}\ ,\ \textit{const-lxm}\ \rangle \\ \text{OUTPUT} & \left[\text{FIRST}\quad \boxed{1}\right] \end{array}\right]$$

- What keeps this from applying to, say, verb lexemes?
- Why is this an *i-rule*?

ARG-ST & ARP

- Given the ARP, what do we need to specify about the valence properties of words?
- Why isn't the ARP a constraint on the type *lexeme*?

The Feature FORM

- Different inflected forms of verbs show up in different syntactic environments. Examples?
- These different forms are syntactically distinguished by the feature FORM, as assigned by lexical rules.
- FORM is also useful in our analyses of coordination and PP selection.

How do we rule these out?

- *Kim eat pizza.
- *Kim seems to eats pizza.
- *Dana helped Leslie [pack and moved].
- *Kim relies for Sandy.
- *Dana walked and Kim.

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- Advice on writing lexical rules
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- The feature FORM
- Reading Questions

Reading Questions

- Where do the morphological functions live?
- To what extent is this a complete theory of morphology? Is this still compatible with multiple theories of morphology?

Reading Questions

- How do the INPUT/OUTPUT features work? Do they fit into the tree somehow?
- If the INPUT and OUTPUT values are the same, then why do we have a Singular Noun Lexical Rule?

Reading Questions

- p250: "For most nouns and verbs, we will assume that there is only one lexical entry."
- p259: "... lexical rules do not change or otherwise operate on lexical sequences. Rather they relate lexical sequences to other lexical sequences."
- Which is it?
- Why is this distinction important? (p259)

Reading Questions

- Where do fully-specified lexemes ever show up?
- How are lexical rules different from the transformations in transformational grammars?
- With derivational rules giving us 'driver' etc, do we still need any lexical entries?
- How do the lexical rules actually streamline anything?

Reading Questions

- What's up with (74), "a lexical sequence that doesn't give rise to any words"?

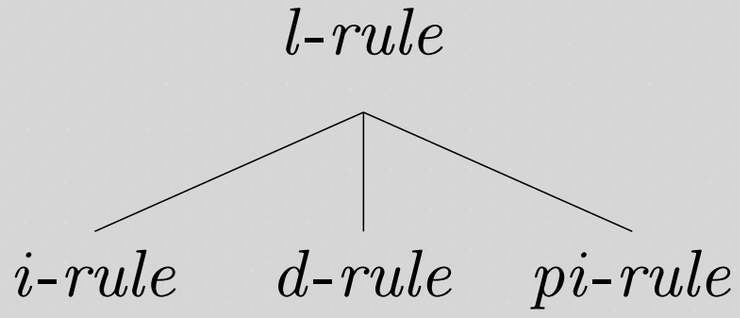
$$\left\langle \text{dog} , \left[\begin{array}{l} \textit{cntn-lxm} \\ \text{SYN} \left[\begin{array}{ll} \text{HEAD} & \left[\begin{array}{ll} \textit{noun} & \\ \text{AGR} & \boxed{1}\left[\text{PER 3rd}\right] \end{array}\right] \\ \text{VAL} & \left[\begin{array}{ll} \text{SPR} & \langle\ \text{NP}[\text{AGR}\ \boxed{1}]\ \rangle \\ \text{COMPS} & \langle\ \text{NP, NP, VP, NP}\ \rangle \end{array}\right] \end{array}\right] \\ \text{SEM} \left[\begin{array}{ll} \text{MODE} & \text{ref} \\ \text{INDEX} & i \\ \text{RESTR} & \left\langle \left[\begin{array}{ll} \text{RELN} & \textbf{dog} \\ \text{INST} & i \end{array}\right] \right\rangle \end{array}\right] \\ \text{ARG-ST} \quad \langle\ \left[\begin{array}{c}\text{DP} \\ \text{COUNT} \quad + \end{array}\right] \rangle \end{array}\right] \right\rangle$$

Reading Questions

- Why is it that in (59), the SEM features in the INPUT and OUTPUT for the category l-rule appear to defeasibly match, but in (75), SEM values are not mentioned and instead SYN values for the d-rules appear to defeasibly match?

- How do we know when to use SYN and SEM or just one or the other? Some rules have only SYN features listed and others only SEM. How do we know which one to use? Does it depend on the rule?

Three Subtypes of *l-rule*



$$\textit{l-rule}: \begin{bmatrix} \text{INPUT} & \textit{l-sequence}\left\langle \text{X} , [\ \text{SEM} \quad / \boxed{2}\ ] \right\rangle \\ \text{OUTPUT} & \textit{l-sequence}\left\langle \text{Y} , [\ \text{SEM} \quad / \boxed{2}\ ] \right\rangle \end{bmatrix}$$

$$\textit{i-rule}: \begin{bmatrix} \text{INPUT} & \left\langle \text{X} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad \boxed{3} \\ \text{ARG-ST} \quad \boxed{\text{A}} \end{bmatrix} \right\rangle \\ \text{OUTPUT} & \left\langle \text{Y} , \begin{bmatrix} \textit{word} \\ \text{SYN} \quad \boxed{3} \\ \text{ARG-ST} \quad \boxed{\text{A}} \end{bmatrix} \right\rangle \end{bmatrix}$$

$$\textit{d-rule}: \begin{bmatrix} \text{INPUT} & \left\langle \text{X} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad / \boxed{3} \end{bmatrix} \right\rangle \\ \text{OUTPUT} & \left\langle \text{Y} , \begin{bmatrix} \textit{lexeme} \\ \text{SYN} \quad / \boxed{3} \end{bmatrix} \right\rangle \end{bmatrix}$$

Reading Questions

- It seems like FORM is added by inflectional rules; it is present on words, but not lexemes. Is that true?
- Head features like FORM and AGR are inherently properties of words and not of lexemes, right?

Reading Questions

- Why do verbs have multiple FORM values, but not other POSs? Like how nouns only have nform, though they can have different forms as in plural vs singular.
- Can FORM only be specified in the ARG-STR or can be put it in SPR or COMPS?
- Why would finite Ss can be stand-alone sentences? Imperative sentences usually have base form instead of fin form, right?

Reading Questions

- What's the deal with RESTR in the 3sing verb LR?

$$\left[\begin{array}{ll} \textit{i-rule} & \\ \text{INPUT} & \left\langle \boxed{3}\,, \left[\begin{array}{ll} \textit{verb-lxm} & \\ \text{SEM} & [\text{RESTR} \quad \boxed{A}] \end{array}\right] \right\rangle \\ \text{OUTPUT} & \left\langle \text{F}_{\textit{3SG}}(\boxed{3})\,, \left[\begin{array}{lll} \text{SYN} & \left[\text{HEAD} & \left[\begin{array}{ll} \text{FORM} & \text{fin} \\ \text{AGR} & \textit{3sing} \end{array}\right]\right] \\ \text{SEM} & [\text{RESTR} & \boxed{A} \oplus \ldots\ ] \\ \text{ARG-ST} & \langle\ [\text{CASE} & \text{nom}]\ ,\ \ldots\ \rangle \end{array}\right] \right\rangle \end{array}\right]$$